FUZZY INTERIOR HYPERIDEALS
IN ORDERED SEMIHYPERGROUPS

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Abstract. We introduce the notions of an interior hyperideal and a fuzzy interior hyperideal of an ordered semihypergroup. Then, we present a characterization of an interior hyperideal in terms of fuzzy interior hyperideals. The notion of an intra-regular ordered semihypergroup is introduced. Then, we show that fuzzy interior hyperideals and fuzzy hyperideals coincide in regular ordered semihypergroups and intra-regular ordered semihypergroups. Finally, we give the concept of a simple ordered semihypergroup and characterize simple ordered semihypergroups by means of fuzzy hyperideals and fuzzy interior hyperideals.

Keywords: ordered semihypergroup, interior hyperideal, fuzzy interior hyperideal, regular ordered semihypergroup, intra-regular ordered semihypergroup, simple ordered semihypergroup.

AMS Mathematics Subject Classification: 06F05, 20N20.

1. Introduction

The notion of algebraic hyperstructures were introduced first by Marty [7] in 1934 as generalizations of algebraic structures and applied to non-commutative groups, so called hypergroups. In 2011, Heidari and Davvaz [4] defined a semihypergroup with a partially order relation called an ordered semihypergroup. Then, Changhai and Davvaz [1] investigated properties of hyperideals in an ordered semihypergroup. After Zadeh [12] defined the notion of a fuzzy set in 1965, Rosenfeld

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applied fuzzification to groups called fuzzy subgroups and then, many researchers considered fuzzification on many algebraic structures, for example, on rings, semigroups, semirings, near-rings, ordered semigroups, semihypergroups and ordered semihypergroups. In [6], the notion of a fuzzy interior ideal of an ordered semigroup was introduced by Kehayopulu and Tsingelis and it was shown that fuzzy interior ideals and fuzzy ideals coincide in a regular ordered semigroup and in an intra-regular ordered semigroup. Then Pibaljommee, Wannatong and Davvaz [9] introduced the notions of fuzzy hyperideals, fuzzy bi-hyperideals and fuzzy quasi-hyperideals of an ordered semihypergroup and proved that in regular ordered semihypergroups, fuzzy bi-hyperideals and fuzzy quasi-hyperideals coincide. Later, Pibaljommee and Davvaz [8] characterized left and right simple ordered semihypergroups, completely regular ordered semihypergroups and strongly regular ordered semihypergroups by means of their fuzzy bi-hyperideals.

In this paper, we generalized the notion of fuzzy interior ideals of an ordered semigroup introduced by Kehayopulu and Tsingelis in [6] to fuzzy interior hyperideals of an ordered semihypergroup.

2. Preliminaries

A hypergroupoid consists of a non-empty set $H$ and a mapping $\circ : H \times H \rightarrow \mathcal{P}^*(H)$ called a hyperoperation, where $\mathcal{P}^*(H)$ denotes the set of all non-empty subsets of $H$ (see, e.g., [2], [3], [11]). We denote by $a \circ b$ the image of the pair $(a, b)$ in $H \times H$.

A hypergroupoid $(H, \circ)$ is called a semihypergroup if it satisfies the associative property, namely,

$$(a \circ b) \circ c = a \circ (b \circ c).$$

For any non-empty subsets $A, B$ of $H$, we denote

$$A \circ B := \bigcup_{a \in A, b \in B} a \circ b.$$

Instead of $\{a\} \circ A$ and $B \circ \{a\}$, we write $a \circ A$ and $B \circ a$, respectively.

Now, we recall the notion of an ordered semihypergroup defined in [4].

**Definition 2.1.** Let $H$ be a non-empty set and $\leq$ be an ordered relation on $H$. The tri-tuple $(H, \circ, \leq)$ is called an ordered semihypergroup if the following conditions are satisfied.

1. $(H, \circ)$ is a semihypergroup.
2. $(H, \leq)$ is a partially ordered set.
3. For every $a, b, c \in H$, $a \leq b$ implies $a \circ c \leq b \circ c$ and $c \circ a \leq c \circ b$, where $a \circ c \leq b \circ c$ means that for every $x \in a \circ c$ there exists $y \in b \circ c$ such that $x \leq y$. 
A non-empty subset $A$ of an ordered semihypergroup $(H, \circ, \leq)$ is called a sub-semihypergroup of $H$ if $(A, \circ, \leq)$ is an ordered semihypergroup.

We note that for every $a, b, c, d, e, f \in H$ with $a \circ b \leq c \circ d$ and $e \leq f$, we obtain

$$a \circ b \circ e \leq c \circ d \circ f.$$ 

For $K \subseteq H$, we denote

$$(K) := \{a \in H \mid a \leq k \text{ for some } k \in K\}.$$ 

**Definition 2.2.** [4] A non-empty subset $A$ of an ordered semihypergroup $(H, \circ, \leq)$ is called a right (resp. left) hyperideal of $H$ if

1. $A \circ H \subseteq A$ (resp. $H \circ A \subseteq A$),
2. for every $a \in H, b \in A$ and $a \leq b$ implies $a \in A$.

If $A$ is both a right hyperideal and a left hyperideal of $H$, then $A$ is called a hyperideal (or two-sided hyperideal) of $H$.

**Definition 2.3.** A subsemihypergroup $A$ of an ordered semihypergroup $(H, \circ, \leq)$ is called an interior hyperideal of $H$ if

1. $H \circ A \circ H \subseteq A$,
2. for every $a \in H, b \in A$ and $a \leq b$ implies $a \in A$.

**Example 1.** Let $H = \{a, b, c, d\}$. Define a hyperoperation $\circ$ on $H$ by the following table:

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
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<td>$a$</td>
<td>${a, b}$</td>
<td>${a, b, c}$</td>
</tr>
</tbody>
</table>

We define an order relation $\leq$ as follows:

$\leq := \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, c), (a, d), (b, d), (c, d)\}.$
The covering relation “≺” and the figure of $H$:

\[ \preceq = \{(a, b), (a, c), (b, d), (c, d)\}, \]

Now, $(H, \circ, \leq)$ is an ordered semihypergroup.

It is easy to see that $\{a\}, \{a, b\}, \{a, b, c\}$ and $H$ are all hyperideals. Moreover, $\{a, c\}$ is an interior hyperideal but not a hyperideal of $H$.

A fuzzy subset [12] of an ordered semihypergroup $(H, \circ, \leq)$ is a function $\mu : H \to [0, 1]$. If $\mu$ satisfies the condition $\min\{\mu(a), \mu(b)\} \leq \inf_{c \in a \circ b} \{\mu(c)\}$ for all $a, b \in H$, then $\mu$ is called a fuzzy subsemihypergroup of $H$.

**Definition 2.4.** [9] Let $(H, \circ, \leq)$ be an ordered semihypergroup. A fuzzy subset $\mu : H \to [0, 1]$ is called a fuzzy right (resp. left) hyperideal of $H$ if

1. for every $a, b \in H$, $a \leq b$ implies $\mu(b) \leq \mu(a)$,

2. for every $a, b \in H$, $\mu(a) \leq \inf_{c \in a \circ b} \{\mu(c)\}$ (resp. $\mu(b) \leq \inf_{c \in a \circ b} \{\mu(c)\}$).

If $\mu$ is both a fuzzy right hyperideal and a fuzzy left hyperideal of $H$, then $\mu$ is called a fuzzy hyperideal of $H$.

**Example 2.** [9] Let $H = \{a, b, c, d, e\}$. Define a hyperoperation $\circ$ on $H$ by the following table:

<table>
<thead>
<tr>
<th></th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
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</thead>
<tbody>
<tr>
<td>$a$</td>
<td>${a, b, d}$</td>
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<td>${a, b, c, d, e}$</td>
</tr>
<tr>
<td>$d$</td>
<td>$a$</td>
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<td>$a$</td>
<td>${a, b, d}$</td>
<td>${a, b, d}$</td>
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<tr>
<td>$e$</td>
<td>$a$</td>
<td>${a, b, d}$</td>
<td>${a, c}$</td>
<td>${a, b, d}$</td>
<td>${a, b, c, d, e}$</td>
</tr>
</tbody>
</table>

We define an order relation $\leq$ as follows:

\[ \leq := \{(a, a), (b, b), (c, c), (d, d), (e, e), (a, c), (a, d), (a, e), (b, d), (b, e), (c, e), (d, e)\}. \]
We give the covering relation "≺" and the figure of $H$:

$$≺ = \{(a, c), (a, d), (b, d), (c, e), (d, e)\},$$

Now, $(H, \circ, \leq)$ is an ordered semihypergroup [9]. We define two fuzzy subsets $\mu$ and $\lambda$ of $H$ as follows:

$$\mu(x) := \begin{cases} 0.7 & \text{if } x = a, b, d \\ 0.3 & \text{if } x = c, e \end{cases}$$
and

$$\lambda(x) := \begin{cases} 0.9 & \text{if } x = a \\ 0.8 & \text{if } x = c \\ 0.5 & \text{if } x = b, d, e \end{cases}$$

We can see that $\mu$ is a fuzzy hyperideal and $\lambda$ is a fuzzy left hyperideal of $H$.

For any fuzzy subset $\mu$ of an ordered semihypergroup $(H, \circ, \leq)$ and $t \in (0, 1]$, the set

$$\mu_t = \{a \in H \mid \mu(a) \geq t\}$$

is called a level subset of $\mu$.

**Theorem 2.5.** [9] Let $(H, \circ, \leq)$ be an ordered semihypergroup and $\mu$ be a fuzzy subset of $H$. Then, $\mu$ is a fuzzy hyperideal of $H$ if and only if for every $t \in [0, 1]$, the non-empty level subset $\mu_t$ is a hyperideal of $H$.

Let $(H, \circ, \leq)$ be an ordered semihypergroup and $\emptyset \neq I \subseteq H$. Then the characteristic function $\mathcal{X}_I : H \to [0, 1]$ of $I$ is defined by

$$\mathcal{X}_I := \begin{cases} 1 & \text{if } x \in I \\ 0 & \text{if } x \notin I \end{cases}$$

**Lemma 2.6.** [9] Let $(H, \circ, \leq)$ be an ordered semihypergroup and $\emptyset \neq I \subseteq H$. Then, $I$ is a hyperideal of $H$ if and only if the characteristic function $\mathcal{X}_I$ is a fuzzy hyperideal of $H$. 
3. Fuzzy interior hyperideals

In this section, we define the concept of a fuzzy interior hyperideal of an ordered semihypergroup and the concept of an intra-regular ordered semihypergroup. Then we show that every fuzzy hyperideal is a fuzzy interior hyperideal. Moreover, in regular ordered semihypergroups and intra-regular ordered semihypergroups fuzzy hyperideals and fuzzy interior hyperideals coincide.

**Definition 3.1.** Let \((H, \circ, \leq)\) be an ordered semihypergroup. A fuzzy subsemihypergroup \(\mu : H \to [0, 1]\) is called a fuzzy interior hyperideal of \(H\) if the following assertions are satisfied:

1. for every \(a, b \in H\), \(a \leq b\) implies \(\mu(b) \leq \mu(a)\),
2. for every \(a, b, d \in H\), \(\mu(a) \leq \inf_{c \in \text{level}} \{\mu(c)\}\).

**Theorem 3.2.** A fuzzy subsemihypergroup \(\mu\) of an ordered semihypergroup \((H, \circ, \leq)\) is a fuzzy interior hyperideal of \(H\) if and only if for every \(t \in [0, 1]\), the non-empty level subset \(\mu_t\) is an interior hyperideal of \(H\).

**Proof.** Assume that \(\mu\) is a fuzzy interior hyperideal of \(H\). Let \(t \in [0, 1]\). Let \(a \in H \circ \mu_t \circ H\). Then \(a \in x \circ c \circ y\) for some \(x, y \in H\), \(c \in \mu_t\). By assumption, \(\inf_{a \in \text{level}} \{\mu(a)\} \geq \mu(c) \geq t\), we have \(\mu(a) \geq t\). It follows that \(H \circ \mu_t \circ H \subseteq \mu_t\). Let \(x, y \in H\) be such that \(x \leq y\). If \(y \in \mu_t\), then \(\mu(y) \geq t\). Since \(\mu\) is a fuzzy interior hyperideal of \(H\), we have \(\mu(x) \geq \mu(y) \geq t\). It follows that \(x \in \mu_t\). Therefore, \(\mu_t\) is an interior hyperideal of \(H\).

Assume that \(\mu_t\) is an interior hyperideal of \(H\) for every \(t \in (0, 1]\). Let \(x, y \in H\) such that \(x \leq y\). If \(\mu(y) := t\), then \(y \in \mu_t\). Since \(\mu_t\) is an interior hyperideal of \(H\) and \(x \leq y\), we have \(x \in \mu_t\). So, \(\mu(x) \geq t = \mu(y)\). Next, we show that \(\mu(a) \leq \inf_{c \in \text{level}} \{\mu(c)\}\) for every \(a, x, y \in H\). Choose \(\mu(a) := s\), then \(a \in \mu_s\). Since \(\mu_s\) is an interior hyperideal of \(H\), we get \(x \circ a \circ y \subseteq \mu_s\). Then for every \(c \in x \circ a \circ y\), we have \(\mu(c) \geq s\) and so \(\mu(a) = s \leq \inf_{c \in \text{level}} \{\mu(c)\}\). Therefore, \(\mu\) is a fuzzy interior hyperideal of \(H\).

**Example 3.** Consider the ordered semihypergroup \((H, \circ, \leq)\) given in Example 1 and define a fuzzy subset \(\mu : H \to [0, 1]\) by:

\[
\mu(x) := \begin{cases} 
0.7 & \text{if } x = a \\
0.5 & \text{if } x = c \\
0.4 & \text{if } x = b \\
0.3 & \text{if } x = d.
\end{cases}
\]

Now, we have \(\{a\}, \{a, c\}, \{a, b, c\}\) and \(H\) are all non-empty level subsets of \(H\) which are interior hyperideals. By Theorem 3.2, \(\mu\) is a fuzzy interior hyperideal of \(H\).
The following result can be directly proved using Theorem 3.2.

**Corollary 3.3.** Let \((H, \circ, \leq)\) be an ordered semihypergroup and \(\emptyset \neq I \subseteq H\). Then, \(I\) is an interior hyperideal of \(H\) if and only if the characteristic function \(X_I\) is a fuzzy interior hyperideal of \(H\).

**Theorem 3.4.** Let \((H, \circ, \leq)\) be an ordered semihypergroup. Then, every fuzzy hyperideal of \(H\) is a fuzzy interior hyperideal of \(H\).

**Proof.** Assume that \(\mu\) is a fuzzy hyperideal of \(H\). Let \(a, b, c \in H\). Since \(\mu\) is a fuzzy left hyperideal of \(H\) and \(\mu\) is a fuzzy right hyperideal of \(H\), we have

\[
\inf_{e \in \text{co}a \circ b} \{\mu(e)\} \geq \inf_{s \in \text{co}b} \{\mu(s)\} \geq \mu(a).
\]

This implies that \(\mu\) is a fuzzy interior hyperideal of \(H\).

**Definition 3.5.** [9] An ordered semihypergroup \((H, \circ, \leq)\) is called *regular* if for each \(a \in H\) there exists \(x \in H\) such that \(a \leq a \circ x \circ a\).

**Theorem 3.6.** Let \((H, \circ, \leq)\) be a regular ordered semihypergroup and \(\mu\) be a fuzzy interior hyperideal of \(H\). Then, \(\mu\) is a fuzzy hyperideal of \(H\).

**Proof.** Assume that \(\mu\) is a fuzzy interior hyperideal of \(H\). Let \(a, b \in H\). Since \(H\) is a regular ordered semihypergroup, there exists \(x \in H\) such that \(a \leq a \circ x \circ a\). Then \(a \circ b \leq a \circ x \circ a \circ b\), that is, for every \(c \in a \circ b\) there exists \(e \in a \circ x \circ a \circ b\) such that \(c \leq e\). Since \(\mu\) is a fuzzy interior hyperideal of \(H\), we have \(\inf_{e \in a \circ b} \{\mu(e)\} \geq \mu(a)\) and \(\inf_{s \in a \circ b} \{\mu(s)\} \geq \mu(a)\) for every \(y \in a \circ x\). Thus,

\[
\inf_{e \in a \circ b} \{\mu(e)\} \geq \mu(e) \geq \inf_{s \in a \circ b} \{\mu(s)\} \geq \mu(a).
\]

This implies that \(\mu\) is a fuzzy right hyperideal of \(H\). In a similar way, we can prove that \(\mu\) is a fuzzy left hyperideal of \(H\).

**Corollary 3.7.** Let \((H, \circ, \leq)\) be a regular ordered semihypergroup and \(\mu\) be a fuzzy subset of \(H\). Then, \(\mu\) is a fuzzy hyperideal of \(H\) if and only if \(\mu\) is a fuzzy interior hyperideal of \(H\).

**Proof.** It is an immediate consequence of Theorem 3.4 and Theorem 3.6.

**Definition 3.8.** An ordered semihypergroup \((H, \circ, \leq)\) is called intra-regular if for each \(a \in H\) there exist \(x, y \in H\) such that \(a \leq x \circ a^2 \circ y\).

**Example 4.** Let \(H = \{a, b, c, d, e\}\). Define a hyperoperation \(\circ\) on \(H\) by the following table:

<table>
<thead>
<tr>
<th>(\circ)</th>
<th>(a)</th>
<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>{a, e}</td>
<td>{b, e}</td>
<td>{a, b, c, e}</td>
<td>(d)</td>
<td>(e)</td>
</tr>
<tr>
<td>(b)</td>
<td>{b, e}</td>
<td>{a, e}</td>
<td>{a, b, c, e}</td>
<td>(d)</td>
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<tr>
<td>(c)</td>
<td>{a, b, c, e}</td>
<td>{a, b, c, e}</td>
<td>{a, b, c, e}</td>
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<td>{a, b, c, e}</td>
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<td>(d)</td>
<td>{a, b, c, e}</td>
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<td>{a, b, c, e}</td>
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<td>(e)</td>
<td>(e)</td>
<td>(e)</td>
<td>{a, b, c, e}</td>
<td>(d)</td>
<td>(e)</td>
</tr>
</tbody>
</table>
We define an order relation \( \leq \) as follows:

\[
\leq := \{(a, a), (b, b), (c, c), (d, d), (e, e), (e, a), (e, b), (e, c), (d, c)\}.
\]

We give the covering relation “\( \prec \)” and the figure of \( H \):

\[
\prec := \{(e, a), (e, b), (e, c), (d, c)\},
\]

Now, \((H, \circ, \leq)\) is an ordered semihypergroup [8].

We can show that \((H, \circ, \leq)\) is an intra-regular ordered semihypergroup.

**Theorem 3.9.** Let \((H, \circ, \leq)\) be an intra-regular ordered semihypergroup and \(\mu\) be a fuzzy interior hyperideal of \(H\). Then, \(\mu\) is a fuzzy hyperideal of \(H\).

**Proof.** Assume that \(\mu\) is a fuzzy interior hyperideal of \(H\). Let \(a, b \in H\). Since \(H\) is an intra-regular ordered semihypergroup, there exist \(x, y \in H\) such that \(a \leq x \circ a^2 \circ y\). Then \(a \circ b \leq x \circ a^2 \circ y \circ b\), that is, for every \(d \in a \circ b\) there exists \(c \in x \circ a^2 \circ y \circ b\) such that \(d \leq c\). Since \(\mu\) is a fuzzy interior hyperideal of \(H\), we have \(\inf_{d \in a \circ b} \{\mu(d)\} \geq \mu(c)\) and \(\inf_{s \in x \circ a \circ b} \{\mu(s)\} \geq \mu(a)\) for every \(h \in a \circ y \circ b\).

Therefore,

\[
\inf_{d \in a \circ b} \{\mu(d)\} \geq \mu(c) \geq \inf_{s \in x \circ a \circ b} \{\mu(s)\} \geq \mu(a).
\]

Hence, \(\mu\) is a fuzzy right hyperideal of \(H\). In a similar way, we can prove that \(\mu\) is a fuzzy left hyperideal of \(H\).

As a consequence of Theorem 3.4 and Theorem 3.9, we have the following result.

**Corollary 3.10.** Let \((H, \circ, \leq)\) be an intra-regular ordered semihypergroup and \(\mu\) be a fuzzy subset of \(H\). Then, \(\mu\) is a fuzzy hyperideal of \(H\) if and only if \(\mu\) is a fuzzy interior hyperideal of \(H\).
4. Simple ordered semihypergroups

In this section, we give a characterization of simple ordered semihypergroups by means of fuzzy hyperideals and fuzzy interior hyperideals. First, we apply the concept of a simple semihypergroup defined in [5] to define the concept of a simple ordered semihypergroup.

**Definition 4.1.** An ordered semihypergroup $(H, \circ, \leq)$ is called *simple* if it has no proper hyperideal.

**Example 5.** Let $H = \{a, b, c\}$. Define a hyperoperation $\circ$ on $H$ by the following table:

<table>
<thead>
<tr>
<th>$\circ$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
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<tr>
<td>$a$</td>
<td>$a$</td>
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<td>$b$</td>
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<td>${a, b}$</td>
<td>${a, c}$</td>
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<tr>
<td>$c$</td>
<td>$a$</td>
<td>${a, b}$</td>
<td>$c$</td>
</tr>
</tbody>
</table>

We define an order relation $\leq$ as follows:

$\leq := \{(a, a), (b, b), (c, c), (a, b)\}$.

We give the covering relation “$\prec$” and the figure of $H$:

Now, $(H, \circ, \leq)$ is an ordered semihypergroup [1]. It is easy to see that $(H, \circ, \leq)$ is simple.

Let $(H, \circ, \leq)$ be an ordered semihypergroup, $a \in H$ and $\mu$ be a fuzzy subset of $H$. Define the set $I_a$ as follows:

$I_a = \{b \in H \mid \mu(b) \geq \mu(a)\}$.

By Theorem 2.5, we obtain the following theorem.

**Theorem 4.2.** Let $(H, \circ, \leq)$ be an ordered semihypergroup and $\mu$ be a fuzzy hyper-ideal of $H$. Then,
(1) $I_a$ is a right hyperideal of $H$ for every $a \in H$,

(2) $I_a$ is a left hyperideal of $H$ for every $a \in H$,

(3) $I_a$ is a hyperideal of $H$ for every $a \in H$.

**Theorem 4.3.** An ordered semihypergroup $(H, \circ, \leq)$ is a simple ordered semihypergroup if and only if every fuzzy hyperideal of $H$ is a constant function.

**Proof.** Assume that $H$ is a simple ordered semihypergroup. Let $\mu$ be a fuzzy hyperideal of $H$ and $a, b \in H$. By Theorem 4.2, we obtain $I_a$ is a hyperideal of $H$. By assumption, this implies that $I_a = H$. Then $b \in I_a$, that is, $\mu(b) \geq \mu(a)$. On the other hand, we get $\mu(a) \geq \mu(b)$. Therefore, $\mu(a) = \mu(b)$.

Conversely, we assume that for every fuzzy hyperideal of $H$ is a constant function. Let $I$ be a hyperideal of $H$ and $x \in H$. By Lemma 2.6, we obtain the characteristic function $X_I$ is a fuzzy hyperideal of $H$. By assumption, $X_I$ is a constant function, that is, $X_I(x) = X_I(b)$ for every $b \in H$. Let $a \in I$. Then $X_I(x) = X_I(a) = 1$, and so $x \in I$. Therefore, $H \subseteq I$.

**Lemma 4.4.** An ordered semihypergroup $(H, \circ, \leq)$ is a simple ordered semihypergroup if and only if for every $a \in H$, $H = (H \circ a \circ H]$.

**Proof.** Assume that $H$ is a simple ordered semihypergroup. We show that for every $a \in H$, $(H \circ a \circ H]$ is a hyperideal of $H$. Let $x \in H$ and $y \in (H \circ a \circ H]$. Then, we can write $y \leq k$ for some $k \in H \circ a \circ H$, so $x \circ y \leq x \circ k$. Since $x \circ k \subseteq H \circ a \circ H$, we have $x \circ y \subseteq (H \circ a \circ H]$. Next, let $x \in H$ and $y \in (H \circ a \circ H]$ be such that $x \leq y$. Then, we can write $x \leq y \leq k$ for some $k \in H \circ a \circ H$ so $x \in (H \circ a \circ H]$. Therefore, $(H \circ a \circ H]$ is a left hyperideal of $H$. In a similar way, we can prove that $(H \circ a \circ H]$ is a right hyperideal of $H$. Hence, $(H \circ a \circ H]$ is a hyperideal of $H$.

Conversely, let $I$ be a hyperideal of $H$ and $a \in I$. Then,

$$H = (H \circ a \circ H] \subseteq (H \circ I \circ H] \subseteq I.$$  

This shows that $H$ is simple.  

**Theorem 4.5.** Let $(H, \circ, \leq)$ be an intra-regular ordered semihypergroup. Then, $H$ is a simple ordered semihypergroup.

**Proof.** Let $a \in H$. Then $a \leq x \circ a^2 \circ y = (x \circ a) \circ a \circ y$ for some $x, y \in H$. This follows that $a \in (H \circ a \circ H]$. By Lemma 4.4, $H$ is simple.

**Theorem 4.6.** Let $(H, \circ, \leq)$ be an ordered semihypergroup. Then, $H$ is a simple ordered semihypergroup if and only if every fuzzy interior hyperideal of $H$ is a constant function.

**Proof.** Assume that $H$ is a simple ordered semihypergroup. Let $\mu$ be a fuzzy interior hyperideal of $H$ and $a, b \in H$. By Lemma 4.4, we have $H = (H \circ b \circ H]$. So, $a \in (H \circ b \circ H]$. Then there exist $x, y \in H$ such that $a \leq x \circ b \circ y$, i.e., there
exists \( e \in x \circ b \circ y \) such that \( a \leq e \). Since \( \mu \) is a fuzzy interior hyperideal of \( H \), we have \( \mu(a) \geq \mu(e) \) and \( \inf_{s \in x \circ b \circ y} \{ \mu(s) \} \geq \mu(b) \). Thus,
\[
\mu(a) \geq \mu(e) \geq \inf_{s \in x \circ b \circ y} \{ \mu(s) \} \geq \mu(b).
\]
Hence, \( \mu(a) \geq \mu(b) \). In a similar way, we can prove that \( \mu(b) \geq \mu(a) \). Therefore, \( \mu(a) = \mu(b) \).

Conversely, assume that every fuzzy interior hyperideal of \( H \) is a constant function. Let \( \mu \) be a fuzzy hyperideal of \( H \). By Theorem 3.4, \( \mu \) is a fuzzy interior hyperideal of \( H \). By assumption, \( \mu \) is a constant function. By Theorem 4.3, \( H \) is a simple ordered semihypergroup.

By Theorem 4.5 and Theorem 4.6, we have the following corollary.

**Corollary 4.7.** Let \((H, \circ, \leq)\) be an intra-regular ordered semihypergroup. Then, every fuzzy interior hyperideal of \( H \) is a constant function.

As a consequence of Theorem 4.3, Lemma 4.4 and Theorem 4.6, we present characterizations of a simple ordered semihypergroup as the following theorem.

**Theorem 4.8.** Let \((H, \circ, \leq)\) be an ordered semihypergroup. Then, the following statements are equivalent:

1. \( H \) is a simple ordered semihypergroup,
2. \( H = (H \circ a \circ H) \) for every \( a \in H \),
3. every fuzzy hyperideal of \( H \) is a constant function,
4. every fuzzy interior hyperideal of \( H \) is a constant function.

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**References**


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