ON A GROUP OF THE FORM 2^{10}:($U_5(2)$:2)

Ayoub B.M. Basheer[1, 2]
Jamshid Moori[3]

Department of Mathematical Sciences
North-West University (Mafikeng)
P.O.Box X2046, Mmabatho 2735
South Africa
e-mails: ayoubbasheer@gmail.com or ayoub.basheer@nwu.ac.za
jamshid.moori@nwu.ac.za

Abstract. The full automorphism group $U_5(2)$:2 of the special unitary group $U_5(2)$ has a 10-dimensional absolutely irreducible module over $GF(2)$. Hence a split extension of the form $\overline{G} = 2^{10}$:($U_5(2)$:2) does exist. In this paper we first determine the conjugacy classes of $\overline{G}$ using the coset analysis technique. The structures of the inertia factor groups were determined. These are the groups $U_5(2)$:2, 2_-^{1+6}:((3^{1+2}:8):2) and $O_5(2)$:2. We then determine the Fischer matrices and apply the Clifford-Fischer theory to compute the ordinary character table of $\overline{G}$. The Fischer matrices $\mathcal{F}_i$ of $\overline{G}$ are all $\mathbb{Z}$-valued, with sizes range between 1 and 5. The full character table of $\overline{G}$, which is 109×109 $\mathbb{C}$-valued matrix is available in the PhD Thesis [1] of the first author, which could be accessed online.

Keywords: Group extensions, unitary group, extra-special p-group, character table, inertia groups, Fischer matrices.
Mathematics Subject Classification (2010): 20C15, 20C40.

1. Introduction

Let $U = U_5(2)$ be the special unitary group consisting of 5×5 matrices over $\mathbb{F}_4$ that preserves a non-singular Hermitian form. The outer automorphism of U is 2 (see the $\mathbb{ATLAS}$ [8]) and thus the full automorphism group of U is a group of the form $U_5(2)$:2. We denote this group by G and we note that $|G| =$ 27 371 520. By the electronic Atlas of Wilson [18], we observe that G has a

[1]Corresponding author.
[2]The first author is currently a postdoctoral fellow at the North-West University, Mafikeng campus.
[3]Support of the North-West University and National Research Foundation (NRF) of South Africa are acknowledged.

10-dimensional absolutely irreducible module over $\mathbb{F}_2$, which is the 5-dimentional Hermitian $\mathbb{F}_4$-vector space involved in the definition of $\overline{G}$. Hence a split extension of the form $\overline{G} := 2^{10}{:}(U_5(2){:}2)$ does exist. In this paper our main aims are to fully study this group, to determine its inertia factor groups (and their respective ordinary character tables) and to compute the Fischer matrices. It will turn out that the character table of $\overline{G}$ is a 109×109 complex matrix and it is partitioned into three blocks corresponding to the three inertia factor groups $H_1 = U_5(2){:}2$, $H_2 = 2_-^{1+6}{:}((3^{1+2}{:}8){:}2)$ and $H_3 = O_5(2){:}2$ (see Section 3).

Clifford-Fischer Theory provides much more interesting information on the group and on the character table, in particular the character table produced by Clifford-Fischer Theory is in a special format that could not be achieved by direct computations using GAP [10] or Magma [7]. Also providing examples of applications of Clifford-Fischer Theory to both split and non-split extensions is sensible choice, since each group requires individual approach. The readers (particulary young researchers) will highly benefit from the theoretical background required for these computations. GAP and Magma are computational tools and would not replace good powerful and theoretical arguments.

For the notation used in this paper and the description of Clifford-Fischer theory technique, we follow [1], [2], [3], [4], [5], [6].

Using the 10-dimensional matrices over $\mathbb{F}_2$ that generate $G = U_5(2){:}2$, given at the electronic ATLAS of Wilson, together with GAP, we were able to construct the group $\overline{G}$ inside $PSL(11, 2)$. The following two elements $\overline{g}_1$ and $\overline{g}_2$ generate $\overline{G}$.

$$\overline{g}_1 = \begin{pmatrix} 1&0&1&1&0&0&0&1&0&0&0\\ 0&1&1&1&0&0&1&1&1&0&0\\ 1&1&1&1&1&1&1&0&0&1&0\\ 1&0&0&1&0&0&1&0&0&0&0\\ 1&1&1&0&1&1&1&1&1&0&0\\ 1&1&1&1&1&0&0&1&1&0&0\\ 1&1&0&0&1&1&1&1&0&0&0\\ 0&0&1&1&0&0&0&1&1&1&0\\ 0&0&1&0&0&0&1&0&1&0&0\\ 1&1&1&1&0&0&0&1&0&0&0\\ 0&1&1&0&1&0&0&0&0&0&1 \end{pmatrix}, \quad \overline{g}_2 = \begin{pmatrix} 1&1&1&1&0&1&1&0&1&0&0\\ 0&1&1&0&0&1&1&1&1&0&0\\ 1&0&1&1&0&0&1&1&1&1&0\\ 0&0&1&0&0&0&1&1&0&0&0\\ 0&0&1&1&0&1&0&0&1&0&0\\ 1&0&1&1&1&1&1&1&0&1&0\\ 0&0&1&1&1&1&1&0&0&0&0\\ 0&1&0&0&0&1&1&0&1&1&0\\ 1&1&0&0&0&1&1&0&0&0&0\\ 1&0&0&0&0&0&1&1&1&1&0\\ 1&0&1&0&0&1&0&0&1&0&1 \end{pmatrix},$$

with $o(\overline{g}_1) = 24,\ o(\overline{g}_2) = 8$ and $o(\overline{g}_1\overline{g}_2) = 20$.

Now having the group $\overline{G}$ constructed in GAP, it is easy to obtain all its normal subgroups. In fact $\overline{G}$ has only two non-trivial proper normal subgroups, one is of order 1024 and the other is of order 14 014 218 240. The normal subgroup of order 1024 is an elementary abelian 2-group and thus is isomorphic to N. Generators $n_1, n_2, \cdots, n_{10}$ of N, in terms of 11-dimensional matrices over $\mathbb{F}_2$ are given in Basheer [1].

In Magma or GAP one can check for the complements of $N = \langle n_1, n_2, \cdots, n_{10}\rangle$ in $\overline{G} = \langle \overline{g}_1, \overline{g}_2\rangle$, where here we obtained only one complement G. The following two elements g_1 and g_2 generate the complement G of N in $\overline{G}$. Note that G is a subgroup of $\overline{G}$ isomorphic to the quotient $\overline{G}/N \cong U_5(2){:}2$ and together with N creates the split extension $\overline{G}$ in consideration.

$$g_1 = \begin{pmatrix} 0&1&0&0&1&1&0&0&0&0&0\\ 0&1&1&1&1&0&1&1&1&0&0\\ 0&1&1&0&1&0&1&0&0&1&0\\ 1&1&1&0&0&1&1&1&0&0&0\\ 1&0&0&1&0&1&1&0&1&0&0\\ 1&1&1&0&0&1&0&1&0&1&0\\ 0&1&1&0&1&0&0&0&1&0&0\\ 1&1&0&0&0&0&0&0&0&1&0\\ 0&0&1&0&0&0&1&1&0&0&0\\ 1&1&1&0&1&0&1&1&0&1&0\\ 0&0&1&1&0&0&1&0&1&1&1 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 1&0&0&0&0&1&0&1&0&0&0\\ 1&1&1&1&1&0&1&0&0&1&0\\ 0&0&1&1&0&0&0&1&1&1&0\\ 1&0&0&0&0&1&0&1&0&1&0\\ 0&0&0&1&1&1&0&1&1&1&0\\ 0&0&0&1&0&1&1&0&1&0&0\\ 1&0&0&0&0&0&0&0&0&1&0\\ 1&0&1&0&0&0&0&1&1&0&0\\ 0&0&0&1&1&0&0&1&1&0&0\\ 0&1&1&0&1&1&1&0&1&1&0\\ 1&1&0&0&0&1&0&0&1&0&1 \end{pmatrix},$$

with $o(g_1) = 16,\ o(g_2) = 16$ and $o(g_1g_2) = 12$.

For the notations used in this paper and the description of Clifford-Fischer theory technique, we follow Basheer [1] and Basheer and Moori [2], [3].

2. Conjugacy Classes of $\overline{G} = 2^{10}{:}(U_5(2){:}2)$

In this section we use the method of the coset analysis technique (see Basheer [1], Basheer and Moori [2], [3], [5] or Moori [14] and [15] for more details)as we are interested to organize the classes of $\overline{G}$ corresponding to the classes of G. We list the conjugacy classes of $\overline{G}$ in Table 1, where in this table:

- g_i is the i^{th} conjugacy class of G as listed in Table 11.14 of [1].
- g_{ij} is a representative of a conjugacy class of $\overline{G}$ correspond to the class g_i of G.
- k_i is the number of orbits $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ on the action of N on the coset $N\overline{g}_i = Ng_i = 2^{10}g_i$. In particular, the action of N on the identity coset N produces 1024 orbits each consists of singleton. Thus $k_1 = 1024$.
- f_{ij} is the number of orbits fused together under the action of $C_G(g_i)$ on $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$. In particular, the action of $C_G(g_1) = G$ on the orbits $Q_{11}, Q_{12}, \cdots, Q_{1k_1}$ affords three orbits of lengths 1, 495 and 528. Thus $f_{11} = 1$, $f_{12} = 495$ and $f_{13} = 528$.
- m_{ij} are weights attached to each class of $\overline{G}$ that will be used later in computing the Fischer matrices of $\overline{G}$. These weights are computed through the formula

$$m_{ij} = [N_{\overline{G}}(N\overline{g}_i) : C_{\overline{G}}(g_{ij})] = |N|\frac{|C_G(g_i)|}{|C_{\overline{G}}(g_{ij})|}. \tag{1}$$

Table 1: The conjugacy classes of $\overline{G} = 2^{10}{:}(U_5(2){:}2)$

$[g_i]_G$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\|[g_{ij}]_{\overline{G}}\|$	$\|C_{\overline{G}}(g_{ij})\|$
$g_1 = 1A$	$k_1 = 1024$	$f_{11} = 1$	$m_{11} = 1$	g_{11}	1	1	28028436480
		$f_{12} = 495$	$m_{12} = 495$	g_{12}	2	495	56623104
		$f_{13} = 528$	$m_{13} = 528$	g_{13}	2	528	53084160
$g_2 = 2A$	$k_2 = 256$	$f_{21} = 1$	$m_{21} = 4$	g_{21}	2	660	42467328
		$f_{22} = 27$	$m_{22} = 108$	g_{22}	2	17820	1572864
		$f_{23} = 36$	$m_{23} = 144$	g_{23}	2	23760	1179648
		$f_{24} = 192$	$m_{24} = 768$	g_{24}	4	126720	221184
$g_3 = 2B$	$k_3 = 64$	$f_{31} = 1$	$m_{31} = 16$	g_{31}	2	47520	589824
		$f_{32} = 3$	$m_{32} = 48$	g_{32}	2	142560	196608
		$f_{33} = 24$	$m_{33} = 384$	g_{33}	4	1140480	24576
		$f_{34} = 36$	$m_{34} = 576$	g_{34}	4	1710720	16384
$g_4 = 2C$	$k_4 = 32$	$f_{41} = 1$	$m_{41} = 32$	g_{41}	2	608256	46080
		$f_{42} = 1$	$m_{42} = 32$	g_{42}	4	608256	46080
		$f_{43} = 15$	$m_{43} = 480$	g_{43}	4	9123840	3072
		$f_{44} = 15$	$m_{44} = 480$	g_{44}	4	9123840	3072
$g_5 = 3A$	$k_5 = 1$	$f_{51} = 1$	$m_{51} = 1024$	g_{51}	3	360448	77760
$g_6 = 3B$	$k_6 = 64$	$f_{61} = 1$	$m_{61} = 16$	g_{61}	3	112640	248832
		$f_{62} = 27$	$m_{62} = 432$	g_{62}	6	3041280	9216
		$f_{63} = 36$	$m_{63} = 576$	g_{63}	6	4055040	6912
$g_7 = 3C$	$k_7 = 16$	$f_{71} = 1$	$m_{71} = 64$	g_{71}	3	4055040	6912
		$f_{72} = 6$	$m_{72} = 384$	g_{72}	6	24330240	1152
		$f_{73} = 9$	$m_{73} = 576$	g_{73}	6	36495360	768
$g_8 = 3D$	$k_8 = 4$	$f_{81} = 1$	$m_{81} = 256$	g_{81}	3	10813440	2592
		$f_{82} = 3$	$m_{82} = 768$	g_{82}	6	32440320	864
$g_9 = 4A$	$k_9 = 64$	$f_{91} = 1$	$m_{91} = 16$	g_{91}	4	190080	147456
		$f_{92} = 3$	$m_{92} = 3$	g_{92}	4	570240	49152
		$f_{93} = 6$	$m_{93} = 96$	g_{93}	4	1140480	24576
		$f_{94} = 6$	$m_{94} = 96$	g_{94}	4	1140480	24576
		$f_{95} = 48$	$m_{95} = 768$	g_{95}	4	9123840	3072
$g_{10} = 4B$	$k_{10} = 16$	$f_{10,1} = 1$	$m_{10,1} = 64$	$g_{10,1}$	4	2880960	12288
		$f_{10,2} = 1$	$m_{10,2} = 64$	$g_{10,2}$	4	2880960	12288
		$f_{10,3} = 2$	$m_{10,3} = 128$	$g_{10,3}$	4	4561920	6144
		$f_{10,4} = 12$	$m_{10,4} = 768$	$g_{10,4}$	4	27371520	1024
$g_{11} = 4C$	$k_{11} = 16$	$f_{11,1} = 1$	$m_{11,1} = 64$	$g_{11,1}$	4	9123840	3072
		$f_{11,2} = 3$	$m_{11,2} = 192$	$g_{11,2}$	4	27371520	1024
		$f_{11,3} = 12$	$m_{11,3} = 768$	$g_{11,3}$	8	109486080	256
$g_{12} = 4D$	$k_{12} = 8$	$f_{12,1} = 1$	$m_{12,1} = 128$	$g_{12,1}$	4	36495360	768
		$f_{12,2} = 1$	$m_{12,2} = 128$	$g_{12,2}$	4	36495360	768
		$f_{12,3} = 3$	$m_{12,3} = 384$	$g_{12,3}$	8	109486080	256
		$f_{12,4} = 3$	$m_{12,4} = 384$	$g_{12,4}$	8	109486080	256
$g_{13} = 5A$	$k_{13} = 4$	$f_{13,1} = 1$	$m_{13,1} = 256$	$g_{13,1}$	5	233570304	120
		$f_{13,2} = 3$	$m_{13,2} = 768$	$g_{13,2}$	10	700710912	40
$g_{14} = 6A$	$k_{14} = 1$	$f_{14,1} = 1$	$m_{14,1} = 1024$	$g_{14,1}$	6	16220160	1728
$g_{15} = 6B$	$k_{15} = 4$	$f_{15,1} = 1$	$m_{15,1} = 256$	$g_{15,1}$	6	5406720	5184
		$f_{15,2} = 3$	$m_{15,2} = 768$	$g_{15,2}$	12	16220160	1728
$g_{16} = 6C$	$k_{16} = 16$	$f_{16,1} = 1$	$m_{16,1} = 64$	$g_{16,1}$	6	4055040	6912
		$f_{16,2} = 3$	$m_{16,2} = 192$	$g_{16,2}$	6	12165120	2304
		$f_{16,3} = 12$	$m_{16,3} = 768$	$g_{16,3}$	12	48660480	576
$g_{17} = 6D$	$k_{17} = 16$	$f_{17,1} = 1$	$m_{17,1} = 64$	$g_{17,1}$	6	4055040	6912
		$f_{17,2} = 6$	$m_{17,2} = 384$	$g_{17,2}$	6	24330240	1152
		$f_{17,3} = 9$	$m_{17,3} = 576$	$g_{17,3}$	6	36495360	768
$g_{18} = 6E$	$k_{18} = 1$	$f_{18,1} = 1$	$m_{18,1} = 1024$	$g_{18,1}$	6	97320960	288
$g_{19} = 6F$	$k_{19} = 4$	$f_{19,1} = 1$	$m_{19,1} = 256$	$g_{19,1}$	6	48660480	576
		$f_{19,2} = 3$	$m_{19,2} = 768$	$g_{19,2}$	12	145981440	192
$g_{20} = 6G$	$k_{20} = 4$	$f_{20,1} = 1$	$m_{20,1} = 256$	$g_{20,1}$	6	64880640	432
		$f_{20,2} = 3$	$m_{20,2} = 768$	$g_{20,2}$	6	194641920	144
$g_{21} = 6H$	$k_{21} = 4$	$f_{21,1} = 1$	$m_{21,1} = 256$	$g_{21,1}$	6	97320960	288
		$f_{21,2} = 3$	$m_{21,2} = 768$	$g_{21,2}$	6	291962880	96
$g_{22} = 6I$	$k_{22} = 2$	$f_{22,1} = 1$	$m_{22,1} = 512$	$g_{22,1}$	6	389283840	72
		$f_{22,2} = 1$	$m_{22,2} = 512$	$g_{22,2}$	12	389283840	72
$g_{23} = 6J$	$k_{23} = 8$	$f_{23,1} = 1$	$m_{23,1} = 128$	$g_{23,1}$	6	97320960	288
		$f_{23,2} = 1$	$m_{23,2} = 128$	$g_{23,2}$	12	97320960	288
		$f_{23,3} = 3$	$m_{23,3} = 384$	$g_{23,3}$	12	291962880	96
		$f_{23,4} = 3$	$m_{23,4} = 384$	$g_{23,4}$	12	291962880	96

continued on next page

Table 1 (continued from previous page)

$[g_i]_G$	k_i	f_{ij}	m_{ij}	$[g_{ij}]_{\overline{G}}$	$o(g_{ij})$	$\lvert[g_{ij}]_{\overline{G}}\rvert$	$\lvert C_{\overline{G}}(g_{ij})\rvert$
$g_{24} = 8A$	$k_{24} = 8$	$f_{24,1} = 1$	$m_{24,1} = 128$	$g_{24,1}$	8	18247680	1536
		$f_{24,2} = 3$	$m_{24,2} = 384$	$g_{24,2}$	8	54743040	512
		$f_{24,3} = 4$	$m_{24,3} = 512$	$g_{24,3}$	8	72990720	384
$g_{25} = 8B$	$k_{25} = 8$	$f_{25,1} = 1$	$m_{25,1} = 128$	$g_{25,1}$	8	18247680	1536
		$f_{25,2} = 3$	$m_{25,2} = 384$	$g_{25,2}$	8	54743040	512
		$f_{25,1} = 4$	$m_{25,3} = 512$	$g_{25,3}$	8	72990720	384
$g_{26} = 8C$	$k_{26} = 8$	$f_{26,1} = 1$	$m_{26,1} = 128$	$g_{26,1}$	8	109486080	256
		$f_{26,2} = 1$	$m_{26,2} = 128$	$g_{26,2}$	8	109486080	256
		$f_{26,3} = 2$	$m_{26,3} = 256$	$g_{26,3}$	8	218972160	128
		$f_{26,4} = 2$	$m_{26,4} = 256$	$g_{26,4}$	8	218972160	128
		$f_{26,5} = 2$	$m_{26,5} = 256$	$g_{26,5}$	8	218972160	128
$g_{27} = 8D$	$k_{27} = 4$	$f_{27,1} = 1$	$m_{27,1} = 256$	$g_{27,1}$	8	218972160	128
		$f_{27,2} = 1$	$m_{27,2} = 256$	$g_{27,2}$	8	218972160	128
		$f_{27,3} = 2$	$m_{27,3} = 512$	$g_{27,3}$	8	437944320	64
$g_{28} = 9A$	$k_{28} = 1$	$f_{28,1} = 1$	$m_{28,1} = 1024$	$g_{28,1}$	9	519045120	54
$g_{29} = 9B$	$k_{29} = 4$	$f_{29,1} = 1$	$m_{29,1} = 256$	$g_{29,1}$	9	259522560	108
		$f_{29,2} = 3$	$m_{29,2} = 768$	$g_{29,2}$	18	778567680	36
$g_{30} = 10A$	$k_{30} = 2$	$f_{30,1} = 1$	$m_{30,1} = 512$	$g_{30,1}$	10	1401421824	20
		$f_{30,2} = 1$	$m_{30,2} = 512$	$g_{30,2}$	20	1401421824	20
$g_{31} = 11A$	$k_{31} = 1$	$f_{31,1} = 1$	$m_{31,1} = 1024$	$g_{31,1}$	11	2548039680	11
$g_{32} = 12A$	$k_{32} = 1$	$f_{32,1} = 1$	$m_{32,1} = 1024$	$g_{32,1}$	12	194641920	144
$g_{33} = 12B$	$k_{33} = 4$	$f_{33,1} = 1$	$m_{33,1} = 256$	$g_{33,1}$	12	389283840	288
		$f_{33,2} = 3$	$m_{33,2} = 768$	$g_{33,2}$	12	291962880	96
$g_{34} = 12C$	$k_{34} = 16$	$f_{34,1} = 1$	$m_{34,1} = 64$	$g_{34,1}$	12	24330240	1152
		$f_{34,2} = 3$	$m_{34,2} = 192$	$g_{34,2}$	12	72990720	384
		$f_{34,3} = 3$	$m_{34,3} = 192$	$g_{34,3}$	12	72990720	384
		$f_{34,4} = 3$	$m_{34,4} = 192$	$g_{34,4}$	12	72990720	384
		$f_{34,5} = 6$	$m_{34,5} = 384$	$g_{34,5}$	12	145981440	192
$g_{35} = 12D$	$k_{35} = 1$	$f_{35,1} = 1$	$m_{35,1} = 1024$	$g_{35,1}$	12	1167851520	24
$g_{36} = 12E$	$k_{36} = 4$	$f_{36,1} = 1$	$m_{36,1} = 256$	$g_{36,1}$	12	291962880	96
		$f_{36,2} = 1$	$m_{36,2} = 256$	$g_{36,2}$	12	291962880	96
		$f_{36,3} = 1$	$m_{36,3} = 256$	$g_{36,3}$	12	291962880	96
		$f_{36,4} = 1$	$m_{36,4} = 256$	$g_{36,4}$	12	291962880	96
$g_{37} = 12F$	$k_{37} = 2$	$f_{37,1} = 1$	$m_{37,1} = 512$	$g_{37,1}$	12	1167851520	24
		$f_{37,2} = 1$	$m_{37,2} = 512$	$g_{37,2}$	12	1167851520	24
$g_{38} = 15A$	$k_{38} = 1$	$f_{38,1} = 1$	$m_{38,1} = 1024$	$g_{38,1}$	15	1868562432	15
$g_{39} = 16A$	$k_{39} = 2$	$f_{39,1} = 1$	$m_{39,1} = 512$	$g_{39,1}$	16	875888640	32
		$f_{39,2} = 1$	$m_{39,2} = 512$	$g_{39,2}$	16	875888640	32
$g_{40} = 16B$	$k_{40} = 2$	$f_{40,1} = 1$	$m_{40,1} = 512$	$g_{40,1}$	16	875888640	32
		$f_{40,2} = 1$	$m_{40,2} = 512$	$g_{40,2}$	16	875888640	32
$g_{41} = 18A$	$k_{41} = 1$	$f_{41,1} = 1$	$m_{41,1} = 1024$	$g_{41,1}$	18	1557135360	18
$g_{42} = 24A$	$k_{42} = 2$	$f_{42,1} = 1$	$m_{42,1} = 512$	$g_{42,1}$	24	583925760	48
		$f_{42,2} = 1$	$m_{42,2} = 512$	$g_{42,2}$	24	583925760	48
$g_{43} = 24B$	$k_{43} = 2$	$f_{43,1} = 1$	$m_{43,1} = 512$	$g_{43,1}$	24	583925760	48
		$f_{43,2} = 1$	$m_{43,2} = 512$	$g_{43,2}$	24	583925760	48

3. Inertia Factor Groups of $\overline{G} = 2^{10}{:}(U_5(2){:}2)$

We have seen in Section 2 that the action of $\overline{G} = 2^{10}{:}(U_5(2){:}2)$ on $N = 2^{10}$ yielded three orbits of lengths 1, 495 and 528. By a theorem of Brauer (see Lemma 4.5.2 of [9]), it follows that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will also produce three orbits. Since $N = 2^{10}$ is a vector space, the action of $\overline{G}$ on $\mathrm{Irr}(2^{10})$ can be viewed as the action of $\overline{G}$ on N^*, where N^* is the dual space of N. In fact we act the group generated by the transposed matrices of the matrix generators of $\overline{G}$ on N. We have found that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ is self-dual to the action of $\overline{G}$ on N, that is the orbit lengths of $\overline{G}$ on $\mathrm{Irr}(N)$ are 1, 495 and 528. This can also be deduced from the associated geometry of classical groups (for example see Liebeck [12]), that is the action of $SU_n(q^2)$ on its natural module $\mathbb{V} = \mathbb{F}_{q^2}^n$ yields three orbits of lengths 1 (consisting of the zero vector) and

$$\begin{cases} (q^n-1)(q^{n-1}+1),\ q^{n-1}(q-1)(q^n-1) & \text{if } n \text{ is even,} \\ (q^n+1)(q^{n-1}-1),\ q^{n-1}(q-1)(q^n+1) & \text{if } n \text{ is odd.} \end{cases}$$

From Grove [11] we know that the number of isotropic vectors together with the zero vector of $\mathbb{V}$ is given by $q^{2n-1} + (-1)^n(q^n - q^{n-1})$. This distinguishes the orbits of isotropic and non-isotropic vectors in each case of n. In the following, we determine the structures of the inertia factor groups of $\overline{G}$.

Let $H_1,\ H_2$ and H_3 be the respective inertia factor groups of representatives of characters from the previous orbits of $\overline{G}$ on $\mathrm{Irr}(N)$. We notice that these inertia factors have indices 1, 495 and 528 respectively in $U_5(2){:}2$. Clearly $H_1 = U_5(2){:}2$ and the character table of this group is given as Table 11.14 of Basheer [1]. By looking at the $\mathbb{ATLAS}$, the group $U_5(2){:}2$ has 7 conjugacy classes of maximal subgroups. Let $M[1], M[2], \cdots, M[7]$ be representatives of these classes of maximal subgroups. That is $M[1] = U_5(2)$, $M[2] = 2_-^{1+6}{:}3_-^{1+2}{:}2S_4$, $M[3] = (3 \times U_4(2)){:}2$, $M[4] = 2^{4+4}{:}(3 \times A_5){:}2$, $M[5] = 3^4{:}(2 \times S_5)$, $M[6] = S_3 \times 3_+^{1+2}{:}2S_4$ and $M[7] = PSL(2,11){:}2$. Note that these maximal subgroups have indices 2, 165, 176, 297, 1408, 3520 and 20736 respectively in G. By considering the indices of H_2 and H_3 in G, we infer that H_2 must be an index 3 subgroup of $M[2]$, while H_3 is either an index 264 subgroup of $M[1]$ or of index 3 in $M[3]$. However the possibility $H_3 \leq M[1]$ is not feasible as we can see from the $\mathbb{ATLAS}$ that $U_5(2)$ does not contain a subgroup of index that is a divisor of 264. This leaves us with the other possibility that H_3 is an index 3 subgroup of $M[3]$. As subgroups of $G = \langle g_1, g_2 \rangle$, the group $M[2]$ is generated by α_1 and α_2, while the group $M[3]$ is generated by β_1 and β_2, where

$$\alpha_1 = \begin{pmatrix} 0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&1&0&1&1&1&1&1&0 \\ 1&1&1&0&0&0&0&1&0&0&0 \\ 1&1&0&1&1&1&0&0&0&0&0 \\ 0&1&0&0&1&0&0&1&1&0&0 \\ 1&1&1&1&0&1&1&0&1&0&0 \\ 1&1&0&0&1&1&1&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&1&0 \\ 0&1&1&0&1&0&0&1&1&0&0 \\ 0&1&1&1&1&1&1&1&0&0&0 \\ 1&0&0&1&1&0&1&1&1&0&1 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0&1&1&0&0&1&0&0&1&0&0 \\ 0&1&1&1&0&0&1&1&0&1&0 \\ 1&1&1&0&0&1&0&1&1&1&0 \\ 0&0&0&1&1&1&0&1&1&0&0 \\ 1&1&1&1&1&0&0&1&1&0&0 \\ 0&1&0&0&1&1&0&0&1&0&0 \\ 0&1&0&0&1&1&0&1&1&1&0 \\ 1&1&0&1&1&0&0&1&0&0&0 \\ 1&0&0&0&1&1&1&0&1&1&0 \\ 0&1&1&0&1&1&1&0&0&0&0 \\ 1&0&1&0&0&1&1&0&0&1&1 \end{pmatrix},$$

$$\beta_1 = \begin{pmatrix} 1&0&0&0&0&1&0&1&1&1&0 \\ 1&0&1&0&0&0&1&1&0&0&0 \\ 0&1&0&1&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&1&0&1&0&0 \\ 0&0&1&0&0&1&1&1&0&1&0 \\ 0&0&1&0&0&1&0&0&0&0&0 \\ 0&1&1&1&0&1&1&1&1&0&0 \\ 0&0&0&0&1&1&1&1&0&0&0 \\ 1&0&0&1&0&1&0&1&1&0&0 \\ 1&1&1&0&0&0&1&0&0&1&0 \\ 0&1&1&0&0&0&0&1&0&1&1 \end{pmatrix}, \quad \beta_2 = \begin{pmatrix} 0&1&0&1&1&1&0&0&0&1&0 \\ 0&1&1&0&1&1&0&1&1&0&0 \\ 1&1&1&0&1&0&1&0&0&1&0 \\ 1&1&1&0&1&0&0&1&1&0&0 \\ 1&1&0&0&1&1&0&0&1&0&0 \\ 1&1&1&0&0&0&1&0&0&1&0 \\ 1&1&0&0&0&0&0&1&1&0&0 \\ 0&0&1&0&0&0&1&0&0&1&0 \\ 0&1&1&0&0&1&1&1&0&1&0 \\ 1&0&0&0&0&0&0&0&0&1&0 \\ 1&0&1&1&0&1&0&1&0&1&1 \end{pmatrix}.$$

Using GAP we were able to locate all the maximal subgroups of $M[2]$ and $M[3]$. We list brief information about these subgroups in Table 2.

Table 2: Some information on the maximal subgroups of $M[2]$ and $M[3]$

Maximal Subgroups of $M[2]$ & $M[3]$	$M[ij]$	$\lvert M[ij]\rvert$	$[M[i]:M[ij]]$	$\lvert\mathrm{Irr}(M[ij])\rvert$
$M[2]=2_-^{1+6}{:}3_-^{1+2}{:}2S_4$	$M[21]$	82944	2	64
	$M[22]$	55296	3	41
	$M[23]$	41472	4	55
	$M[24]$	18432	9	50
	$M[25]$	2592	64	36
$M[3]=(3\times U_4(2)){:}2$	$M[31]$	77760	2	60
	$M[32]$	51840	3	25
	$M[33]$	5760	27	30
	$M[34]$	4320	36	33
	$M[35]$	3888	40	39
	$M[36]$	3888	40	42
	$M[37]$	3456	45	48

From Table 2 we can see that the two groups H_2 and H_3 are in the conjugacy classes of maximal subgroups containing $M[22]$ and $M[32]$ respectively.

As subgroups of $G=\langle g_1,g_2\rangle$, the group $M[22]$ is generated by μ_1 and μ_2, while the group $M[32]$ is generated by ζ_1 and ζ_2, where

$$\mu_1=\begin{pmatrix} 0&1&0&1&0&1&0&1&1&1&0\\ 0&1&0&1&0&1&1&0&1&1&0\\ 1&1&0&0&0&1&0&0&0&0&0\\ 0&0&0&1&1&1&0&0&0&1&0\\ 1&1&0&1&1&1&0&0&0&0&0\\ 1&1&1&0&0&0&1&1&1&1&0\\ 0&1&0&0&0&1&0&0&1&0&0\\ 0&1&0&1&1&0&1&1&0&1&0\\ 0&1&0&1&0&0&0&0&1&1&0\\ 0&1&1&1&1&1&1&1&0&1&0\\ 0&0&0&1&0&0&0&0&1&1&1 \end{pmatrix},\quad \mu_2=\begin{pmatrix} 0&1&1&0&1&0&0&1&1&0&0\\ 1&1&1&0&0&1&1&1&0&0&0\\ 1&1&0&0&0&0&0&1&1&0&0\\ 0&1&1&0&0&1&1&1&0&1&0\\ 0&0&1&0&0&0&0&0&1&0&0\\ 0&1&1&0&0&0&0&1&1&0&0\\ 0&0&0&1&0&0&1&0&1&0&0\\ 0&0&1&0&0&1&0&0&0&0&0\\ 1&0&1&1&0&1&1&1&0&0&0\\ 0&0&1&1&1&1&0&1&1&0&0\\ 1&0&1&1&0&0&0&0&0&0&1 \end{pmatrix},$$

with $o(\mu_1)=24,\ o(\mu_2)=16$ and $o(\mu_1\mu_2)=12$,

$$\zeta_1=\begin{pmatrix} 1&1&1&0&0&0&1&0&0&1&0\\ 0&1&0&1&1&1&0&0&1&1&0\\ 0&0&1&1&0&0&1&0&0&0&0\\ 0&0&0&1&1&0&1&1&1&1&0\\ 1&1&1&1&0&1&0&0&1&0&0\\ 1&1&1&0&1&1&1&1&1&0&0\\ 0&0&1&1&1&0&0&0&0&0&0\\ 1&0&0&0&1&1&0&1&1&1&0\\ 1&0&0&0&0&1&1&1&1&0&0\\ 0&0&1&1&0&0&0&1&1&0&0\\ 0&0&0&0&1&0&1&1&1&1&1 \end{pmatrix},\quad \zeta_2=\begin{pmatrix} 1&1&0&0&0&0&0&0&1&1&0\\ 1&0&0&0&0&1&1&1&1&1&0\\ 0&1&0&0&0&1&0&1&1&0&0\\ 0&0&1&0&1&1&1&0&0&1&0\\ 1&1&1&1&0&1&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&1&1&0\\ 1&1&1&1&1&0&0&1&1&1&0\\ 0&1&1&1&1&1&0&1&1&1&0\\ 0&1&1&0&0&0&1&0&1&1&0\\ 0&0&0&1&1&1&0&1&1&1&0\\ 1&1&1&0&0&1&1&0&0&1&1 \end{pmatrix},$$

with $o(\zeta_1)=3,\ o(\zeta_2)=4$ and $o(\zeta_1\zeta_2)=10$.

Remark 1. With the aid of GAP we were able to determine the structures of all the normal subgroups of H_2 and H_3 and the corresponding quotient groups. In fact we have found that $H_2\cong 2_-^{1+6}{:}((3^{1+2}{:}8){:}2)$, while the group $H_3=M[32]\cong O_5(3){:}2\cong U_4(2){:}2\cong Sp(4,3){:}2$.

We recall that knowledge of the appropriate character tables of inertia factor groups is crucial in calculating the full character table of any group extension. Since in our extensions $\overline{G}$, the normal subgroup N is abelian and the extension splits, it follows by applications of Mackey's Theorem (see for example Theorem 3.3.4 of Whitley [17]), that every character of N is extendible to an ordinary character of its respective inertia group $\overline{H}_k$. Thus all the character tables of the inertia factor groups that we will use to construct the character tables of $\overline{G}$ are the ordinary ones. The character table of $H_2=2_-^{1+6}{:}((3^{1+2}{:}8){:}2)$ is not in a library of GAP. One can use the generators μ_1 and μ_2 of H_2 to generate H_2 inside Magma [7] or GAP and then obtain its character table. In fact the character table of H_2 appears as Table 11.13 of [1]. The character table of H_3 is stored in GAP, or one can use any set of generators of $U_4(2)$:2 in the forms of 6, 8 or 14-dimensional $\mathbb{F}_2$-representations supplied at [18] to generate this group in either Magma or GAP and then obtain its character table. Alternatively one can use the generators ζ_1 and ζ_2 to find the character table of H_3. Note that from Table 1 the group $\overline{G}$ has 109 conjugacy classes. By the description of Section 3 of [2] the 109 irreducible characters of $\overline{G}$ are distributed into three blocks of characters correspond to the inertia factor groups. That is $|\mathrm{Irr}(\overline{G})| = |\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)|$. From Tables 11.14, 11.15 and 11.16 of [1], we can see that $|\mathrm{Irr}(H_1)| + |\mathrm{Irr}(H_2)| + |\mathrm{Irr}(H_3)| = 43 + 41 + 25 = 109 = |\mathrm{Irr}(\overline{G})|$.

In Tables 3 and 4 we list respectively the fusions of classes of H_2 and H_3 into classes of G.

Table 3: The fusion of classes of H_2 into classes of G

Class of H_2	$\hookrightarrow$ Class of $U_5(2){:}2$	Class of H_2	$\hookrightarrow$ Class of $U_5(2){:}2$
$1a = g_{121}$	$1A$	$8a = g_{24,21}$	$8A$
$2a = g_{221}$	$2A$	$8b = g_{25,21}$	$8B$
$2b = g_{222}$	$2A$	$8c = g_{26,21}$	$8C$
$2c = g_{321}$	$2B$	$8d = g_{27,21}$	$8D$
$2d = g_{322}$	$2B$	$8e = g_{27,22}$	$8D$
$2e = g_{421}$	$2C$	$8f = g_{26,22}$	$8C$
$3a = g_{721}$	$3C$	$8g = g_{24,22}$	$8A$
$3b = g_{621}$	$3B$	$8h = g_{25,22}$	$8B$
$4a = g_{921}$	$4A$	$8i = g_{26,23}$	$8C$
$4b = g_{10,21}$	$4B$	$12a = g_{33,21}$	$12B$
$4c = g_{11,21}$	$4C$	$12b = g_{34,21}$	$12C$
$4d = g_{12,21}$	$4D$	$12c = g_{36,21}$	$12E$
$4e = g_{922}$	$4A$	$12d = g_{34,22}$	$12C$
$4f = g_{10,22}$	$4B$	$12e = g_{36,22}$	$12E$
$4g = g_{923}$	$4A$	$12f = g_{36,23}$	$12E$
$4h = g_{10,23}$	$4B$	$12g = g_{34,23}$	$12C$
$6a = g_{15,21}$	$6B$	$16a = g_{39,21}$	$16A$
$6b = g_{16,21}$	$6C$	$16b = g_{40,21}$	$16B$
$6c = g_{19,21}$	$6F$	$24a = g_{42,21}$	$24A$
$6d = g_{17,21}$	$6D$	$24b = g_{43,21}$	$24B$
$6e = g_{23,21}$	$6J$		

Table 4: The fusion of classes of H_3 into classes of G

Class of H_3	$\hookrightarrow$ Class of $U_5(2){:}2$	Class of H_3	$\hookrightarrow$ Class of $U_5(2){:}2$
$1a = g_{131}$	$1A$	$6a = g_{14,31}$	$6D$
$2a = g_{431}$	$2C$	$6b = g_{20,31}$	$6G$
$2b = g_{231}$	$2A$	$6c = g_{23,31}$	$6J$
$2c = g_{331}$	$2B$	$6d = g_{16,31}$	$6C$
$2d = g_{432}$	$2C$	$6e = g_{22,31}$	$6I$
$3a = g_{731}$	$3C$	$6f = g_{21,31}$	$6H$
$3b = g_{831}$	$3D$	$6g = g_{23,32}$	$6J$
$3c = g_{631}$	$3B$	$8a = g_{26,31}$	$8C$
$4a = g_{12,31}$	$4D$	$9a = g_{29,31}$	$9B$
$4b = g_{933}$	$4A$	$10a = g_{30,31}$	$10A$
$4c = g_{12,32}$	$4D$	$12a = g_{37,31}$	$12F$
$4d = g_{11,31}$	$4C$	$12b = g_{34,31}$	$12C$
$5a = g_{13,31}$	$5A$		

4. Fischer matrices of $\overline{G} = 2^{10}$:$(U_5(2)$:2)

We recall from [1, 2] that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers of g_{ij}, $1 \le j \le c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 1 we supplied $|C_{\overline{G}}(g_{ij})|$ and m_{ij}, $1 \le i \le 43$, $1 \le j \le c(g_i)$. Also the fusions of the classes of H_2 and H_3 into classes of G are given in Tables 3 and 4 respectively. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 1 that the sizes of the Fischer matrices of $\overline{G}$ range between 1 and 5 for every $i \in \{1, 2, ..., 43\}$.

We have used the arithmetical properties of the Fischer matrices, given in Proposition 3.6 of [2], to calculate some of the entries of these matrices and also to build an algebraic system of equations. For example since the extension splits and $N = 2^{10}$ is abelian, then every coset $N\overline{g}_i$ (or just Ng_i) is a split coset (see Schiffer [16]) and it results that $a_{i1}^{(k,m)} = \frac{|C_{U_5(2):2}(g_i)|}{|C_{H_k}(g_{ikm})|}$, $\forall i \in \{1, 2, ..., 43\}$. With the help of the symbolic mathematical package Maxima [13], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which we list below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}
$o(g_{1j})$		1	2	2
$\|C_{\overline{G}}(g_{1j})\|$		28028436480	56623104	53084160
(k,m)	$\|C_{H_k}(g_{1km})\|$			
$(1,1)$	27371520	1	1	1
$(2,1)$	55296	495	15	-17
$(3,1)$	51840	528	-16	16
m_{1j}		1	495	528

$\mathcal{F}_2$

g_2		g_{21}	g_{22}	g_{23}	g_{24}
$o(g_{2j})$		2	2	2	4
$\|C_{\overline{G}}(g_{2j})\|$		42467328	1572864	1179648	221184
(k,m)	$\|C_{H_k}(g_{2km})\|$				
$(1,1)$	165888	1	1	1	1
$(2,1)$	55296	3	3	3	-1
$(2,2)$	1536	108	-20	12	0
$(3,1)$	1152	144	16	-16	0
m_{2j}		4	108	144	768

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}	g_{34}
$o(g_{3j})$		2	2	4	4
$\|C_{\overline{G}}(g_{3j})\|$		589824	196608	24576	16384
(k,m)	$\|C_{H_k}(g_{3km})\|$				
$(1,1)$	9216	1	1	1	1
$(2,1)$	1536	6	6	2	-2
$(2,2)$	1024	9	9	-3	1
$(3,1)$	192	48	-16	0	0
m_{3j}		16	48	384	576

$\mathcal{F}_4$

g_4		g_{41}	g_{42}	g_{43}	g_{44}
$o(g_{4j})$		2	4	4	4
$\|C_{\overline{G}}(g_{4j})\|$		46080	46080	3072	3072
(k,m)	$\|C_{H_k}(g_{4km})\|$				
$(1,1)$	1440	1	1	1	1
$(2,1)$	96	15	15	-1	-1
$(3,1)$	1440	1	-1	1	-1
$(3,2)$	96	15	-15	-1	1
m_{4j}		32	32	480	480

$\mathcal{F}_5$

g_5		g_{51}
$o(g_{5j})$		3
$\|C_{\overline{G}}(g_{5j})\|$		77760
(k,m)	$\|C_{H_k}(g_{5km})\|$	
$(1,1)$	77760	1
m_{5j}		1024

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}
$o(g_{6j})$		3	6	6
$\|C_{\overline{G}}(g_{6j})\|$		248832	9216	6912
(k,m)	$\|C_{H_k}(g_{6km})\|$			
$(1,1)$	3888	1	1	1
$(2,1)$	144	27	3	-5
$(3,1)$	108	36	-4	4
m_{6j}		16	432	576

$\mathcal{F}_7$

g_7		g_{71}	g_{72}	g_{73}
$o(g_{7j})$		3	6	6
$\|C_{\overline{G}}(g_{7j})\|$		6912	1152	768
(k,m)	$\|C_{H_k}(g_{7km})\|$			
$(1,1)$	432	1	1	1
$(2,1)$	48	9	-3	1
$(3,1)$	72	6	2	-2
m_{7j}		64	384	576

$\mathcal{F}_8$

g_8		g_{81}	g_{82}
$o(g_{8j})$		3	6
$\vert C_{\overline{G}}(g_{8j})\vert$		2592	864
(k,m)	$\vert C_{H_k}(g_{8km})\vert$		
$(1,1)$	648	1	1
$(3,1)$	216	3	−1
m_{8j}		256	768

$\mathcal{F}_9$

g_9		g_{91}	g_{92}	g_{93}	g_{94}	g_{95}
$o(g_{9j})$		4	4	4	4	4
$\vert C_{\overline{G}}(g_{9j})\vert$		147456	49152	24576	24576	3072
(k,m)	$\vert C_{H_k}(g_{9km})\vert$					
$(1,1)$	2304	1	1	1	1	1
$(2,1)$	768	3	3	3	3	−1
$(2,2)$	192	12	12	−4	−4	0
$(2,3)$	96	24	−8	−8	8	0
$(3,1)$	96	24	−8	8	−8	0
m_{9j}		16	48	96	96	768

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{10,4}$
$o(g_{10j})$		4	4	4	4
$\vert C_{\overline{G}}(g_{10j})\vert$		12288	12288	6144	1024
(k,m)	$\vert C_{H_k}(g_{10km})\vert$				
$(1,1)$	768	1	1	1	1
$(2,1)$	256	3	3	3	−1
$(2,2)$	192	4	4	−4	0
$(2,3)$	96	8	−8	0	0
m_{10j}		64	64	128	768

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$	$g_{11,3}$
$o(g_{11j})$		4	4	8
$\vert C_{\overline{G}}(g_{11j})\vert$		3072	1024	256
(k,m)	$\vert C_{H_k}(g_{11km})\vert$			
$(1,1)$	192	1	1	1
$(2,1)$	64	3	3	−1
$(3,1)$	16	12	−4	0
m_{11j}		64	192	768

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{12,4}$
$o(g_{12j})$		4	4	8	8
$\vert C_{\overline{G}}(g_{12j})\vert$		768	768	256	256
(k,m)	$\vert C_{H_k}(g_{12km})\vert$				
$(1,1)$	96	1	1	1	1
$(2,1)$	32	3	3	−1	−1
$(3,1)$	96	1	−1	1	−1
$(3,2)$	32	3	−3	−1	1
m_{12j}		128	128	384	384

$\mathcal{F}_{13}$

g_{13}		$g_{13,1}$	$g_{13,2}$
$o(g_{13j})$		5	10
$\vert C_{\overline{G}}(g_{13j})\vert$		120	40
(k,m)	$\vert C_{H_k}(g_{13km})\vert$		
$(1,1)$	30	1	1
$(3,1)$	10	3	−1
m_{13j}		256	768

$\mathcal{F}_{14}$

g_{14}		$g_{14,1}$
$o(g_{14j})$		6
$\vert C_{\overline{G}}(g_{14j})\vert$		1728
(k,m)	$\vert C_{H_k}(g_{14km})\vert$	
$(1,1)$	1728	1
m_{14j}		1024

$\mathcal{F}_{15}$

g_{15}		$g_{15,1}$	$g_{15,2}$
$o(g_{15j})$		6	12
$\vert C_{\overline{G}}(g_{15j})\vert$		5184	1728
(k,m)	$\vert C_{H_k}(g_{15km})\vert$		
$(1,1)$	1296	1	1
$(2,1)$	432	3	−1
m_{15j}		256	768

$\mathcal{F}_{16}$

g_{16}		$g_{16,1}$	$g_{16,2}$	$g_{16,3}$
$o(g_{16j})$		6	6	12
$\vert C_{\overline{G}}(g_{16j})\vert$		6912	2304	576
(k,m)	$\vert C_{H_k}(g_{16km})\vert$			
$(1,1)$	432	1	1	1
$(2,1)$	144	3	3	−1
$(3,1)$	36	12	−4	0
m_{16j}		64	192	768

$\mathcal{F}_{17}$

g_{17}		$g_{17,1}$	$g_{17,2}$	$g_{17,3}$
$o(g_{17j})$		6	6	6
$\vert C_{\overline{G}}(g_{17j})\vert$		6912	1152	768
(k,m)	$\vert C_{H_k}(g_{17km})\vert$			
$(1,1)$	432	1	1	1
$(2,1)$	48	9	−3	1
$(3,1)$	72	6	2	−2
m_{17j}		64	384	576

$\mathcal{F}_{18}$

g_{18}		$g_{18,1}$
$o(g_{18j})$		6
$\vert C_{\overline{G}}(g_{18j})\vert$		288
(k,m)	$\vert C_{H_k}(g_{18km})\vert$	
$(1,1)$	288	1
m_{18j}		1024

$\mathcal{F}_{19}$

g_{19}		$g_{19,1}$	$g_{19,2}$
$o(g_{19j})$		6	12
$\vert C_{\overline{G}}(g_{19j})\vert$		576	192
(k,m)	$\vert C_{H_k}(g_{19km})\vert$		
$(1,1)$	144	1	1
$(2,1)$	48	3	−1
m_{19j}		256	768

$\mathcal{F}_{20}$

g_{20}		$g_{20,1}$	$g_{20,2}$
$o(g_{20j})$		6	6
$\vert C_{\overline{G}}(g_{20j})\vert$		432	144
(k,m)	$\vert C_{H_k}(g_{20km})\vert$		
$(1,1)$	108	1	1
$(3,1)$	36	3	−1
m_{20j}		256	768

$\mathcal{F}_{21}$

g_{21}		$g_{21,1}$	$g_{21,2}$
$o(g_{21j})$		6	6
$\vert C_{\overline{G}}(g_{21j})\vert$		288	96
(k,m)	$\vert C_{H_k}(g_{21km})\vert$		
$(1,1)$	72	1	1
$(3,1)$	24	3	−1
m_{21j}		256	768

$\mathcal{F}_{22}$

g_{22}		$g_{22,1}$	$g_{22,2}$
$o(g_{22j})$		6	12
$\vert C_{\overline{G}}(g_{22j})\vert$		72	72
(k,m)	$\vert C_{H_k}(g_{22km})\vert$		
$(1,1)$	36	1	1
$(3,1)$	36	1	−1
m_{22j}		512	512

$\mathcal{F}_{23}$

g_{23}		$g_{23,1}$	$g_{23,2}$	$g_{23,3}$	$g_{23,4}$
$o(g_{23j})$		6	12	12	12
$\vert C_{\overline{G}}(g_{23j})\vert$		288	288	96	96
(k,m)	$\vert C_{H_k}(g_{23km})\vert$				
$(1,1)$	36	1	1	1	1
$(2,1)$	12	3	3	−1	−1
$(3,1)$	36	1	−1	1	−1
$(3,2)$	12	3	−3	−1	1
m_{23j}		128	128	384	384

$\mathcal{F}_{24}$

g_{24}		$g_{24,1}$	$g_{24,2}$	$g_{24,3}$
$o(g_{24j})$		8	8	8
$\lvert C_{\overline{G}}(g_{24j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{24km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{24j}		128	384	512

$\mathcal{F}_{25}$

g_{25}		$g_{25,1}$	$g_{25,2}$	$g_{25,3}$
$o(g_{25j})$		8	8	8
$\lvert C_{\overline{G}}(g_{25j})\rvert$		1536	512	384
(k,m)	$\lvert C_{H_k}(g_{25km})\rvert$			
$(1,1)$	192	1	1	1
$(2,1)$	192	1	1	-1
$(2,2)$	32	6	-2	0
m_{25j}		128	384	512

$\mathcal{F}_{26}$

g_{26}		$g_{26,1}$	$g_{26,2}$	$g_{26,3}$	$g_{26,4}$	$g_{26,5}$
$o(g_{26j})$		8	8	8	8	8
$\lvert C_{\overline{G}}(g_{26j})\rvert$		256	256	128	128	128
(k,m)	$\lvert C_{H_k}(g_{26km})\rvert$					
$(1,1)$	32	1	1	1	1	1
$(2,1)$	32	1	1	-1	1	-1
$(2,2)$	32	1	1	1	-1	-1
$(2,3)$	32	1	1	-1	-1	1
$(3,1)$	8	4	-4	0	0	0
m_{26j}		128	128	256	256	256

$\mathcal{F}_{27}$

g_{27}		$g_{27,1}$	$g_{27,2}$	$g_{27,3}$
$o(g_{27j})$		8	8	8
$\lvert C_{\overline{G}}(g_{27j})\rvert$		128	128	64
(k,m)	$\lvert C_{H_k}(g_{27km})\rvert$			
$(1,1)$	32	1	1	1
$(2,1)$	32	1	1	-1
$(2,2)$	16	2	-2	0
m_{27j}		256	256	512

$\mathcal{F}_{28}$

g_{28}		$g_{28,1}$
$o(g_{28j})$		9
$\lvert C_{\overline{G}}(g_{28j})\rvert$		54
(k,m)	$\lvert C_{H_k}(g_{28km})\rvert$	
$(1,1)$	54	1
m_{28j}		1024

$\mathcal{F}_{29}$

g_{29}		$g_{29,1}$	$g_{29,2}$
$o(g_{29j})$		9	18
$\lvert C_{\overline{G}}(g_{29j})\rvert$		108	36
(k,m)	$\lvert C_{H_k}(g_{29km})\rvert$		
$(1,1)$	27	1	1
$(3,1)$	9	3	-1
m_{29j}		256	768

$\mathcal{F}_{30}$

g_{30}		$g_{30,1}$	$g_{30,2}$
$o(g_{30j})$		10	20
$\lvert C_{\overline{G}}(g_{30j})\rvert$		20	20
(k,m)	$\lvert C_{H_k}(g_{30km})\rvert$		
$(1,1)$	10	1	1
$(3,1)$	10	1	-1
m_{30j}		512	512

$\mathcal{F}_{31}$

g_{31}		$g_{31,1}$
$o(g_{31j})$		11
$\lvert C_{\overline{G}}(g_{31j})\rvert$		11
(k,m)	$\lvert C_{H_k}(g_{31km})\rvert$	
$(1,1)$	11	1
m_{31j}		1024

$\mathcal{F}_{32}$

g_{32}		$g_{32,1}$
$o(g_{32j})$		12
$\lvert C_{\overline{G}}(g_{32j})\rvert$		144
(k,m)	$\lvert C_{H_k}(g_{32km})\rvert$	
$(1,1)$	144	1
m_{32j}		1024

$\mathcal{F}_{33}$

g_{33}		$g_{33,1}$	$g_{33,2}$
$o(g_{33j})$		12	12
$\lvert C_{\overline{G}}(g_{33j})\rvert$		288	96
(k,m)	$\lvert C_{H_k}(g_{33km})\rvert$		
$(1,1)$	72	1	1
$(2,1)$	24	3	-1
m_{33j}		256	768

$\mathcal{F}_{34}$

g_{34}		$g_{34,1}$	$g_{34,2}$	$g_{34,3}$	$g_{34,4}$	$g_{34,5}$
$o(g_{34j})$		12	12	12	12	12
$\lvert C_{\overline{G}}(g_{34j})\rvert$		1152	384	384	384	192
(k,m)	$\lvert C_{H_k}(g_{34km})\rvert$					
$(1,1)$	72	1	1	1	1	1
$(2,1)$	24	3	-1	-1	3	-1
$(2,2)$	24	3	-1	3	-1	-1
$(2,3)$	24	3	3	-1	-1	-1
$(3,1)$	12	6	-2	-2	-2	2
m_{34j}		64	192	192	192	384

$\mathcal{F}_{35}$

g_{35}		$g_{35,1}$
$o(g_{35j})$		12
$\lvert C_{\overline{G}}(g_{35j})\rvert$		24
(k,m)	$\lvert C_{H_k}(g_{35km})\rvert$	
$(1,1)$	24	1
m_{35j}		1024

$\mathcal{F}_{36}$

g_{36}		$g_{36,1}$	$g_{36,2}$	$g_{36,3}$	$g_{36,4}$
$o(g_{36j})$		12	12	12	12
$\lvert C_{\overline{G}}(g_{36j})\rvert$		96	96	96	96
(k,m)	$\lvert C_{H_k}(g_{36km})\rvert$				
$(1,1)$	24	1	1	1	1
$(2,1)$	24	1	-1	1	-1
$(2,2)$	24	1	1	-1	-1
$(2,3)$	24	1	-1	-1	1
m_{36j}		256	256	256	256

$\mathcal{F}_{37}$

g_{37}		$g_{37,1}$	$g_{37,2}$
$o(g_{37j})$		12	12
$\lvert C_{\overline{G}}(g_{37j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{37km})\rvert$		
$(1,1)$	12	1	1
$(3,1)$	12	1	-1
m_{37j}		512	512

$\mathcal{F}_{38}$

g_{38}		$g_{38,1}$
$o(g_{38j})$		15
$\lvert C_{\overline{G}}(g_{38j})\rvert$		15
(k,m)	$\lvert C_{H_k}(g_{38km})\rvert$	
$(1,1)$	15	1
m_{38j}		1024

$\mathcal{F}_{39}$

g_{39}		$g_{39,1}$	$g_{39,2}$
$o(g_{39j})$		16	16
$\lvert C_{\overline{G}}(g_{39j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{39km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{39j}		512	512

$\mathcal{F}_{40}$

g_{40}		$g_{40,1}$	$g_{40,2}$
$o(g_{40j})$		16	16
$\lvert C_{\overline{G}}(g_{40j})\rvert$		32	32
(k,m)	$\lvert C_{H_k}(g_{40km})\rvert$		
$(1,1)$	16	1	1
$(2,1)$	16	1	-1
m_{40j}		512	512

$\mathcal{F}_{41}$

g_{41}		$g_{41,1}$
$o(g_{41j})$		18
$\lvert C_{\overline{G}}(g_{41j})\rvert$		18
(k,m)	$\lvert C_{H_k}(g_{41km})\rvert$	
$(1,1)$	18	1
m_{41j}		1024

$\mathcal{F}_{42}$

g_{42}		$g_{42,1}$	$g_{42,2}$
$o(g_{42j})$		24	24
$\lvert C_{\overline{G}}(g_{42j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{42km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{42j}		512	512

$\mathcal{F}_{43}$

g_{43}		$g_{43,1}$	$g_{43,2}$
$o(g_{43j})$		24	24
$\lvert C_{\overline{G}}(g_{43j})\rvert$		48	48
(k,m)	$\lvert C_{H_k}(g_{43km})\rvert$		
$(1,1)$	24	1	1
$(2,1)$	24	1	-1
m_{43j}		512	512

5. The character table of $\overline{G} = 2^{10}{:}(U_5(2){:}2)$

From Sections 2, 3, 4 and the Appendix of Basheer [1], we have

- the conjugacy classes of $\overline{G}$ (Table 1),
- the fusions of classes of the inertia factors H_2 and H_3 into classes of G (Tables 3 and 4 respectively),
- the character tables of the inertia factors H_1, H_2 and H_3 (Tables 11.14, 11.15 and 11.16 of [1] respectively),
- the Fischer matrices of $\overline{G}$ (see Section).

By [1] or [2], it follows that the full character table of $\overline{G}$ can be constructed easily. One can apply similar arguments used in [2, 3] to obtain the character table of $\overline{G}$, which is a 109×109 $\mathbb{C}$-valued matrix, partitioned into 129 parts $\mathcal{K}_{ik}\mathcal{F}_{ik}$, where $1 \leq i \leq 43,\ 1 \leq k \leq 3$. The full character table of $\overline{G}$, in the format of Clifford-Fischer theory appears as Table 11.17 of [1].

Acknowledgments. The first author would like to thank his supervisor (second author) for his advice and support. The financial support from the National Research Foundation (NRF) of South Africa and the North-West University are also acknowledged.

References

[1] Basheer, A.B.M., *Clifford-Fischer Theory Applied to Certain Groups Associated with Symplectic, Unitary and Thompson Groups*, PhD Thesis, University of KwaZulu-Natal, Pietermaitzburg, 2012.

[2] Basheer, A.B.M., Moori, J., *Fischer matrices of Dempwolff group* $2^{5\cdot}GL(5,2)$, International Journal of Group Theory, 1 (4) (2012), 43-63.

[3] Basheer, A.B.M., Moori, J., *On the non-split extension group* $2^{6\cdot}Sp(6,2)$, Bulletin of the Iranian Mathematical Society, 39 (6) (2013), 1189-1212.

[4] Basheer, A.B.M., Moori, J., *On the non-split extension* $2^{2n\cdot}Sp(2n,2)$, Bulletin of the Iranian Mathematical Society, 41 No. 2 (2015), 499-518.

[5] Basheer, A.B.M., Moori, J., *On a maximal subgroup of the Thompson simple group*, Mathematical Communications, 20 (2015), 201 - 218.

[6] Basheer, A.B.M., Moori, J., *A survey on Clifford-Fischer theory*, London Mathematical Society Lecture Notes Series, 422, published by Cambridge University Press, 2015, 160-172.

[7] Bosma, W., Cannon, J.J., *Handbook of Magma Functions*, Department of Mathematics, University of Sydeny, November 1994.

[8] Conway, J.H., Curtis, R.T., Norton, S.P., Parker, R.A., Wilson, R.A., *Atlas of Finite Groups*, Clarendon Press, Oxford, 1985.

[9] Gorenstein, D., *Finite Groups*, Harper and Row Publishers, New York, 1968.

[10] The GAP Group, *GAP – Groups, Algorithms, and Programming, Version 4.4.10*; 2007. (http://www.gap-system.org)

[11] Grove, L.C., *Classical Groups and Geometric Algebra*, American Mathematical Society, Graduate Text in Mathematics, 39 2002.

[12] Liebeck, M.W., *The affine permutation groups of rank three*, Proceedings of the London Mathematical Society, 54 (1987), 477-516.

[13] Maxima, a Computer Algebra System. Version 5.18.1; 2009. (http://maxima.sourceforge.net)

[14] Moori, J., *On the Groups* G^+ *and* $\overline{G}$ *of the form* $2^{10}{:}M_{22}$ *and* $2^{10}{:}\overline{M}_{22}$, PhD Thesis, University of Birmingham, 1975.

[15] Moori, J., *On certain groups associated with the smallest Fischer group*, J. London Math. Soc., 2 (1981), 61-67.

[16] Schiffer, U., *Cliffordmatrizen*, Diplomarbeit, Lehrstul D Fur Matematik, RWTH, Aachen, 1995.

[17] Whitely, N.S., *Fischer Matrices and Character Tables of Group Extensions*, MSc Thesis, University of Natal, Pietermaritzburg, 1993.

[18] Wilson, R.A. et al., *Atlas of finite group representations*, (http://brauer.maths.qmul.ac.uk/Atlas/v3/)

Accepted: 11.11.2016