FUZZY INTERIOR HYPERIDEALS IN ORDERED SEMIHYPERGROUPS

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Abstract. We introduce the notions of an interior hyperideal and a fuzzy interior hyperideal of an ordered semihypergroup. Then, we present a characterization of an interior hyperideal in terms of fuzzy interior hyperideals. The notion of an intra-regular ordered semihypergroup is introduced. Then, we show that fuzzy interior hyperideals and fuzzy hyperideals coincide in regular ordered semihypergroups and intra-regular ordered semihypergroups. Finally, we give the concept of a simple ordered semihypergroup and characterize simple ordered semihypergroups by means of fuzzy hyperideals and fuzzy interior hyperideals.

Keywords: ordered semihypergroup, interior hyperideal, fuzzy interior hyperideal, regular ordered semihypergroup, intra-regular ordered semihypergroup, simple ordered semihypergroup.

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1. Introduction

The notion of algebraic hyperstructures were introduced first by Marty [7] in 1934 as generalizations of algebraic structures and applied to non-commutative groups, so called hypergroups. In 2011, Heidari and Davvaz [4] defined a semihypergroup with a partially order ralation called an ordered semihypergroup. Then, Changphas and Davvaz [1] investigated properties of hyperideals in an ordered semihypergroup. After Zadeh [12] defined the notion of a fuzzy set in 1965, Rosenfeld

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[10] applied fuzzification to groups called fuzzy subgroups and then, many researchers considered fuzzification on many algebraic structures, for example, on rings, semigroups, semirings, near-rings, ordered semigroups, semihypergroups and ordered semihypergroups. In [6], the notion of a fuzzy interior ideal of an ordered semigroup was introduced by Kehayopulu and Tsingelis and it was shown that fuzzy interior ideals and fuzzy ideals coincide in a regular ordered semigroup and in an intra-regular ordered semigroup. Then Pibaljommee, Wannatong and Davvaz [9] introduced the notions of fuzzy hyperideals, fuzzy bi-hyperideals and fuzzy quasi-hyperideals of an ordered semihypergroup and proved that in regular ordered semihypergroups, fuzzy bi-hyperideals and fuzzy quasi-hyperideals coincide. Later, Pibaljommee and Davvaz [8] characterized left and right simple ordered semihypergroups, completely regular ordered semihypergroups and strongly regular ordered semihypergroups by means of their fuzzy bi-hyperideals.

In this paper, we generalized the notion of fuzzy interior ideals of an ordered semigroup introduced by Kehayopulu and Tsingelis in [6] to fuzzy interior hyperideals of an ordered semihypergroup.

2. Preliminaries

A *hypergroupoid* consists of a non-empty set H and a mapping $\circ : H \times H \to \mathcal{P}^*(H)$ called a *hyperoperation*, where $\mathcal{P}^*(H)$ denotes the set of all non-empty subsets of H (see, e.g., [2], [3], [11]). We denote by $a \circ b$ the image of the pair (a, b) in $H \times H$.

A hypergroupoid $(H, \circ)$ is called a *semihypergroup* if it satisfies the associative property, namely,

$$(a \circ b) \circ c = a \circ (b \circ c).$$

For any non-empty subsets A, B of H, we denote

$$A \circ B := \bigcup_{a \in A, b \in B} a \circ b.$$

Instead of $\{a\} \circ A$ and $B \circ \{a\}$, we write $a \circ A$ and $B \circ a$, respectively.

Now, we recall the notion of an ordered semihypergroup defined in [4].

Definition 2.1. Let H be a non-empty set and $\leq$ be an ordered relation on H. The tri-tuple $(H, \circ, \leq)$ is called an *ordered semihypergroup* if the following conditions are satisfied.

(1) $(H, \circ)$ is a semihypergroup.

(2) $(H, \leq)$ is a partially ordered set.

(3) For every $a, b, c \in H$, $a \leq b$ implies $a \circ c \leq b \circ c$ and $c \circ a \leq c \circ b$, where $a \circ c \leq b \circ c$ means that for every $x \in a \circ c$ there exists $y \in b \circ c$ such that $x \leq y$.

A non-empty subset A of an ordered semihypergroup $(H, \circ, \leq)$ is called a *subsemihypergroup* of H if $(A, \circ, \leq)$ is an ordered semihypergroup.

We note that for every $a, b, c, d, e, f \in H$ with $a \circ b \leq c \circ d$ and $e \leq f$, we obtain

$$a \circ b \circ e \leq c \circ d \circ f.$$

For $K \subseteq H$, we denote

$$(K] := \{a \in H \mid a \leq k \text{ for some } k \in K\}.$$

Definition 2.2. [4] A non-empty subset A of an ordered semihypergroup $(H, \circ, \leq)$ is called a *right* (*resp. left*) *hyperideal* of H if

(1) $A \circ H \subseteq A$ (resp. $H \circ A \subseteq A$),

(2) for every $a \in H$, $b \in A$ and $a \leq b$ implies $a \in A$.

If A is both a right hyperideal and a left hyperideal of H, then A is called a *hyperideal* (or *two-sided hyperideal*) of H.

Definition 2.3. A subsemihypergroup A of an ordered semihypergroup $(H, \circ, \leq)$ is called an *interior hyperideal* of H if

(1) $H \circ A \circ H \subseteq A$,

(2) for every $a \in H$, $b \in A$ and $a \leq b$ implies $a \in A$.

Example 1. Let $H = \{a, b, c, d\}$. Define a hyperoperation $\circ$ on H by the following table:

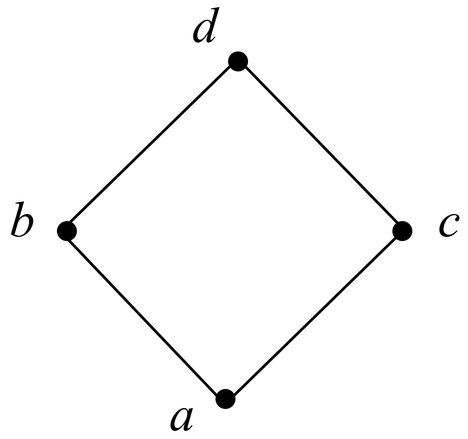
$\circ$	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	$\{a, b\}$
d	a	a	$\{a, b\}$	$\{a, b, c\}$

We define an order relation $\leq$ as follows:

$$\leq := \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, c), (a, d), (b, d), (c, d)\}.$$

The covering relation " $\prec$ " and the figure of H:

$$\prec=\{(a,b),(a,c),(b,d),(c,d)\},$$



Now, $(H,\circ,\leq)$ is an ordered semihypergroup.

It is easy to see that $\{a\},\{a,b\},\{a,b,c\}$ and H are all hyperideals. Moreover, $\{a,c\}$ is an interior hyperideal but not a hyperideal of H.

A fuzzy subset [12] of an ordered semihypergroup $(H,\circ,\leq)$ is a function $\mu: H\to[0,1]$. If μ satisfies the condition $\min\{\mu(a),\mu(b)\}\leq \inf\limits_{c\in a\circ b}\{\mu(c)\}$ for all $a,b\in H$, then μ is called a *fuzzy subsemihypergroup* of H.

Definition 2.4. [9] Let $(H,\circ,\leq)$ be an ordered semihypergroup. A fuzzy subset $\mu: H\to[0,1]$ is called a *fuzzy right* (*resp. left*) *hyperideal* of H if

(1) for every $a,b\in H$, $a\leq b$ implies $\mu(b)\leq\mu(a)$,

(2) for every $a,b\in H$, $\mu(a)\leq\inf\limits_{c\in a\circ b}\{\mu(c)\}$ (resp. $\mu(b)\leq\inf\limits_{c\in a\circ b}\{\mu(c)\}$).

If μ is both a fuzzy right hyperideal and a fuzzy left hyperideal of H, then μ is called a *fuzzy hyperideal* of H.

Example 2. [9] Let $H=\{a,b,c,d,e\}$. Define a hyperoperation $\circ$ on H by the following table:

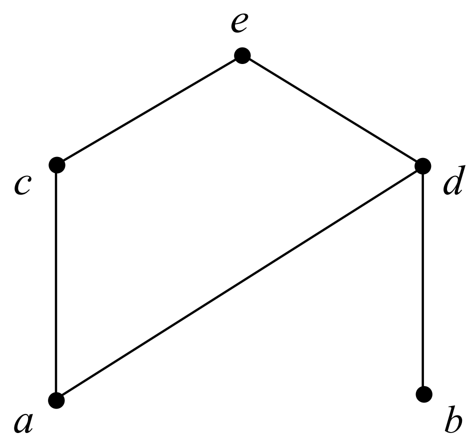
$\circ$	a	b	c	d	e
a	a	$\{a,b,d\}$	a	$\{a,b,d\}$	$\{a,b,d\}$
b	a	b	a	$\{a,b,d\}$	$\{a,b,d\}$
c	a	$\{a,b,d\}$	$\{a,c\}$	$\{a,b,d\}$	$\{a,b,c,d,e\}$
d	a	$\{a,b,d\}$	a	$\{a,b,d\}$	$\{a,b,d\}$
e	a	$\{a,b,d\}$	$\{a,c\}$	$\{a,b,d\}$	$\{a,b,c,d,e\}$

We define an order relation $\leq$ as follows:

$$\leq:=\{(a,a),(b,b),(c,c),(d,d),(e,e),(a,c),(a,d),(a,e),(b,d),(b,e),(c,e),(d,e)\}.$$

We give the covering relation " $\prec$ " and the figure of H:

$$\prec = \{(a,c),(a,d),(b,d),(c,e),(d,e)\},$$



Now, $(H,\circ,\leq)$ is an ordered semihypergroup [9]. We define two fuzzy subsets μ and λ of H as follows:

$$\mu(x) := \begin{cases} 0.7 & \text{if } x = a,b,d \\ 0.3 & \text{if } x = c,e \end{cases} \quad \text{and} \quad \lambda(x) := \begin{cases} 0.9 & \text{if } x = a \\ 0.8 & \text{if } x = c \\ 0.5 & \text{if } x = b,d,e. \end{cases}$$

We can see that μ is a fuzzy hyperideal and λ is a fuzzy left hyperideal of H.

For any fuzzy subset μ of an ordered semihypergroup $(H,\circ,\leq)$ and $t \in (0,1]$, the set

$$\mu_t = \{a \in H \mid \mu(a) \geq t\}$$

is called a *level subset* of μ.

Theorem 2.5. [9] *Let $(H,\circ,\leq)$ be an ordered semihypergroup and μ be a fuzzy subset of H. Then, μ is a fuzzy hyperideal of H if and only if for every $t \in [0,1]$, the non-empty level subset μ_t is a hyperideal of H.*

Let $(H,\circ,\leq)$ be an ordered semihypergroup and $\emptyset \neq I \subseteq H$. Then the *characteristic function* $\mathcal{X}_I : H \to [0,1]$ of I is defined by

$$\mathcal{X}_I := \begin{cases} 1 & \text{if } x \in I \\ 0 & \text{if } x \notin I. \end{cases}$$

Lemma 2.6. [9] *Let $(H,\circ,\leq)$ be an ordered semihypergroup and $\emptyset \neq I \subseteq H$. Then, I is a hyperideal of H if and only if the characteristic function $\mathcal{X}_I$ is a fuzzy hyperideal of H.*

3. Fuzzy interior hyperideals

In this section, we define the concept of a fuzzy interior hyperideal of an ordered semihypergroup and the concept of an intra-regular ordered semihypergroup. Then we show that every fuzzy hyperideal is a fuzzy interior hyperideal. Moreover, in regular ordered semihypergroups and intra-regular ordered semihypergroups fuzzy hyperideals and fuzzy interior hyperideals coincide.

Definition 3.1. Let $(H, \circ, \leq)$ be an ordered semihypergroup. A fuzzy subsemihypergroup $\mu : H \to [0, 1]$ is called a *fuzzy interior hyperideal* of H if the following assertions are satisfied:

(1) for every $a, b \in H, a \leq b$ implies $\mu(b) \leq \mu(a)$,

(2) for every $a, b, d \in H$, $\mu(a) \leq \inf\limits_{c \in b \circ a \circ d} \{\mu(c)\}$.

Theorem 3.2. *A fuzzy subsemihypergroup μ of an ordered semihygroup $(H, \circ, \leq)$ is a fuzzy interior hyperideal of H if and only if for every $t \in [0, 1]$, the non-empty level subset μ_t is an interior hyperideal of H.*

Proof. Assume that μ is a fuzzy interior hyperideal of H. Let $t \in [0, 1]$. Let $a \in H \circ \mu_t \circ H$. Then $a \in x \circ c \circ y$ for some $x, y \in H$, $c \in \mu_t$. By assumption, $\inf\limits_{a \in x \circ c \circ y} \{\mu(a)\} \geq \mu(c) \geq t$, we have $\mu(a) \geq t$. It follows that $H \circ \mu_t \circ H \subseteq \mu_t$. Let $x, y \in H$ be such that $x \leq y$. If $y \in \mu_t$, then $\mu(y) \geq t$. Since μ is a fuzzy interior hyperideal of H, we have $\mu(x) \geq \mu(y) \geq t$. It follows that $x \in \mu_t$. Therefore, μ_t is an interior hyperideal of H.

Assume that μ_t is an interior hyperideal of H for every $t \in (0, 1]$. Let $x, y \in H$ such that $x \leq y$. If $\mu(y) := t$, then $y \in \mu_t$. Since μ_t is an interior hyperideal of H and $x \leq y$, we have $x \in \mu_t$. So, $\mu(x) \geq t = \mu(y)$. Next, we show that $\mu(a) \leq \inf\limits_{c \in x \circ a \circ y} \{\mu(c)\}$ for every $a, x, y \in H$. Choose $\mu(a) := s$, then $a \in \mu_s$. Since μ_s is an interior hyperideal of H, we get $x \circ a \circ y \subseteq \mu_s$. Then for every $c \in x \circ a \circ y$, we have $\mu(c) \geq s$ and so $\mu(a) = s \leq \inf\limits_{c \in x \circ a \circ y} \{\mu(c)\}$. Therefore, μ is a fuzzy interior hyperideal of H. ∎

Example 3. Consider the ordered semihypergroup $(H, \circ, \leq)$ given in Example 1 and define a fuzzy subset $\mu : H \to [0, 1]$ by:

$$\mu(x) := \begin{cases} 0.7 & \text{if } x = a \\ 0.5 & \text{if } x = c \\ 0.4 & \text{if } x = b \\ 0.3 & \text{if } x = d. \end{cases}$$

Now, we have $\{a\}$, $\{a, c\}$, $\{a, b, c\}$ and H are all non-empty level subsets of H which are interior hyperideals. By Theorem 3.2, μ is a fuzzy interior hyperideal of H.

The following result can be directly proved using Theorem 3.2.

Corollary 3.3. *Let $(H, \circ, \leq)$ be an ordered semihypergroup and $\emptyset \neq I \subseteq H$. Then, I is an interior hyperideal of H if and only if the characteristic function $\mathcal{X}_I$ is a fuzzy interior hyperideal of H.*

Theorem 3.4. *Let $(H, \circ, \leq)$ be an ordered semihypergroup. Then, every fuzzy hyperideal of H is a fuzzy interior hyperideal of H.*

Proof. Assume that μ is a fuzzy hyperideal of H. Let $a, b, c \in H$. Since μ is a fuzzy left hyperideal of H and μ is a fuzzy right hyperideal of H, we have

$$\inf_{e \in c \circ a \circ b}\{\mu(e)\} \geq \inf_{s \in a \circ b}\{\mu(s)\} \geq \mu(a).$$

This implies that μ is a fuzzy interior hyperideal of H. ∎

Definition 3.5. [9] An ordered semihypergroup $(H, \circ, \leq)$ is called *regular* if for each $a \in H$ there exists $x \in H$ such that $a \leq a \circ x \circ a$.

Theorem 3.6. *Let $(H, \circ, \leq)$ be a regular ordered semihypergroup and μ be a fuzzy interior hyperideal of H. Then, μ is a fuzzy hyperideal of H.*

Proof. Assume that μ is a fuzzy interior hyperideal of H. Let $a, b \in H$. Since H is a regular ordered semihypergroup, there exists $x \in H$ such that $a \leq a \circ x \circ a$. Then $a \circ b \leq a \circ x \circ a \circ b$, that is, for every $c \in a \circ b$ there exists $e \in a \circ x \circ a \circ b$ such that $c \leq e$. Since μ is a fuzzy interior hyperideal of H, we have $\inf\limits_{c \in a \circ b}\{\mu(c)\} \geq \mu(e)$ and $\inf\limits_{s \in y \circ a \circ b}\{\mu(s)\} \geq \mu(a)$ for every $y \in a \circ x$. Thus,

$$\inf_{c \in a \circ b}\{\mu(c)\} \geq \mu(e) \geq \inf_{s \in y \circ a \circ b}\{\mu(s)\} \geq \mu(a).$$

This implies that μ is a fuzzy right hyperideal of H. In a similar way, we can prove that μ is a fuzzy left hyperideal of H. ∎

Corollary 3.7. *Let $(H, \circ, \leq)$ be a regular ordered semihypergroup and μ be a fuzzy subset of H. Then, μ is a fuzzy hyperideal of H if and only if μ is a fuzzy interior hyperideal of H.*

Proof. It is an immediate consequence of Theorem 3.4 and Theorem 3.6. ∎

Definition 3.8. *An ordered semihypergroup $(H, \circ, \leq)$ is called intra-regular if for each $a \in H$ there exist $x, y \in H$ such that $a \leq x \circ a^2 \circ y$.*

Example 4. Let $H = \{a, b, c, d, e\}$. Define a hyperoperation $\circ$ on H by the following table:

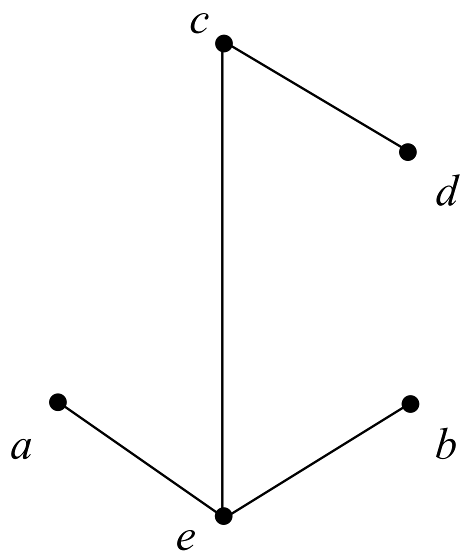
$\circ$	a	b	c	d	e
a	$\{a, e\}$	$\{b, e\}$	$\{a, b, c, e\}$	d	e
b	$\{b, e\}$	$\{a, e\}$	$\{a, b, c, e\}$	d	e
c	$\{a, b, c, e\}$	$\{a, b, c, e\}$	$\{a, b, c, e\}$	$\{a, b, c, e\}$	$\{a, b, c, e\}$
d	$\{a, b, c, e\}$	$\{a, b, c, e\}$	$\{a, b, c, e\}$	$\{a, b, c, e\}$	$\{a, b, c, e\}$
e	e	e	$\{a, b, c, e\}$	d	e

We define an order relation $\leq$ as follows:

$$\leq := \{(a,a),(b,b),(c,c),(d,d),(e,e),(e,a),(e,b),(e,c),(d,c)\}.$$

We give the covering relation " $\prec$ " and the figure of H:

$$\prec = \{(e,a),(e,b),(e,c),(d,c)\},$$



Now, $(H,\circ,\leq)$ is an ordered semihypergroup [8].
We can show that $(H,\circ,\leq)$ is an intra-regular ordered semihypergroup.

Theorem 3.9. *Let $(H,\circ,\leq)$ be an intra-regular ordered semihypergroup and μ be a fuzzy interior hyperideal of H. Then, μ is a fuzzy hyperideal of H.*

Proof. Assume that μ is a fuzzy interior hyperideal of H. Let $a,b \in H$. Since H is an intra-regular ordered semihypergroup, there exist $x,y \in H$ such that $a \leq x \circ a^2 \circ y$. Then $a \circ b \leq x \circ a^2 \circ y \circ b$, that is, for every $d \in a \circ b$ there exists $c \in x \circ a^2 \circ y \circ b$ such that $d \leq c$. Since μ is a fuzzy interior hyperideal of H, we have $\inf\limits_{d \in a\circ b}\{\mu(d)\} \geq \mu(c)$ and $\inf\limits_{s \in x\circ a\circ h}\{\mu(s)\} \geq \mu(a)$ for every $h \in a \circ y \circ b$. Therefore,

$$\inf_{d \in a\circ b}\{\mu(d)\} \geq \mu(c) \geq \inf_{s \in x\circ a\circ h}\{\mu(s)\} \geq \mu(a).$$

Hence, μ is a fuzzy right hyperideal of H. In a similar way, we can prove that μ is a fuzzy left hyperideal of H. ∎

As a consequence of Theorem 3.4 and Theorem 3.9, we have the following result.

Corollary 3.10. *Let $(H,\circ,\leq)$ be an intra-regular ordered semihypergroup and μ be a fuzzy subset of H. Then, μ is a fuzzy hyperideal of H if and only if μ is a fuzzy interior hyperideal of H.*

4. Simple ordered semihypergroups

In this section, we give a characterization of simple ordered semihypergroups by means of fuzzy hyperideals and fuzzy interior hyperideals. First, we apply the concept of a simple semihypergroup defined in [5] to define the concept of a simple ordered semihypergroup.

Definition 4.1. An ordered semihypergroup $(H, \circ, \leq)$ is called *simple* if it has no a proper hyperideal.

Example 5. Let $H = \{a, b, c\}$. Define a hyperoperation $\circ$ on H by the following table:

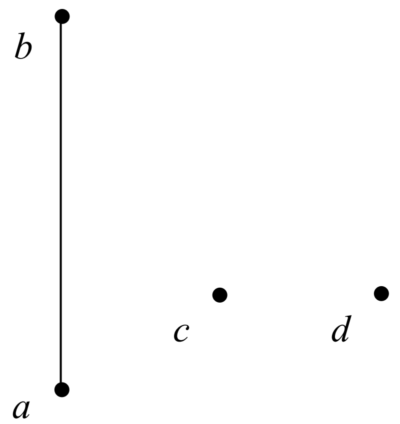
$\circ$	a	b	c
a	a	$\{a, b\}$	$\{a, c\}$
b	a	$\{a, b\}$	$\{a, c\}$
c	a	$\{a, b\}$	c

We define an order relation $\leq$ as follows:

$$\leq := \{(a, a), (b, b), (c, c), (a, b)\}.$$

We give the covering relation " $\prec$ " and the figure of H:

$$\prec = \{(a, b)\},$$



Now, $(H, \circ, \leq)$ is an ordered semihypergroup [1]. It is easy to see that $(H, \circ, \leq)$ is simple.

Let $(H, \circ, \leq)$ be an ordered semihypergroup, $a \in H$ and μ be a fuzzy subset of H. Define the set I_a as follows:

$$I_a = \{b \in H \mid \mu(b) \geq \mu(a)\}.$$

By Theorem 2.5, we obtain the following theorem.

Theorem 4.2. *Let $(H, \circ, \leq)$ be an ordered semihypergroup and μ be a fuzzy hyperideal of H. Then,*

(1) *I_a is a right hyperideal of H for every $a \in H$,*

(2) *I_a is a left hyperideal of H for every $a \in H$,*

(3) *I_a is a hyperideal of H for every $a \in H$.*

Theorem 4.3. *An ordered semihypergroup $(H, \circ, \leq)$ is a simple ordered semihypergroup if and only if every fuzzy hyperideal of H is a constant function.*

Proof. Assume that H is a simple ordered semihypergroup. Let μ be a fuzzy hyperideal of H and $a, b \in H$. By Theorem 4.2, we obtain I_a is a hyperideal of H. By assumption, this implies that $I_a = H$. Then $b \in I_a$, that is, $\mu(b) \geq \mu(a)$. On the other hand, we get $\mu(a) \geq \mu(b)$. Therefore, $\mu(a) = \mu(b)$.

Conversely, we assume that for every fuzzy hyperideal of H is a constant function. Let I be a hyperideal of H and $x \in H$. By Lemma 2.6, we obtain the characteristic function $\mathcal{X}_I$ is a fuzzy hyperideal of H. By assumption, $\mathcal{X}_I$ is a constant function, that is, $\mathcal{X}_I(x) = \mathcal{X}_I(b)$ for every $b \in H$. Let $a \in I$. Then $\mathcal{X}_I(x) = \mathcal{X}_I(a) = 1$, and so $x \in I$. Therefore, $H \subseteq I$. ∎

Lemma 4.4. *An ordered semihypergroup $(H, \circ, \leq)$ is a simple ordered semihypergroup if and only if for every $a \in H$, $H = (H \circ a \circ H]$.*

Proof. Assume that H is a simple ordered semihypergroup. We show that for every $a \in H$, $(H \circ a \circ H]$ is a hyperideal of H. Let $x \in H$ and $y \in (H \circ a \circ H]$. Then, we can write $y \leq k$ for some $k \in H \circ a \circ H$, so $x \circ y \leq x \circ k$. Since $x \circ k \subseteq H \circ a \circ H$, we have $x \circ y \subseteq (H \circ a \circ H]$. Next, let $x \in H$ and $y \in (H \circ a \circ H]$ be such that $x \leq y$. Then, we can write $x \leq y \leq k$ for some $k \in H \circ a \circ H$ so $x \in (H \circ a \circ H]$. Therefore, $(H \circ a \circ H]$ is a left hyperideal of H. In a similar way, we can prove that $(H \circ a \circ H]$ is a right hyperideal of H. Hence, $(H \circ a \circ H]$ is a hyperideal of H.

Conversely, let I be a hyperideal of H and $a \in I$. Then,

$$H = (H \circ a \circ H] \subseteq (H \circ I \circ H] \subseteq I.$$

This shows that H is simple. ∎

Theorem 4.5. *Let $(H, \circ, \leq)$ be an intra-regular ordered semihypergroup. Then, H is a simple ordered semihypergroup.*

Proof. Let $a \in H$. Then $a \leq x \circ a^2 \circ y = (x \circ a) \circ a \circ y$ for some $x, y \in H$. This follows that $a \in (H \circ a \circ H]$. By Lemma 4.4, H is simple. ∎

Theorem 4.6. *Let $(H, \circ, \leq)$ be an ordered semihypergroup. Then, H is a simple ordered semihypergroup if and only if every fuzzy interior hyperideal of H is a constant function.*

Proof. Assume that H is a simple ordered semihypergroup. Let μ be a fuzzy interior hyperideal of H and $a, b \in H$. By Lemma 4.4, we have $H = (H \circ b \circ H]$. So, $a \in (H \circ b \circ H]$. Then there exist $x, y \in H$ such that $a \leq x \circ b \circ y$, i.e, there exists $e \in x \circ b \circ y$ such that $a \leq e$. Since μ is a fuzzy interior hyperideal of H, we have $\mu(a) \geq \mu(e)$ and $\inf\limits_{s \in x \circ b \circ y} \{\mu(s)\} \geq \mu(b)$. Thus,

$$\mu(a) \geq \mu(e) \geq \inf_{s \in x \circ b \circ y} \{\mu(s)\} \geq \mu(b).$$

Hence, $\mu(a) \geq \mu(b)$. In a similar way, we can prove that $\mu(b) \geq \mu(a)$. Therefore, $\mu(a) = \mu(b)$.

Conversely, assume that every fuzzy interior hyperideal of H is a constant function. Let μ be a fuzzy hyperideal of H. By Theorem 3.4, μ is a fuzzy interior hyperideal of H. By assumption, μ is a constant function. By Theorem 4.3, H is a simple ordered semihypergroup. ∎

By Theorem 4.5 and Theorem 4.6, we have the following corollary.

Corollary 4.7. *Let $(H, \circ, \leq)$ be an intra-regular ordered semihypergroup. Then, every fuzzy interior hyperideal of H is a constant function.*

As a consequence of Theorem 4.3, Lemma 4.4 and Theorem 4.6, we present characterizations of a simple ordered semihypergroup as the following theorem.

Theorem 4.8. *Let $(H, \circ, \leq)$ be an ordered semihypergroup. Then, the following statements are equivalent:*

(1) *H is a simple ordered semihypergroup,*

(2) *$H = (H \circ a \circ H]$ for every $a \in H$,*

(3) *every fuzzy hyperideal of H is a constant function,*

(4) *every fuzzy interior hyperideal of H is a constant function.*

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References

[1] Changphas, T., Davvaz, B., *Properties of hyperideals in ordered semihypergroup*, Ital. J. Pure Appl. Math., 33 (2014), 425-432.

[2] Corsini, P., *Prolegomena of hypergroup theory*, Aviani Editore, 1993.

[3] Corsini, P., Leoreanu, V., *Applications of hyperstructure theory*, Kluwer Academic Publishers, Dordrecht, Hardbound, 2003.

[4] Heidari, D., Davvaz, B., *On ordered hyperstructures*, U.P.B. Sci. Bull. Series A, 73 (2011), 85-96.

[5] Jafarabadi, H.M., Sarmin, N.H., Molaei, M.R., *Simple Semi Hypergroups*, Aust. J. Basic Appl. Sci., 5 (4) (2011), 51-55.

[6] Kehayopulu, N., Tsingelis, M., *Fuzzy interior ideals in ordered semigroups*, Lobachevskii J. Math., 21 (2006), 65-71.

[7] Marty, F., *Sur une généralization de la notion de groupe*, 8ème Congrès Math. Scandinaves, Stockholm, 1934, 45-49.

[8] Pibaljommee, B., Davvaz, B., *Characterization of (fuzzy) bi-hyperideals in ordered semihypergroups*, J. Intell. Fuzzy Syst., 28 (2015), 2141-2148.

[9] Pibaljommee, B., Wannatong, K., Davvaz, B., *An investigation on fuzzy hyperideals of ordered semihypergroups*, Quasigroups and Related Systems, 23 (2015), 297-308.

[10] Rosenfeld, A., *Fuzzy groups*, J. Math. Anal. Appl., 35 (1971), 512-517.

[11] Vougiouklis, T., *Hyperstructures and their representations*, Hadronic Press, Florida, 1994.

[12] Zadeh, L.A., *Fuzzy sets*, Inform. Control, 8 (1965), 338-353.

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