INTUITIONISTIC PERMEABLE VALUES IN BCK/BCI-ALGEBRAS

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Abstract. Intuitionistic permeable values in BCK/BCI-algebras are introduced, and several properties are investigated. A relation between an intuitionistic permeable S-value and an intuitionistic permeable I-value is discussed. Conditions for the intuitionistic lower (upper) level set to be S-energetic and I-energetic are considered. Conditions for a couple of numbers to be an intuitionistic permeable S-value are studied.

Keywords: S-energetic subset, I-energetic subset, intuitionistic permeable S-value, intuitionistic permeable I-value.

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1. Introduction

The notions of energetic (resp. right vanished, right stable) subsets and (anti) permeable values in BCK/BCI-algebras are introduced by Jun et al. [2]. Using the notion of (anti) fuzzy subalgebras/ideals of BCK/BCI-algebras, they investigated relations among subalgebras/ideals, energetic subsets, (anti) permeable values, right vanished subsets and right stable subsets.

In this paper, we introduce the notion of intuitionistic permeable values in BCK/BCI-algebras, and investigate several properties. We provide a relation between an intuitionistic permeable S-value and an intuitionistic permeable I-value. We study conditions for the intuitionistic lower (upper) level set to be

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S-energetic and I-energetic. We consider conditions for a couple of numbers to be an intuitionistic permeable S-value. We show that if an intuitionistic fuzzy set $A = (f_A, g_A)$ in a BCK-algebra X is an intuitionistic fuzzy ideal of X, then the nonempty intuitionistic upper level set of $A = (f_A, g_A)$ is a right stable subset of X and the nonempty intuitionistic lower level set of $A = (f_A, g_A)$ is a right vanished subset of X.

2. Preliminaries

The BCK/BCI-algebra is an important class of logical algebras introduced by K. Iséki and has been extensively investigated by many researchers.

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

(I) $(\forall x, y, z \in X)\ (((x * y) * (x * z)) * (z * y) = 0)$,

(II) $(\forall x, y \in X)\ ((x * (x * y)) * y = 0)$,

(III) $(\forall x \in X)\ (x * x = 0)$,

(IV) $(\forall x, y \in X)\ (x * y = 0, y * x = 0 \Rightarrow x = y)$.

If a BCI-algebra X satisfies the following identity

(V) $(\forall x \in X)\ (0 * x = 0)$,

then X is called a *BCK-algebra*. Any BCK/BCI-algebra X satisfies the following axioms:

$$(\forall x \in X)\,(x * 0 = x)\,, \tag{2.1}$$

$$(\forall x, y, z \in X)\,(x \leq y \Rightarrow x * z \leq y * z,\ z * y \leq z * x)\,, \tag{2.2}$$

$$(\forall x, y, z \in X)\,((x * y) * z = (x * z) * y)\,, \tag{2.3}$$

$$(\forall x, y, z \in X)\,((x * z) * (y * z) \leq x * y) \tag{2.4}$$

where $x \leq y$ if and only if $x * y = 0$. A nonempty subset S of a BCK/BCI-algebra X is called a *subalgebra* of X if $x * y \in S$ for all $x, y \in S$. A subset I of a BCK/BCI-algebra X is called an *ideal* of X if it satisfies

$$0 \in I, \tag{2.5}$$

$$(\forall x \in X)\,(\forall y \in I)\,(x * y \in I \Rightarrow x \in I)\,. \tag{2.6}$$

We refer the reader to the books [1], [4] for further information regarding BCK/BCI-algebras.

An intuitionistic fuzzy set $A = (f_A, g_A)$ in X is called an *intuitionistic fuzzy subalgebra* of X (see [3]) if the following condition is valid.

$$(\forall x, y \in X)\begin{pmatrix} f_A(x * y) \geq \min\{f_A(x), f_A(y)\} \\ g_A(x * y) \leq \max\{g_A(x), g_A(y)\} \end{pmatrix}. \tag{2.7}$$

An intuitionistic fuzzy set $A = (f_A, g_A)$ in X is called an *intuitionistic fuzzy ideal* of X (see [3]) if the following conditions are valid.

$$(\forall x \in X)\,(f_A(0) \geq f_A(x),\ g_A(0) \leq g_A(x))\,, \tag{2.8}$$

$$(\forall x, y \in X)\begin{pmatrix} f_A(x) \geq \min\{f_A(x * y), f_A(y)\} \\ g_A(x) \leq \max\{g_A(x * y), g_A(y)\} \end{pmatrix}. \tag{2.9}$$

Note that every intuitionistic fuzzy ideal is an intuitionistic fuzzy subalgebra in a BCK-algebra (see [3]).

Every intuitionistic fuzzy ideal $A = (f_A, g_A)$ of X satisfies the following condition (see [3]).

$$(\forall x, y \in X)\,(x * y = 0\ \Rightarrow\ f_A(x) \geq f(y),\ g_A(x) \leq g_A(y))\,. \tag{2.10}$$

3. Intuitionistic permeable values

In what follows, let X denote a BCK/BCI-algebra unless otherwise specified.

Definition 3.1. ([2]) A non-empty subset A of X is said to be *S-energetic* if it satisfies

$$(\forall a, b \in X)\,(a * b \in A\ \Rightarrow\ \{a, b\} \cap A \neq \emptyset)\,. \tag{3.1}$$

Definition 3.2. Let $A = (f_A, g_A)$ be an intuitionistic fuzzy set in X and $(t_A, s_A) \in [0, 1] \times [0, 1]$.

We say that (t_A, s_A) is an *intuitionistic permeable S-value* for $A = (f_A, g_A)$ if the following assertion is valid:

$$(\forall x, y \in X)\begin{pmatrix} f_A(x * y) \geq t_A\ \Rightarrow \max\{f_A(x), f_A(y)\} \geq t_A \\ g_A(x * y) \leq s_A\ \Rightarrow \min\{g_A(x), g_A(y)\} \leq s_A \end{pmatrix}. \tag{3.2}$$

Example 3.3. (1) Let $X = \{0, 1, 2, 3\}$ be a BCK-algebra with the following Cayley table:

$*$	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	2	1	0

Let $A = (f_A, g_A)$ be an intuitionistic fuzzy set in X defined by

$$f_A = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.3 & 0.5 & 0.4 & 0.6 \end{pmatrix}, \quad g_A = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0.6 & 0.4 & 0.3 & 0.2 \end{pmatrix}.$$

If we take $(t_A, s_A) \in (0.3, 1] \times [0, 0.6)$, then it is easy to check that (t_A, s_A) is an intuitionistic permeable S-value for $A = (f_A, g_A)$.

(2) Let $X = \{0, 1, 2, a, b\}$ be a BCI-algebra with the following Cayley table:

$*$	0	1	2	a	b
0	0	0	0	a	a
1	1	0	1	b	a
2	2	2	0	a	a
a	a	a	a	0	0
b	b	a	b	1	0

Let $A = (f_A, g_A)$ be an intuitionistic fuzzy set in X defined by

$$f_A = \begin{pmatrix} 0 & 1 & 2 & a & b \\ 0.2 & 0.5 & 0.3 & 0.4 & 0.5 \end{pmatrix}$$

and

$$g_A = \begin{pmatrix} 0 & 1 & 2 & a & b \\ 0.7 & 0.4 & 0.6 & 0.2 & 0.3 \end{pmatrix}.$$

If we take $(t_A, s_A) \in (0.2, 1] \times (0.3, 0.7)$, then we know that (t_A, s_A) is an intuitionistic permeable S-value for $A = (f_A, g_A)$.

For an intuitionistic fuzzy set $A = (f_A, g_A)$ in X and $(t_A, s_A) \in [0, 1] \times [0, 1]$, the intuitionistic upper (resp. lower) level sets are defined as follows:

$$U(f_A; t_A) := \{x \in X \mid f_A(x) \geq t_A\},\ L(g_A; s_A) := \{x \in X \mid g_A(x) \leq s_A\},$$

Theorem 3.4. *If (t_A, s_A) is an intuitionistic permeable S-value for an intuitionistic fuzzy set $A = (f_A, g_A)$ in X, then the intuitionistic upper level set $U(f_A; t_A)$ and the intuitionistic lower level set $L(g_A; s_A)$ are S-energetic subsets of X when they are nonempty.*

Proof. Assume that $U(f_A; t_A) \neq \emptyset \neq L(g_A; s_A)$ for $(t_A, s_A) \in [0, 1] \times [0, 1]$. Let $a, b, x, y \in X$ be such that $a * b \in U(f_A; t_A)$ and $x * y \in L(g_A; s_A)$. Then $f_A(a * b) \geq t_A$ and $g_A(x * y) \leq s_A$. It follows from (3.2) that

$$\max\{f_A(a), f_A(b)\} \geq t_A, \tag{3.3}$$

$$\min\{g_A(x), g_A(y)\} \leq s_A. \tag{3.4}$$

The fact (3.3) implies that $f_A(a) \geq t_A$ or $f_A(b) \geq t_A$. Thus $\{a, b\} \cap U(f_A; t_A) \neq \emptyset$. From the fact (3.4), we have $\{x, y\} \cap L(g_A; s_A) \neq \emptyset$. Therefore $U(f_A; t_A)$ and $L(g_A; s_A)$ are S-energetic subsets of X. ■

Note from [2, Theorem 3.3] that if A is an S-energetic subset of X, then $X \setminus A$ is a subalgebra of X. Hence we have the following corollary.

Corollary 3.5. *If (t_A, s_A) is an intuitionistic permeable S-value for an intuitionistic fuzzy set $A = (f_A, g_A)$ in X, then $X \setminus U(f_A; t_A)$ and $X \setminus L(g_A; s_A)$ are subalgebras of X when they are nonempty.*

Theorem 3.6. *If $A = (f_A, g_A)$ is an intuitionistic fuzzy subalgebra of X, then $X \setminus U(f_A;t)$ and $X \setminus L(g_A;t)$ are S-energetic subsets of X for all $t \in [0,1]$ whenever they are nonempty.*

Proof. Let $x, y \in X$ be such that $x * y \in X \setminus U(f_A;t)$ and $x * y \in X \setminus L(g_A;t)$ for $t \in [0,1]$. If $\{x,y\} \cap (X \setminus U(f_A;t)) = \emptyset$, then $x, y \in U(f_A;t)$ and so $x * y \in U(f_A;t)$ since $U(f_A;t)$ is a subalgebra of X. Assume that $\{x,y\} \cap (X \setminus L(g_A;t)) = \emptyset$. Then $x, y \in L(g_A;t)$ and so $x * y \in L(g_A;t)$ since $L(g_A;t)$ is a subalgebra of X. This is a contradiction. Therefore $X \setminus U(f_A;t)$ and $X \setminus L(g_A;t)$ are S-energetic subsets of X for all $t \in [0,1]$. ∎

Corollary 3.7. *If $A = (f_A, g_A)$ is an intuitionistic fuzzy ideal of a BCK-algebra X, then $X \setminus U(f_A;t)$ and $X \setminus L(g_A;t)$ are S-energetic subsets of X for all $t \in [0,1]$ whenever they are nonempty.*

Definition 3.8. An intuitionistic fuzzy set $A = (f_A, g_A)$ in X is called an *anti-intuitionistic fuzzy subalgebra* if the following condition is valid.

$$(\forall x, y \in X) \begin{pmatrix} f_A(x * y) \leq \max\{f_A(x), f_A(y)\} \\ g_A(x * y) \geq \min\{g_A(x), g_A(y)\} \end{pmatrix}. \tag{3.5}$$

Theorem 3.9. *For an intuitionistic fuzzy set $A = (f_A, g_A)$ in X, let $(t_A, s_A) \in [0,1] \times [0,1]$ be such that $U(f_A;t_A) \neq \emptyset \neq L(g_A;s_A)$. If $A = (f_A, g_A)$ is an anti-intuitionistic fuzzy subalgebra of X, then (t_A, s_A) is an intuitionistic permeable S-value for $A = (f_A, g_A)$.*

Proof. Let $x, y \in X$ be such that $f_A(x * y) \geq t_A$ and $g_A(x * y) \leq s_A$. Using (3.5), we have

$$t_A \leq f_A(x * y) \leq \max\{f_A(x), f_A(y)\}$$

and

$$s_A \geq g_A(x * y) \geq \min\{g_A(x), g_A(y)\}.$$

Therefore (t_A, s_A) is an intuitionistic permeable S-value for $A = (f_A, g_A)$. ∎

Theorem 3.10. *If $A = (f_A, g_A)$ is an anti-intuitionistic fuzzy subalgebra of X, then $U(f_A;t_A)$ and $L(g_A;s_A)$ are S-energetic subsets of X for all $(t_A, s_A) \in [0,1] \times [0,1]$ with $U(f_A;t_A) \neq \emptyset \neq L(g_A;s_A)$.*

Proof. Assume that $U(f_A;t_A) \neq \emptyset \neq L(g_A;s_A)$ for $(t_A, s_A) \in [0,1] \times [0,1]$. Let $x, y \in X$ be such that $x * y \in U(f_A;t_A)$. Then $t_A \leq f_A(x * y) \leq \max\{f_A(x), f_A(y)\}$, and so $f_A(x) \geq t_A$ or $f_A(y) \geq t_A$. Thus $\{x,y\} \cap U(f_A;t_A) \neq \emptyset$. If $a * b \in L(g_A;s_A)$ for $a, b \in X$, then $s_A \geq g_A(a * b) \geq \min\{g_A(a), g_A(b)\}$. It follows that $g_A(a) \leq s_A$ or $g_A(b) \leq s_A$, that is, $a \in L(g_A;s_A)$ or $b \in L(g_A;s_A)$. Hence $\{a,b\} \cap L(g_A;s_A) \neq \emptyset$. This completes the proof. ∎

The following example shows that the converse of Theorem 3.10 is not true in general, that is, there exist an intuitionistic fuzzy set $A = (f_A, g_A)$ in X and $(t_A, s_A) \in [0,1] \times [0,1]$ such that

1. $U(f_A; t_A)$ and $L(g_A; s_A)$ are S-energetic subsets of X.

2. $A = (f_A, g_A)$ is not an anti-intuitionistic fuzzy subalgebra of X.

Example 3.11. Let $X = \{0, a, b, c, d\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c	d
0	0	0	0	0	0
a	a	0	0	0	0
b	b	a	0	0	a
c	c	b	a	0	b
d	d	a	a	a	0

Let $A = (f_A, g_A)$ be an intuitionistic fuzzy set in X defined by

$$f_A = \begin{pmatrix} 0 & a & b & c & d \\ 0.3 & 0.4 & 0.4 & 0.2 & 0.4 \end{pmatrix}$$

and

$$g_A = \begin{pmatrix} 0 & a & b & c & d \\ 0.5 & 0.4 & 0.4 & 0.3 & 0.2 \end{pmatrix}.$$

Let $(t_A, s_A) \in (0.3, 0.4] \times [0.2, 0.4)$. Then $U(f_A; t_A) = \{a, b, d\}$ and

$$L(g_A; s_A) = \begin{cases} \{c, d\} & \text{if } s_A \in [0.3, 0.4), \\ \{d\} & \text{if } s_A \in [0.2, 0.3) \end{cases}$$

are S-energetic subsets of X. But $A = (f_A, g_A)$ is not an anti-intuitionistic fuzzy subalgebra of X.

Definition 3.12. ([2]) A non-empty subset A of X is said to be *I-energetic* if it satisfies:

$$(\forall x, y \in X)\,(y \in A \;\Rightarrow\; \{x, y * x\} \cap A \neq \emptyset)\,. \tag{3.6}$$

Definition 3.13. Let $A = (f_A, g_A)$ be an intuitionistic fuzzy set in X and $(t_A, s_A) \in [0, 1] \times [0, 1]$. We say that (t_A, s_A) is an *intuitionistic permeable I-value* for $A = (f_A, g_A)$ if the following assertion is valid:

$$(\forall x, y \in X) \begin{pmatrix} f_A(x) \geq t_A \;\Rightarrow \max\{f_A(x * y), f_A(y)\} \geq t_A \\ g_A(x) \leq s_A \;\Rightarrow \min\{g_A(x * y), g_A(y)\} \leq s_A \end{pmatrix}. \tag{3.7}$$

Example 3.14. In Example 3.3(2), the number $(t_A, s_A) \in (0.2, 1] \times (0.3, 0.7)$ is an intuitionistic permeable I-value for $A = (f_A, g_A)$.

Theorem 3.15. *If $A = (f_A, g_A)$ is an anti-intuitionistic fuzzy subalgebra of a BCK-algebra X, then every intuitionistic permeable I-value for $A = (f_A, g_A)$ is an intuitionistic permeable S-value for $A = (f_A, g_A)$.*

Proof. Note that $f_A(0) \leq f_A(x)$ and $g_A(0) \geq g_A(x)$ for all $x \in X$.

Let $(t_A, s_A) \in [0,1] \times [0,1]$ be an intuitionistic permeable I-value for $A = (f_A, g_A)$. Assume that $f_A(x * y) \geq t_A$ and $g_A(x * y) \leq s_A$ for all $x, y \in X$. Using (3.7), (2.3), (III) and (V), we have

$$t_A \leq \max\{f_A((x*y)*x), f_A(x)\} = \max\{f_A((x*x)*y), f_A(x)\}$$
$$= \max\{f_A(0*y), f_A(x)\} = \max\{f_A(0), f_A(x)\} = f_A(x)$$

and

$$s_A \geq \min\{g_A((x*y)*x), g_A(x)\} = \min\{g_A((x*x)*y), g_A(x)\}$$
$$= \min\{g_A(0*y), g_A(x)\} = \min\{g_A(0), g_A(x)\} = g_A(x).$$

It follows that

$$\max\{f_A(x), f_A(y)\} \geq f_A(x) \geq t_A \text{ and } \min\{g_A(x), g_A(y)\} \leq g_A(x) \leq s_A.$$

Therefore (t_A, s_A) is an intuitionistic permeable S-value for $A = (f_A, g_A)$. ∎

Corollary 3.16. *If $A = (f_A, g_A)$ is an anti-intuitionistic fuzzy ideal of a BCK-algebra X, then every intuitionistic permeable I-value for $A = (f_A, g_A)$ is an intuitionistic permeable S-value for $A = (f_A, g_A)$.*

The converse of Theorem 3.15 is not true in general as seen in the following example.

Example 3.17. Let $X = \{0, a, b, c\}$ be a BCK-algebra with the following Cayley table:

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Let $A = (f_A, g_A)$ be an intuitionistic fuzzy set in X defined by

$$f_A = \begin{pmatrix} 0 & a & b & c \\ 0.2 & 0.4 & 0.5 & 0.5 \end{pmatrix}, \quad g_A = \begin{pmatrix} 0 & a & b & c \\ 0.3 & 0.2 & 0.4 & 0.2 \end{pmatrix}.$$

If we take $(t_A, s_A) \in (0.4, 0.5] \times [0.2, 0.3)$, then $U(f_A; t_A) = \{b, c\}$ and $L(g_A; s_A) = \{a, c\}$. It is easy to check that (t_A, s_A) is an an intuitionistic permeable S-value for $A = (f_A, g_A)$. Note that $f_A(b) = 0.5 \geq t_A$ and $g_A(a) = 0.2 \leq s_A$. Hence $\max\{f_A(b*a), f_A(a)\} = 0.4 < t_A$ and $\min\{g_A(a*b), g_A(b)\} = 0.3 > s_A$. This shows that (t_A, s_A) is not an intuitionistic permeable I-value for $A = (f_A, g_A)$.

Theorem 3.18. *If $A = (f_A, g_A)$ is an intuitionistic fuzzy ideal of X, then $X \setminus U(f_A; t_A)$ and $X \setminus L(g_A; s_A)$ are empty or I-energetic subsets of X for all $(t_A, s_A) \in [0,1] \times [0,1]$.*

Proof. Assume that $X\setminus U(f_A;t_A)$ and $X\setminus L(g_A;s_A)$ are nonempty for all $(t_A,s_A)\in[0,1]\times[0,1]$. If $y\in X\setminus U(f_A;t_A)$ and $b\in X\setminus L(g_A;s_A)$, then

$$t_A > f_A(y) \geq \min\{f_A(y*x), f_A(x)\} \tag{3.8}$$

$$s_A < g_A(b) \leq \max\{g_A(b*a), g_A(a)\} \tag{3.9}$$

for all $a,x\in X$. From (3.8), we have $f_A(y*x)<t_A$ or $f_A(x)<t_A$, that is, $y*x\in X\setminus U(f_A;t_A)$ or $x\in X\setminus U(f_A;t_A)$. Hence $\{x,y*x\}\cap(X\setminus U(f_A;t_A))\neq\emptyset$. From (3.9), we get $g_A(b*a)>s_A$ or $g_A(a)>s_A$, that is, $b*a\in X\setminus L(g_A;s_A)$ or $a\in X\setminus L(g_A;s_A)$. Thus $\{a,b*a\}\cap(X\setminus L(g_A;s_A))\neq\emptyset$. Therefore $X\setminus U(f_A;t_A)$ and $X\setminus L(g_A;s_A)$ are I-energetic subsets of X for all $(t_A,s_A)\in[0,1]\times[0,1]$. ■

Theorem 3.19. *If (t_A,s_A) is an intuitionistic permeable I-value for an intuitionistic fuzzy set $A=(f_A,g_A)$ in X, then the intuitionistic upper level set $U(f_A;t_A)$ and the intuitionistic lower level set $L(g_A;s_A)$ are empty or I-energetic subsets of X.*

Proof. Assume that $U(f_A;t_A)$ and $L(g_A;s_A)$ are nonempty for all $(t_A,s_A)\in[0,1]\times[0,1]$. Let $y\in U(f_A;t_A)$. Then $f_A(y)\geq t_A$, and so

$$\max\{f_A(y*x), f_A(x)\}\geq t_A \text{ for all } x\in X.$$

It follows that $f_A(y*x)\geq t_A$ or $f_A(x)\geq t_A$, that is, $y*x\in U(f_A;t_A)$ or $x\in U(f_A;t_A)$. Hence $\{x,y*x\}\cap U(f_A;t_A)\neq\emptyset$. If $b\in L(g_A;s_A)$, then $g_A(b)\leq s_A$ and thus

$$\min\{g_A(b*a), g_A(a)\}\leq s_A$$

for all $a\in X$. It follows that $g_A(b*a)\leq s_A$ or $g_A(a)\leq s_A$ and so that

$$\{a,b*a\}\cap L(g_A;s_A)\neq\emptyset.$$

Therefore $U(f_A;t_A)$ and $L(g_A;s_A)$ are I-energetic subsets of X. ■

Theorem 3.20. *Let $A=(f_A,g_A)$ be an intuitionistic fuzzy subalgebra of X which satisfies the following condition:*

$$(x*y)*z=0 \;\Rightarrow\; \begin{pmatrix} f_A(x)\geq\min\{f_A(y),f_A(z)\} \\ g_A(x)\leq\max\{g_A(y),g_A(z)\} \end{pmatrix} \tag{3.10}$$

for all $x,y,z\in X$. Then $X\setminus U(f_A;t_A)$ and $X\setminus L(g_A;s_A)$ are empty or I-energetic subsets of X for all $(t_A,s_A)\in[0,1]\times[0,1]$.

Proof. Assume that $A=(f_A,g_A)$ is an intuitionistic fuzzy subalgebra of X satisfying the condition (3.10). Note that $f_A(0)\geq f_A(x)$ and $g_A(0)\leq g_A(x)$ for all $x\in X$. Since $(x*(x*y))*y=0$ for all $x,y\in X$, it follows from (3.10) that $f_A(x)\geq\min\{f_A(x*y),f_A(y)\}$ and $g_A(x)\leq\max\{g_A(x*y),g_A(y)\}$. Hence $A=(f_A,g_A)$ is an intuitionistic fuzzy ideal of X. Using Theorem 3.18, we know that $X\setminus U(f_A;t_A)$ and $X\setminus L(g_A;s_A)$ are I-energetic subsets of X for all $(t_A,s_A)\in[0,1]\times[0,1]$ whenever they are nonempty. ■

Definition 3.21. ([2]) Let Q be a non-empty subset of X. Then Q is said to be *right vanished* if it satisfies:

$$(\forall a, b \in X)\,(a * b \in Q \;\Rightarrow\; a \in Q)\,. \tag{3.11}$$

Q is said to be *right stable* if $Q * X := \{a * x \mid a \in Q,\ x \in X\} \subseteq Q$.

Theorem 3.22. *If $A = (f_A, g_A)$ is an intuitionistic fuzzy ideal of a BCK-algebra X, then*

(1) *The nonempty intuitionistic upper level set of $A = (f_A, g_A)$ is a right stable subset of X.*

(2) *The nonempty intuitionistic lower level set of $A = (f_A, g_A)$ is a right stable subset of X.*

Proof. (1) Let $y \in U(f_A; t_A)$ Then $f_A(y) \geq t_A$. Since $(y*x)*y = 0$ for all $x, y \in X$, it follows from (2.10) that $f_A(y * x) \geq f_A(y) \geq t_A$, that is, $y * x \in U(f_A; t_A)$ for all $x \in X$. Thus $U(f_A; t_A)$ is a right stable subset of X.

(2) If $b \in L(g_A; s_A)$, then $g_A(b) \leq s_A$ and so $g_A(b * x) \leq g_A(b) \leq s_A$ for all $x \in X$ by (2.10). Hence $b * x \in L(g_A; s_A)$ for all $x \in X$, and therefore $L(g_A; s_A)$ is a right stable subset of X. ∎

4. Conclusion

We have introduced the notion of intuitionistic permeable values in BCK/BCI-algebras, and have investigated several properties. We have discussed the relation between an intuitionistic permeable S-value and an intuitionistic permeable I-value, and have presented conditions for the intuitionistic lower (upper) level set to be S-energetic and I-energetic. We have considered conditions for a couple of numbers to be an intuitionistic permeable S-value, and have shown that if an intuitionistic fuzzy set $A = (f_A, g_A)$ in a BCK-algebra X is an intuitionistic fuzzy ideal of X, then the nonempty intuitionistic upper level set of $A = (f_A, g_A)$ is a right stable subset of X and the nonempty intuitionistic lower level set of $A = (f_A, g_A)$ is a right vanished subset of X. Based on this paper, we will apply the notion of this article to other algebraic structures, for example, semigroup theory, (pseudo) MV-algebras, (pseudo) BL-algebra, etc.

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