THE STRUCTURE OF FINITE GROUPS WITH c^*-NORMAL SUBGROUPS

Ping Kang[1]

Department of Mathematics
Tianjin Polytechnic University
Tianjin, 300387
People's Republic of China
e-mail: jnkp1980@163.com

Abstract. Let H be a subgroup of a finite group G. H is said to be c^*-normal in G if there exists a normal subgroup K of G such that $G = HK$ and $H \cap K$ is s-quasinormally embedded in G. We fix in every non-cyclic Sylow subgroup P of G some subgroup D satisfying $1 < |D| < |P|$ and study the structure of G under the assumption that every subgroup H of P with $|H| = |D|$ is c^*-normal in G. Some recent results are generalized and unified.

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1. Introduction

All groups considered in this paper are finite. We use conventional notions and notation in [3]. G always means a group, $|G|$ denotes the order of G and $\pi(G)$ denotes the set of all primes dividing $|G|$. Let $\mathscr{F}$ be a class of groups. We call $\mathscr{F}$ a formation, provided that (1) if $G \in \mathscr{F}$ and $H \trianglelefteq G$, then $G/H \in \mathscr{F}$, and (2) if G/M and G/N are in $\mathscr{F}$, then $G/(M \cap N)$ is in $\mathscr{F}$ for any normal subgroups M, N of G. A formation $\mathscr{F}$ is said to be saturated if $G/\Phi(G) \in \mathscr{F}$ implies that $G \in \mathscr{F}$. In this paper, $\mathscr{U}$ will denote the class of all supersolvable groups. Clearly, $\mathscr{U}$ is a saturated formation.

A subgroup H of G is called s-quasinormal (or s-permutable, π-quasinormal) in G provided H permutes with all Sylow subgroups of G, i.e, $HP = PH$ for any Sylow subgroup P of G. This concept was introduced by Kegel in [6] and has been studied extensively by Deskins [2] and Schmidt [14]. More recently, Ballester-Bolinches and Pedraza-Aquilera [1] generalized s-quasinormal subgroups to s-quasinormally embedded subgroups. A subgroup H is said to

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be an s-quasinormally embedded in G if for each prime p dividing the order of H, a Sylow p-subgroup of H is also a Sylow p-subgroup of some s-quasinormal subgroup of G. Clearly, every s-quasinormal subgroup of G is an s-quasinormally embedded subgroup of G, but the converse does not hold. The authors of [18] obtained many interesting results. For example, they prove:

Theorem 1.1 *Let p be the smallest prime dividing the order of a group G and P a Sylow p-subgroup of G. Then G is p-nilpotent if and only if there is a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is s-quasinormally embedded in G.*

As another generalization of the normality, Wang [15] introduced the following concept: A subgroup H of G is called c-normal in G if there is a normal subgroup K such that $G = HK$ and $H \cap K \leq H_G$, where H_G is the normal core of H in G. Many authors consider some subgroups of a Sylow subgroup of a group when investigating the structure of G, such as in [5], [7]-[8], [17], etc. In [5], Jaraden and Skiba provide the following result.

Theorem 1.2 *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$, the class of all supersolvable groups and G a group with E as a normal subgroup of G such that $G/E \in \mathscr{F}$. Suppose that every non-cyclic Sylow subgroup P of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c-normal in G, where $F^*(E)$ is the generalized Fitting subgroup of E. Then $G \in \mathscr{F}$.*

There are examples to show s-quasinormally embedded and c-normal are two different properties of subgroups in [16]. There is no inclusion-relationship between the two concepts. Hence it is meaningful to unify and generalize the two concepts and related results.

In [16], the authors introduce a new subgroup embedding property called c^*-normal which is a generalization of both c-normality and s-quasinormal embedding.

Definition 1.1 A subgroup H of a group G is called c^*-normal in G if there exists a normal subgroup K of G such that $G = HK$ and $H \cap K$ is s-quasinormally embedded in G.

It is clear from Definition 1.1 that each of c-normality and s-quasinormal embedding implies weakly c^*-normal. But the converses do not hold.

The aim of this article is to unify and improve above Theorems using c^*-normal subgroups. The following result can unify and generalize some related ones including the above two theorems.

Theorem 1.3 (i.e., Theorem 3.4) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$, the class of all supersolvable groups and G a group with E as a normal subgroup of G such that $G/E \in \mathscr{F}$. Suppose that every non-cyclic Sylow subgroup P of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G, where $F^*(E)$ is the generalized Fitting subgroup of E. Then $G \in \mathscr{F}$.*

2. Basic definitions and preliminary results

In this section, we collect some known results that are useful later.

Lemma 2.1. ([16]) *Let H be a subgroup of a group G.*

(i) *If H is c^*-normal in G and $H \leq M \leq G$, then H is c^*-normal in M;*

(ii) *Let $N \lhd G$ and $N \leq H$. Then H is c^*-normal in G if and only if H/N is c^*-normal in G/N;*

(iii) *Let p be a set of primes, H a p-subgroup of G and N a normal π'-subgroup of G. If H is c^*-normal in G, then HN/N is c^*-normal in G/N.*

By Corollary 3.2 in [16], we have the following result.

Lemma 2.2. ([16]) *Let p be the smallest prime dividing the order of a group G and P be a Sylow p-subgroup of G. If all maximal subgroups of P are c^*-normal in G. Then G is p-nilpotent.*

Lemma 2.3. *Suppose that H is a p-subgroup for some prime p and H is not s-quasinormally embedded in G. Assume that H is c^*-normal in G. Then G has a normal subgroup M such that $|G : M| = p$ and $G = HM$.*

Proof. By the hypothesis, G has a normal subgroup T such that $HT = G$ and $T \cap H < H$. Hence G has a proper normal subgroup K such that $T \leq K$. Since G/K is a p-group, G has a normal maximal subgroup M such that $HM = G$ and $|G : M| = p$. ∎

Lemma 2.4.([9]) *Let H be a nilpotent subgroup of a group G. Then the following statements are equivalent:*

(i) *H is s-quasinormal in G;*

(ii) *$H \leq F(G)$ and H is s-quasinormally embedded in G.*

Lemma 2.5. *Suppose that $H \leq O_p(G)$ and that H is c^*-normal in G. Then H is weakly s-permutable in G.*

Proof. By the hypothesis, G has a normal subgroup B such that $HB = G$ and $H \cap B$ is s-quasinormally embedded in G. Note that $H \cap B \leq H \leq O_p(G)$, then by Lemma 2.4 $H \cap B$ is s-quasinormal in G, and thus $H \cap B \le H_{sG}$. Hence H is weakly s-permutable in G. ∎

By Lemma 2.11 of [13] and Lemma 2.5, we have the following.

Lemma 2.6. *Let N be an elementary abelian normal p-subgroup of a group G. If there exists a subgroup D in N such that $1 < |D| < |N|$ and every subgroup H of N with $|H| = |D|$ is c^*-normal in G, then there exists a maximal subgroup M of N such that M is normal in G.*

By Lemma 2.12 of [13] and Lemma 2.5, we have the following.

Lemma 2.7. *Let $\mathscr{F}$ be a saturated formation containing all nilpotent groups and let G be a group with solvable $\mathscr{F}$-residual $P = G^{\mathscr{F}}$. Suppose that every maximal subgroup of G not containing P belongs to $\mathscr{F}$. Then P is a p-group for some prime p. In addition, if every cyclic subgroup of P with prime order or order 4 (if $p = 2$ and P is non-abelian) not having a supersolvable supplement in G is c^*-normal in G, then $|P/\Phi(P)| = p$.*

The generalized Fitting subgroup $F^*(G)$ of G is the unique maximal normal quasinilpotent subgroup of G. Its definition and important properties can be found in [4, X, 13]. We would like to give the following basic facts we will use in our proof.

Lemma 2.8. ([4, X, 13]) *Let G be a group and M a subgroup of G.*

(i) *If M is normal in G, then $F^*(M) \le F^*(G)$;*

(ii) *$F^*(G) \ne 1$ if $G \ne 1$; in fact, $F^*(G)/F(G) = Soc(F(G)C_G(F(G))/F(G))$;*

(iii) *$F^*(F^*(G)) = F^*(G) \ge F(G)$; if $F^*(G)$ is solvable, then $F^*(G) = F(G)$.*

Lemma 2.9. ([13]) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$, the class of all supersolvable groups and G a group with E as a normal subgroup of G such that $G/E \in \mathscr{F}$. Suppose that every non-cyclic Sylow subgroup P of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is weakly s-permutable in G, where $F^*(E)$ is the generalized Fitting subgroup of E. Then $G \in \mathscr{F}$.*

3. Main results

In this section, we will prove our main results.

Theorem 3.1 *Let p be the smallest prime dividing the order of a group G and P be a Sylow p-subgroup of G. If P has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G. Then G is p-nilpotent.*

Proof. Suppose that the theorem is false and let G be a counterexample of minimal order. We will derive a contradiction in several steps.

Step 1. $O_{p'}(G) = 1$.

If $O_{p'}(G) \neq 1$, then by Lemma 2.1, $G/O_{p'}(G)$ satisfies the hypotheses of the theorem. Thus $G/O_{p'}(G)$ is p-nilpotent by the choice of G. Then G is p-nilpotent, a contradiction.

Step 2. $|D| > p$.

Suppose that $|D| = p$. Then by Lemma 2.1, G is a minimal non-p-nilpotent group, so $G = [P]Q$, where $P \in \mathrm{Syl}_p(G)$ and $Q \in \mathrm{Syl}_q(G)$, $p \neq q$. Set $\Phi = \Phi(P)$ and let X/Φ be subgroup of P/Φ of order p, $x \in X \backslash \Phi$ and $L = \langle x \rangle$. Then L is order p or 4. L is c^*-normal in G. Lemma 2.7 implies that $|P/\Phi| = p$, it follows that G is p-nilpotent, a contradiction.

Step 3. $|P : D| > p$.

By Lemma 2.3.

Step 4. If $N \leq P$ and N is minimal normal in G, then $|N| \leq |D|$.

Suppose that $|N| > |D|$. Since $N \leq O_p(G)$, N is elementary abelian. By Lemma 2.6, N has a maximal subgroup which is normal in G, contrary to the minimality of N.

Step 5. Suppose that $N \leq P$ and N is minimal normal in G. Then G/N is p-nilpotent.

If $|N| < |D|$, G/N satisfies the hypotheses of the theorem by Lemma 2.1. Thus G/N is p-nilpotent by the minimal choice of G. So we may suppose that $|N| = |D|$ by Step 4. Let $N \leq K \leq P$ such that $|K/N| = p$. By Step 2, N is non-cyclic, so K is also non-cyclic, it follows that K has a maximal subgroup $L \neq N$ and $K = LN$. So L is c^*-normal in G (note that $|L| = |D|$), it follows that $K/N = LN/N$ is c^*-normal in G/N. If P/N is abelian, then G/N satisfies hypothesis. Next suppose that that P/N is a non-abelian 2-group. So every subgroup of P of order $2|D|$ is c^*-normal in G. In this case one can show as above that every subgroup X of P containing N and such that $|X : N| = 4$ is c^*-normal in G. Therefore G/N also satisfies the hypothesis. By the minimal choice of G, G/N is p-nilpotent.

Step 6. $O_p(G) = 1$.

Suppose that $O_p(G) \neq 1$. Take a minimal normal subgroup N of G contained in $O_p(G)$. By Step 5, G/N is p-nilpotent. It is easy to see that N is the unique minimal normal subgroup of G contained in $O_p(G)$. Furthermore, $O_p(G) \cap \Phi(G) = 1$. Hence $O_p(G)$ is an elementary abelian p-group. On the other hand, G has a maximal subgroup M such that $G = MN$ and $M \cap N = 1$. It is easy to deduce that $O_p(G) \cap M = 1$, $N = O_p(G)$ and $M \cong G/N$ is p-nilpotent. Then G can be written as $G = N(M \cap P)M_{p'}$, where $M_{p'}$ is the normal p-complement of M. Pick a maximal subgroup S of $M_p = P \cap M$. Then $NSM_{p'}$ is a subgroup of G with index p. Since p is the minimal prime in $\pi(G)$, we know that $NSM_{p'}$ is normal in G. Now by Step 3 and the induction, we have $NSM_{p'}$ is p-nilpotent. Therefore, G is p-nilpotent, a contradiction. Thus $O_p(G) = 1$.

Step 7. The minimal normal subgroup L of G is not p-nilpotent.

If L is p-nilpotent, then it follows from the fact that $L_{p'}$ char $L \lhd G$ that $L_{p'} \leq O_{p'}(G) = 1$. Thus L is a p-group. Whence $L \leq O_p(G) = 1$ by Step 6, a contradiction.

Step 8. G is a non-abelian simple group.

Suppose that G is not a simple group. Take a minimal normal subgroup L of G. Then $L < G$. If $|L|_p > |D|$, then L is p-nilpotent by the minimal choice of G, contrary to Step 7. If $|L|_p \leq |D|$. Take $P_* \geq L \cap P$ such that $|P_*| = p|D|$. Hence P_* is a Sylow p-subgroup of P_*L. Since every maximal subgroup of P_* is of order $|D|$, every maximal subgroup of P_* is weakly c^*-normal in G by hypotheses, thus in P_*L by Lemma 2.1. Now applying Lemma 2.2, we get P_*L is p-nilpotent. Therefore, L is p-nilpotent, contrary to Step 7.

Step 9. All subgroups of P of order $|D|$ and $2|D|$ (if P is a non-abelian 2-group and $|P : D| > 2$) are s-quasinormally embedded in G.

Let $H \leq P$ and $|H| = |D|$ or $2|D|$. If H is not s-quasinormally embedded in G, by Lemma 2.3, there is a normal subgroup M of G such that $|G : M| = p$. By Step 3, M is p-nilpotent, it follows that G is p-nilpotent, a contradiction.

Step 10. The final contradiction.

By Theorem 1.1, G is p-nilpotent, a contradiction. The contradiction completes the proof. ∎

The following corollary is immediate from Theorem 3.1.

Corollary 3.2 *Suppose that G is a group. If every non-cyclic Sylow subgroup of G has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G, then G has a Sylow tower of supersolvable type.*

Theorem 3.3 *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$, the class of all supersolvable groups and G a group with E as a normal subgroup of G such that $G/E \in \mathscr{F}$. Suppose that every non-cyclic Sylow subgroup of E has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G. Then $G \in \mathscr{F}$.*

Proof. Suppose that P is a non-cyclic Sylow p-subgroup of E, $\forall p \in \pi(E)$. Since P has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G by hypotheses, thus in E by Lemma 2.1. Applying Corollary 3.2, we conclude that E has a Sylow tower of supersolvable type. Let q be the maximal prime divisor of $|E|$ and $Q \in \mathrm{Syl}_q(E)$. Then $Q \trianglelefteq G$. Since $(G/Q, E/Q)$ satisfies the hypotheses of the theorem, by induction, $G/Q \in \mathscr{F}$. For any subgroup H of Q with $|H| = |D|$, since $Q \leq O_q(G)$, H is weakly s-permutable in G by Lemma 2.5. Since $F^*(Q) = Q$ by Lemma 2.8, we get $G \in \mathscr{F}$ by applying Lemma 2.9. ∎

Theorem 3.4 *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$, the class of all supersolvable groups and G a group with E as a normal subgroup of G such that $G/E \in \mathscr{F}$. Suppose that every non-cyclic Sylow subgroup of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G. Then $G \in \mathscr{F}$.*

Proof. We distinguish two cases:

Case 1. $\mathscr{F} = \mathscr{U}$.

Let G be a minimal counter-example.

Step 1. Every proper normal subgroup N of G containing $F^*(E)$ (if it exists) is supersolvable.

If N is a proper normal subgroup of G containing $F^*(E)$, then $N/N \cap E \cong NE/E$ is supersolvable. By Lemma 2.8 (iii), $F^*(E) = F^*(F^*(E)) \leq F^*(E \cap N) \leq F^*(E)$, so $F^*(E \cap N) = F^*(E)$. For any Sylow subgroup P of $F^*(E \cap N) = F^*(E)$, P has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G by hypotheses, thus in N by Lemma 2.1. So N and $N \cap H$ satisfy the hypotheses of the theorem, the minimal choice of G implies that N is supersolvable.

Step 2. $E = G$.

If $E < G$, then $E \in \mathscr{U}$ by Step 1. Hence $F^*(E) = F(E)$ by Lemma 2.8. It follows that every Sylow subgroup of $F^*(E)$ is normal in G. By Lemma 2.5, every non-cyclic Sylow subgroup of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is weakly s-permutable in G. Applying Lemma 2.9 for the special case $\mathscr{F} = \mathscr{U}$, $G \in \mathscr{U}$, a contradiction.

Step 3. $F^*(G) = F(G) < G$.

If $F^*(G) = G$, then $G \in \mathscr{F}$ by Theorem 3.3, contrary to the choice of G. So $F^*(G) < G$. By Step 1, $F^*(G) \in \mathscr{U}$ and $F^*(G) = F(G)$ by Lemma 2.8.

Step 4. The final contradiction.

Since $F^*(G) = F(G)$, each non-cyclic Sylow subgroup of $F^*(G)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is weakly s-permutable in G by Lemma 2.5. Applying Lemma 2.9, $G \in \mathscr{U}$, a contradiction.

Case 2. $\mathscr{F} \neq \mathscr{U}$.

By hypotheses, every non-cyclic Sylow subgroup of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is c^*-normal in G, thus in E Lemma 2.1. Applying Case 1, $E \in \mathscr{U}$. Then $F^*(E) = F(E)$ by Lemma 2.8. It follows that each Sylow subgroup of $F^*(E)$ is normal in G. By Lemma 2.5, each non-cyclic Sylow subgroup of $F^*(E)$ has a subgroup D such that $1 < |D| < |P|$ and every subgroup H of P with order $|H| = |D|$ or with order $2|D|$ (if P is a nonabelian 2-group and $|P : D| > 2$) is weakly s-permutable in G. Applying Lemma 2.9, $G \in \mathscr{F}$. These complete the proof of the theorem. ■

The following corollaries are immediate from Theorem 3.4.

Corollary 3.5 (17, Theorem 3.1) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$. Suppose that G is a group with a normal subgroup H such that $G/H \in \mathscr{F}$. Then $G \in \mathscr{F}$ if and only if every maximal subgroup of any Sylow subgroup of $F^*(H)$ is c-normal in G.*

Corollary 3.6 (17, Theorem 3.2) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$. Suppose that G is a group with a normal subgroup H such that $G/H \in \mathscr{F}$. Then $G \in \mathscr{F}$ if and only if every cyclic subgroup of any Sylow subgroup of $F^*(H)$ of prime order or order 4 is c-normal in G.*

Corollary 3.7 (12, Theorem 1.1) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$. Suppose that G is a group with a normal subgroup E such that $G/E \in \mathscr{F}$. Then $G \in \mathscr{F}$ if and only if every maximal subgroup of any Sylow subgroup of $F^*(E)$ is π-quasinormally embedded in G.*

Corollary 3.8 (12, Theorem 1.2) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$. Suppose that G is a group with a normal subgroup E such that $G/E \in \mathscr{F}$. Then $G \in \mathscr{F}$ if and only if every cyclic subgroup of any Sylow subgroup of $F^*(E)$ of prime order or order 4 is π-quasinormally embedded in G.*

Corollary 3.9 (10, Theorem 3.4) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$. Suppose that G is a group with a normal subgroup H such that $G/H \in \mathscr{F}$. Then $G \in \mathscr{F}$ if and only if every maximal subgroup of any Sylow subgroup of $F^*(H)$ is s-quasinormal in G.*

Corollary 3.10 (11, Theorem 3.3) *Let $\mathscr{F}$ be a saturated formation containing $\mathscr{U}$. Suppose that G is a group with a normal subgroup H such that $G/H \in \mathscr{F}$.*

Then $G \in \mathscr{F}$ if and only if every cyclic subgroup of any Sylow subgroup of $F^*(H)$ of prime order or order 4 is s-quasinormal in G.

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