ADAPTIVE CONTROLLER FOR GENERAL NETWORKED MANIPULATORS WITH UNCERTAIN DYNAMICS

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Abstract. In this paper, two adaptive tracking control algorithms for redundant and non-redundant networked nonidentical robot manipulators are proposed, where the networked topology graph is direct and has a spanning tree. It is shown that, the distributed feedback controllers can be achieve end-effectors time-varying positions, velocities and sub-task tracking for manipulators with parametric uncertainty in the Lagrange dynamics. Moreover, the designed distributed tracking control algorithms in this paper is even a small portion of followers can get the leader's position information. Simulation example is presented to illustrate the performance of the proposed controller.
Keywords: redundant manipulators, Lagrange dynamics, dynamic friction, adaptive tracking, uncertain dynamics.

1. Introduction

Lagrange dynamics represent a class of mechanical systems including formation flying spacecraft, autonomous underwater vehicle, robotic manipulators etc. [5], [7], [9], [19], [20], [23]. Recently, coordinated motion and coordinated control of networked Lagrange systems have become important topics and numerous applications including coverage control, consensus, spacecraft formation control, adaptive tracking control, flocking control etc. [5], [17], [20], but it is also very challenging due to the highly inherent nonlinearity of the Lagrange systems [5]. Nair and Leonard [14] addressed stable synchronization of a network of rotating and translating rigid bodies and designed control laws so that the exponential stability of Lagrangian dynamics is proved. Ren [16] presented some distributed, leaderless, model-independent consensus algorithms for Lagrange systems relied on undirected communication topology. Wu and Zhou [25], [26] studied impulsive synchronization motion in networked open-loop multibody systems formulated by

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Lagrange dynamics with undirected graph. Chung et al. [6] studied the cooperative robot control and concurrent synchronization of nonidentical Lagrange systems.

In most applications of robot, the assumption of having exact models of robot systems also means that the robot is not able to adapt to any changes and uncertainties in its models environment. Meanwhile, the exact dynamics of robot manipulators is hard to be obtained due to the imprecision and measurement of manipulator parameters and interactions between manipulator and environments, such as load variation, friction and external disturbances. In order to compensate the nonlinearity and parametric uncertainty, many robot adaptive controllers have been proposed [4], [11], [15], [24]. By using linear parametrization method of Lagrange dynamics, Chopra et al. [4] proposed adaptive coordination control passivity-based architecture and guarantee state synchronization of master-slave robots in free motion. Nuño, et al. [15] investigated the adaptive synchronization of networked nonidentical Lagrange systems with unknown parameters. It is challenging problem in control field to find an effective control scheme to achieve accurate tracking of the desired motion or leader, recently, tracking and cooperative control of multiple robot manipulators have become an attractive area of research owing to its important role in the assembly automation and flexible manufacturing system. The tracking control of the Lagrange systems is relatively challenging due to the highly coupled dynamics, nonlinearities and uncertainties in the system models [3], [13]. Mei et al. [13] investigated the distributed coordinated tracking problem for networked Lagrange systems on undirected graphs. Chen and Lewis [3] developed the cooperative tracking control for networked Lagrange vehicle systems with directed graph topology.

The assigned task to robot manipulators are specified in task space, by using the inverse of kinematic of the non-redundant robot manipulator, the end-effector adaptive tracking controller with uncertainty in the kinematic and dynamic models is developed in [1], [2], [8], [18]. Nevertheless, the redundant manipulator having more degree of freedom than the number of independent task coordinates is being widely used in various applications in industry. The control of redundant robot manipulators is particulary interesting, and has become a very active active research because of the extra degrees of freedom of redundant manipulator allow the robot manipulator to perform more dextrous manipulation and increase flexibility for the task execution. This fact is commonly referred to as self-motion since it is not observed in the operational space. To achieve accurate end-effector tracking while allowing the self-motion of the system to be available for performance augmentation [12], [21], Zergerolu et al. [21] considered the nonlinear tracking control of kinematically redundant robot manipulators, specifically, the designed controller realize the subtasks tracking effectively. In [12], based on the neural network learning of the parametric uncertainties in the dynamical model, nonlinear tracking control of redundant robot manipulator is considered, the end-effector tracking as well as subtask tracking are achieved effectively.

However, in some special engineering case, more than one manipulator is needed to cooperate with each other to perform the desired operation in task-space by utilizing synchronous track motions. The above mentioned tracking control strategies can only deal with the single robot manipulator [1], [2], [8], [12], [18], [21], the robotic network and its topological structure is not considered. Recently, based on the input-output passivation framework, Wang [25] investigated the adaptive synchronization of networked non-redundant and identical robotic systems with uncertainties in kinematics and dynamics. As to a networked robot manipulators systems which may include the non-identical structures (such as different physical parameters or different degree of freedom), furthermore, these non-identical robots are used to achieve the same designed task. By using linear parametrization approach, passivity-based synchronization control algorithms of heterogeneous redundant robotic manipulators with uncertainties in dynamics are proposed [10]. However, the end-effectors adaptive synchronization control algorithms of networked robot manipulators in [10], [22] are relied on passivity feature and strongly connected graph theory, and all the needed information of leader or desired is accessed to manipulator.

With the aforementioned background, in this paper based on combination of linear parametrization approach and directed graph theory, our main interest is in in end-effector adaptive tracking control of nonidentical networked manipulators systems under the assumptions that the graph contains a spanning tree and only a small part of the followers can get access to the leader's position vector in the task space. Two adaptive tracking algorithms for redundant and non-redundant networked robot manipulators with unknown parametric uncertainties dynamics are proposed. Moreover, the proposed algorithms work to track time-varying positions and velocities vector of the leader, do not require the computation of inverse kinematics and do not place restriction on the self-motion of the manipulator, hence, the extra degrees of freedom are available for sub-tasks.

The outline of the paper is organized as follows: in Section 2, notations, properties, assumptions, dynamical model and its decomposition of networked manipulator systems are shown; in Section 3, under direct graph theory, adaptive tracking control algorithms for redundant and non-redundant manipulators with uncertainty dynamics are designed; in Section 4, an example of application and simulation for three link networked redundant manipulators are presented and validated the results; final conclusion is given in Section 5.

2. Preliminary and problem statement

Let $\mathbb{R} = (-\infty, +\infty)$ be the set of real numbers, $\mathbb{N} = \{1, 2, ...\}$ be the set of nonnegative integer numbers. For the vector $u \in \mathbb{R}^n$, u^T denotes its transpose. The norm of the vector u is defined as $\|u\| = \sqrt{u^T u}$. $\mathbb{R}^{n\times n}$ stands for $n \times n$ the set of real matrices. I_n denotes identity matrix of n order. Let $A \otimes B$ be the Kronecker product of two matrices A and B. Assume matrix A is symmetric, the negative (positive) definite matrix is denoted by $A < 0, (> 0)$. Let $\mathbb{G} = (V, E, A)$ be a directed graph of order N ($N \geq 2$) with the set of nodes $V = \{v_1, v_2, ..., v_N\}$, an edge $e_{ij} \in E \subset V \times V$ if and only if manipulators i and j exchange information, and a adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ with nonnegative adjacency elements a_{ij}. The edge (i, j) in the edge set of a directed graph denotes that agent j can obtain information from agent i, but not necessarily vice versa. A is defined as $a_{ij} > 0$ if $(j,i) \in E$, otherwise $a_{ij} = 0$ for all $i \neq j$, and $a_{ii} = 0$ for all $i \in V$. Let Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ associated with A be defined as $l_{ii} = \sum_{k=1,k\neq i}^{N} a_{ik}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

Assume there are N followers, marking agents 1 to N, the $(N+1)$th manipulator indexed by 0 is treated as leader, the general graph $\overline{\mathbb{G}}$ associated with the considered $N+1$ manipulators, the Laplacian matrix of directed $\overline{\mathbb{G}}$ is denoted as $\overline{L}$. Let $B = \mathrm{diag}(b_{1d}, b_{2d}, ..., b_{Nd})$, where $b_{id} > 0$ if $0 \in N_i$, otherwise $b_{id} = 0$, where $N_i = \{j : (j,i) \in E\}$ denote the set of manipulators transmitting their information to the ith manipulators, suppose that leader can't obtain the followers information. define $H = L + B$, thus, there exists positive definite matrix P such that $Q \triangleq HP + PH^T > 0$.

The dynamics equation of n-degree N manipulators can be described as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + F_i\dot{q}_i = \tau_i, \quad i = 1,2,...,N, \tag{1}$$

where $q_i \in \mathbb{R}^n$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is inertia matrix of the ith manipulator, $C_i(q_i,\dot{q}_i) \in \mathbb{R}^{n\times n}$ is Coriolis and centripetal matrix, $G_i(q_i) \in \mathbb{R}^n$ is gravitational vector of the ith manipulator, F_i is a diagonal matrix of viscous and dynamic friction coefficients, $\tau_i \in \mathbb{R}^n$ denotes ith manipulator's control input generalized vector.

Three properties and assumptions [17] of dynamics system (1) are given as follows:

Property 2.1. *The inertia matrix M_i is symmetric and positive definite, and has the following relationship $k_{\overline{m}i}I_n \leq M_i(q_i) \leq k_{\underline{m}i}I_n$, where $k_{\overline{m}i}$, $k_{\underline{m}i}$ are positive constants.*

Property 2.2. *The matrix $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew-symmetric, i.e., $\zeta^T[\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)]\zeta = 0$, for $\forall \zeta, q_i, \dot{q}_i \in \mathbb{R}^n$.*

Property 2.3. *The general form of dynamics system (1) can be linearly parameterized in constant unknown parameters $\Theta_i = (\Theta_{i1}, \Theta_{i2}, ..., \Theta_{id})^T \in \mathbb{R}^d$ as*

$$M_i(q_i)\dot{\xi}_i + C_i(q_i,\dot{q}_i)\xi_i + G_i(q_i) + F_i\xi_i = Y_i(q_i,\dot{q}_i,\xi_i,\dot{\xi}_i)\Theta_i,$$

where $Y_i(q_i,\dot{q}_i,\xi_i,\dot{\xi}_i)$ is a known regressor matrix depending on $q_i, \dot{q}_i, \xi_i, \dot{\xi}_i$, suppose if the parameters of Y_i are limited, Y_i is bounded $i = 1,2,...,N$.

Assumption 2.1. *The end-effector position vector $x_d(t)$, velocity $\dot{x}_d(t)$ and acceleration $\ddot{x}_d(t)$ of the leader are bounded.*

The relation between joint space and task space of the ith manipulator is defined as the following kinematics

$$x_i = f_i(q_i), \tag{2}$$

where $f_i(q_i) \in \mathbb{R}^n \to \mathbb{R}^m$ denotes the nonlinear forward kinematics transformation. Suppose $x_d(t) \in \mathbb{R}^m$ and $\dot{x}_d(t) \in \mathbb{R}^m$ be the end-effector position and velocity of the leader.

The differential relation between joint space and task space can be calculated as:

$$\dot{x}_i = J_i(q_i)\dot{q}_i,$$
$$\ddot{x}_i = \dot{J}_i(q_i)\dot{q}_i + J_i(q_i)\ddot{q}_i, \ i = 1, 2, ..., N, \tag{3}$$

where $J_i(q_i) \in \mathbb{R}^{m\times n}$ Jacobian matrix, $J_i(q_i)$ and $\dot{J}_i(q_i)$ are described as follows:

$$J_i(q_i) = \frac{\partial f_i(q_i)}{\partial q_i},$$
$$\dot{J}_i(q_i) = \frac{dJ_i(q_i)}{dt} = \sum_{j=1}^{n} \frac{\partial J_i(q_i)}{\partial q_{ij}}\dot{q}_{ij}. \tag{4}$$

Suppose that (1) is redundant dynamic system, i.e., $n > m$. For convenience, the pseudo-inverse of $J_i(q_i)$, denoted by $J_i^+(q_i) \in \mathbb{R}^{n\times m}$ define as follows:

$$J_i^+ = J_i^T(J_iJ_i^T)^{-1}, \tag{5}$$

where the pseudo-inverse J_i^+ satisfies the following properties:

$$J_iJ_i^+ = I_m,\ J_iJ_i^+J_i = J_i,\ J_i^+J_iJ_i^+ = J_i^+,\ (J_iJ_i^+)^T = J_iJ_i^+,$$
$$(J_i^+J_i)^T = J_i^+J_i,\ (I_n - J_i^+J_i)J_i^+ = \mathbf{0},\ J_i(I_n - J_i^+J_i) = \mathbf{0}.$$

3. Main results

In this section, we present two distribute coordinated adaptive tracking control algorithms that account for networked redundant and non-redundant robot manipulators in the presence of Lagrange dynamics uncertain parameters being updated online by the proposed parameter updated laws. Additionally, the designed tracking control algorithms allow the redundancy of Lagrange system to perform the sub-task defined by at least one motion optimization index such as maintaining and avoidance of mechanical joint limits.

Before moving on, introducing assistant variables variables:

$$\begin{aligned}
p_i &= \underbrace{J_i^+\Big[\dot{x}_d - \Big(\sum_{j\in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d)\Big)\Big]}_{task-space\ tracking\ control} + \underbrace{(I_n - J_i^+J_i)\phi_i}_{sub-task\ tracking\ control},\\
\dot{p}_i &= J_i^+\Big[\ddot{x}_d - \Big(\sum_{j\in N_i} a_{ij}(\dot{x}_i - \dot{x}_j) + b_{id}(\dot{x}_i - \dot{x}_d)\Big)\Big]\\
&\quad + \dot{J}_i^+\Big[\dot{x}_d - \Big(\sum_{j\in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d)\Big)\Big]\\
&\quad + \dot{\phi}_i - \dot{J}_i^+J_i\phi_i - J_i^+\dot{J}_i\phi_i - J_i^+J_i\dot{\phi}_i,\\
s_i &= J_i^+\Big[-\dot{x}_d + \sum_{j\in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d)\Big] - (I_n - J_i^+J_i)\phi_i + \dot{q}_i,
\end{aligned} \tag{6}$$

where $p_i = \dot{q}_i - s_i$, $\ddot{q}_i = \dot{p}_i + \dot{s}_i$, $\phi_i \in \mathbb{R}^n$ is an auxiliary signal that is constructed in view of the sub-task of control targets, for instance mechanical limit avoidance or eluding obstacles. $b_{id} > 0$ if follower i can obtain leader's information, or $b_{id} = 0$.

Let $r_i = J_i s_i$, $e_i = x_i - x_d$, thus

$$
\begin{aligned}
r_i &= -\dot{x}_d + \sum_{j \in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d) + J_i \dot{q}_i \\
&= \dot{e}_i + \sum_{j \in N_i} a_{ij}(e_i - e_j) + b_{id} e_i, \qquad (7)
\end{aligned}
$$

The subtask tracking error denoted by $e_{si} \in \mathbb{R}^n$ which is defined as follows:

$$
\begin{aligned}
e_{si} &= (I_n - J_i^+ J_i)(\dot{q}_i - \phi_i) \\
&= (I_n - J_i^+ J_i) J_i^+ \Big[-\dot{x}_d + \sum_{j \in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d) \Big] \\
&- (I_n - J_i^+ J_i)(I_n - J_i^+ J_i)\phi_i + (I_n - J_i^+ J_i)\dot{q}_i \qquad (8) \\
&= (I_n - J_i^+ J_i) s_i.
\end{aligned}
$$

The properties of J_i indicate the regulation of s_i implies the regulation of tracking error e_{si}, so the subtask tracking control can be realized.

The inverse kinematics do not computer by J_i , we first consider the coordinated regulation algorithm as

$$
\begin{aligned}
\tau_i &= \widehat{M}_i(q_i)\dot{p}_i + \widehat{C}_i(q_i, \dot{q}_i)p_i + \widehat{G}_i(q_i) + \widehat{F}_i p_i - J_i^T \sum_{j=1}^{N} P_{ij} e_i \\
&= \underbrace{Y_i(q_i, \dot{q}_i, p_i, \dot{p}_i)\widehat{\Theta}_i}_{adaptive\ feedback\ control} - \underbrace{J_i^T \sum_{j=1}^{N} P_{ij} e_i}_{tracking\ control}, \qquad (9)
\end{aligned}
$$

where $P = (P_{ij})_{N \times N}$ is a candidate positive definite matrix, since the value of the dynamic parameter Θ_i is hard to be known exactly in practice, one defines $\widehat{\Theta}_i$ as the estimate of Θ_i. $\widehat{M}_i$, $\widehat{C}_i, \widehat{G}_i$ and $\widehat{F}_i$ represent estimation of $M_i(q_i, \widehat{\Theta}_i)$, $C_i(q_i, \dot{q}_i, \widehat{\Theta}_i)$, $G_i(q_i, \widehat{\Theta}_i)$ and $F_i(\widehat{\Theta}_i)$, respectively.

The parameter estimation is selected as the following adaptive law:

$$
\dot{\widehat{\Theta}}_i = -\Gamma_i^{-1} Y_i^T s_i, \qquad (10)
$$

where Γ_i is a constant diagonal positive definite matrix.

Theorem 3.1. *If control input torque τ_i and adaptive compensator are designed as* (9) *and* (10)*, assume that the topology $\overline{\mathbb{G}}$ is directed, and at least one follower can get leader's information, then $x_i(t) \to x_d(t)$ and $\dot{x}_i(t) \to \dot{x}_d(t)$ as $t \to \infty$, $i = 1, 2, ..., N$.*

Proof. The matrix or vector of $q, M, C, G, F, \tau, J, x, J^+, s, r, p, e, e_s, \Theta$ and Γ are the same definition in [10], [22], [25], [26].

From the distributed algorithm (9), system (1) can be written in the compact form as follows:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F\dot{q} = Y(q,\dot{q},p,\dot{p})\widehat{\Theta} - J^T(P \otimes I_m)e. \tag{11}$$

By $\ddot{q} = \dot{p} + \dot{s}$, (11) can be reformulated

$$M\dot{s} = Y\widetilde{\Theta} - J^T(P \otimes I_m)e - Cs - Fs. \tag{12}$$

where $\widetilde{\Theta} = \widehat{\Theta} - \Theta$ denote the estimation error, clearly $\dot{\widetilde{\Theta}} = \dot{\widehat{\Theta}}$.

Consider the following candidate Lyapunov function

$$V = \frac{1}{2}\big(s^T M s + e^T(P \otimes I_m)e + \widetilde{\Theta}^T \Gamma \widetilde{\Theta}\big). \tag{13}$$

The derivative of V along the trajectory of (12) is

$$\begin{aligned} \dot{V} &= \frac{1}{2}s^T\dot{M}s + s^T M\dot{s} + e^T(P \otimes I_m)\dot{e} + \widetilde{\Theta}^T\Gamma\dot{\widetilde{\Theta}} \\ &= \frac{1}{2}s^T\dot{M}s + s^T\big[Y\widetilde{\Theta} - J^T(P \otimes I_m)e - Cs - Fs\big] \\ &\quad + e^T(P \otimes I_m)\dot{e} + \widetilde{\Theta}^T\Gamma\dot{\widetilde{\Theta}} \\ &= \frac{1}{2}s^T\dot{M}s - s^TCs + s^TY\widetilde{\Theta} - s^TJ^T(P \otimes I_m)e - s^TFs \\ &\quad + e^T(P \otimes I_m)[r - (H \otimes I_m)e] + \widetilde{\Theta}^T\Gamma(-\Gamma^{-1}Y^Ts). \end{aligned} \tag{14}$$

By Property 2, we have

$$\begin{aligned} \dot{V} &= s^TY\widetilde{\Theta} - r^T(P \otimes I_m)e - s^TFs + e^T(P \otimes I_m)r \\ &\quad - e^T\big[(PH) \otimes I_m\big]e - \widetilde{\Theta}^TY^Ts = -s^TFs - \tfrac{1}{2}e^T(Q \otimes I)e. \end{aligned} \tag{15}$$

It is important to note that F is a positive definite diagonal matrix and there exist matrix $P > 0$ such that $Q > 0$, hence, $V(t) \geq 0$, $\dot{V}(t) \leq 0$.

Therefore, $V(t) \in L_\infty$, then $s \in L_\infty$, $e \in L_\infty$, $\widetilde{\Theta} \in L_\infty$, from (3) and (6) $r \in L_\infty, x \in L_\infty,$, and then by $J \in L_\infty$ and (7), $\dot{e} \in L_\infty\, \dot{q} \in L_\infty$. By(12), $\dot{s} \in L_\infty$, and the bounded of $\ddot{x}_d \in L_\infty$ and $J^+ \in L_\infty$, so $\dot{J}^+ \in L_\infty$, hence $\ddot{V}(t) \in L_\infty$. By the Barbalat's Lemma, $\dot{V}(t) \to \infty$ as $t \to \infty$, then $x_i(t) \to x_d(t)$, $\dot{x}_i(t) \to \dot{x}_d(t)$ as $t \to \infty$.

In the meantime, from $e_s = (I_{nN} - J^+J)s$, since $s \to 0$, then $e_s \to 0$, i.e., the sub-task tracking can be realize. ∎

Second, consider the system (1) with non-redundant degree case, i.e., $m = n$ and $J^+ = J^{-1}$. In this case, the auxiliary variables (6) can be rewritten as follows:

$$
\begin{aligned}
y_i &= J_i^{-1}\Big[\dot{x}_d - \Big(\sum_{j\in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d)\Big)\Big],\\
\dot{y}_i &= J_i^{-1}\Big[\ddot{x}_d - \Big(\sum_{j\in N_i} a_{ij}(\dot{x}_i - \dot{x}_j) + b_{id}(\dot{x}_i - \dot{x}_d)\Big)\Big]\\
&\quad + \dot{J}_i^{-1}\Big[\dot{x}_d - \Big(\sum_{j\in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d)\Big)\Big],\\
s_i &= J_i^{-1}\Big[-\dot{x}_d + \sum_{j\in N_i} a_{ij}(x_i - x_j) + b_{id}(x_i - x_d)\Big] + \dot{q}_i, \qquad (16)
\end{aligned}
$$

We propose an adaptive control as

$$\tau_i = Y_i(q_i, \dot{q}_i, y_i, \dot{y}_i)\widehat{\Theta}_i - J_i^T \sum_{j=1}^{N} P_{ij} e_i, \qquad (17)$$

where Θ_i, e_i, P_{ij} and the updated adaptive law are the same as that of (9) and (10).

Similar to the proof of Theorem 3.1, it is obviously easy to get following results for the case of non-redundant manipulators systems.

Corollary 3.1. *Assume the communication graph $\overline{\mathbb{G}}$ is directed and and at least one follower can obtain the leader's information. For networked manipulator system* (1), *under adaptive controller* (17), *then $x_i(t) \to x_d(t)$ and $\dot{x}_i(t) \to \dot{x}_d(t)$ as $t \to \infty$.*

Remark 3.1. Theorem 3.1 is suitable for the systems with nonidentical manipulators, where the network can contain redundant and non-redundant manipulators simultaneously, i.e., all the manipulators can have different degree of freedom. So our result is general and practical.

Remark 3.2. Noted that from the distributed algorithms (9) and (6), (17) and (16), only small number of followers' end-effectors need to get information of x_d, however all information of the desired trajectory needs to be accessed to all manipulators in [25], [26].

4. Illustrative example

For a better illustration of the proposed adaptive control algorithm we put forward an example in numerical simulation. For simplicity, planar motion of nonidentical redundant manipulators with the same degree of freedom is consider in the task space. The simulations are displayed with networked planar manipulators with three-link (see Fig. 1(a)). The joint q_{i1}, q_{i2} and q_{i3}, spiale O_1, O_2, O_3 and moments of inertias I_{i1}, I_{i2} and I_{i3} are shown in Fig. 1(a). The general communication graph is shown in Fig. 1(b).

Example 4.1. *Consider networked three-link manipulators with communication topology Fig.* 1(b)*, each manipulator nonlinear dynamics follows the Lagrange systems:*

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + F_i\dot{q}_i = \tau_i,\ i = 1, ..., 4, \tag{18}$$

where $q_i = (q_{i1}, q_{i2}, q_{i3})^T \in \mathbb{R}^3$*, the matrices* $M_i(q_i)$*,* $C_i(q_i,\dot{q}_i)$*,* Y_i*, vector* $G_i(q_i)$ *and constant parameter* Θ_i *are represented as shown below.*

Let

$$M_i(q_i) = \begin{bmatrix} M_{i11} & M_{i12} & M_{i13} \\ M_{i21} & M_{i22} & M_{i23} \\ M_{i31} & M_{i32} & M_{i33} \end{bmatrix},\quad C_i(q_i,\dot{q}_i) = \begin{bmatrix} C_{i11} & C_{i12} & C_{i13} \\ C_{i21} & C_{i22} & C_{i23} \\ C_{i31} & C_{i32} & C_{i33} \end{bmatrix},$$

$$G_i(q_i) = \begin{bmatrix} G_{i1} \\ G_{i2} \\ G_{i3} \end{bmatrix},$$

where the entries of $M_i(q_i)$, $C_i(q_i,\dot{q}_i)$ and $G_i(q_i)$ are given as $M_{i11} = \Theta_{i1} + 2(\Theta_{i2}\mathrm{cos}_{i2} + \Theta_{i3}\mathrm{cos}_{i3} + \Theta_{i4}\mathrm{cos}_{i23})$, $M_{i12} = M_{i21} = \Theta_{i5} + \Theta_{i2}\mathrm{cos}_{i2} + 2\Theta_{i3}\mathrm{cos}_{i3} + \Theta_{i4}\mathrm{cos}_{i23}$, $M_{i13} = M_{i31} = \Theta_{i6} + \Theta_{i3}\mathrm{cos}_{i3} + \Theta_{i4}\mathrm{cos}_{i23}$, $M_{i22} = \Theta_{i5} + 2\Theta_{i3}\mathrm{cos}_{i3}$, $M_{i23} = M_{i32} = \Theta_{i6} + \Theta_{i3}\mathrm{cos}_{i3}$, $M_{i33} = \Theta_{i6}$. $C_{i11} = -\Theta_{i2}\mathrm{sin}_{i2}\dot{q}_{i2} - \Theta_{i4}\mathrm{sin}_{i23}(\dot{q}_{i2} + \dot{q}_{i3}) - \Theta_{i3}\mathrm{sin}_{i3}\dot{q}_{i3}$, $C_{i12} = -\Theta_{i2}\mathrm{sin}_{i2}(\dot{q}_{i1} + \dot{q}_{i2}) - \Theta_{i4}\mathrm{sin}_{i23}(\dot{q}_{i1} + \dot{q}_{i2} + \dot{q}_{i3}) - \Theta_{i3}\mathrm{sin}_{i3}\dot{q}_{i3}$, $C_{i13} = -\Theta_{i3}\mathrm{sin}_{i3}(\dot{q}_{i1} + \dot{q}_{i2} + \dot{q}_{i3}) - \Theta_{i4}\mathrm{sin}_{i23}(\dot{q}_{i1} + \dot{q}_{i2} + \dot{q}_{i3})$, $C_{i21} = \Theta_{i2}\mathrm{sin}_{i2}\dot{q}_{i1} + \Theta_{i4}\mathrm{sin}_{i23}\dot{q}_{i1} - \Theta_{i3}\mathrm{sin}_{i3}\dot{q}_{i3}$, $C_{i22} = -\Theta_{i3}\mathrm{sin}_{i3}\dot{q}_{i3}$, $C_{i23} = -\Theta_{i3}\mathrm{sin}_{i3}(\dot{q}_{i1} + \dot{q}_{i2} + \dot{q}_{i3})$, $C_{i31} = \Theta_{i3}\mathrm{sin}_{i3}(\dot{q}_{i1} + \dot{q}_{i2}) + \Theta_{i4}\mathrm{sin}_{i23}\dot{q}_{i1}$, $C_{i32} = \Theta_{i3}\mathrm{sin}_{i3}(\dot{q}_{i1} + \dot{q}_{i2})$, $C_{i33} = 0$, $G_{i1} = \Theta_{i7}\mathrm{sin}_{i1} + \Theta_{i8}\mathrm{sin}_{i12} + \Theta_{i9}\mathrm{sin}_{i123}$, $G_{i2} = \Theta_{i8}\mathrm{sin}_{i12} + \Theta_{i9}\mathrm{sin}_{i123}$, $G_{i3} = \Theta_{i9}\mathrm{sin}_{i123}$.

Let

$$Y_i(q_i,\dot{q}_i,p_i,\dot{p}_i) = \begin{bmatrix} Y_{i1}^T \\ Y_{i2}^T \\ Y_{i3}^T \end{bmatrix},\ \Theta_i = \begin{bmatrix} \Theta_{i1} \\ \vdots \\ \Theta_{i12} \end{bmatrix} \text{ and } F_i = \begin{bmatrix} \Theta_{i10} & 0 & 0 \\ 0 & \Theta_{i11} & 0 \\ 0 & 0 & \Theta_{i12} \end{bmatrix},$$

where the vector of unknown estimate parameters $\Theta_i \in \mathbb{R}^{12}$ and regressor matrix $Y_i(q_i,\dot{q}_i,p_i,\dot{p}_i) \in \mathbb{R}^{3\times 12}$ are given as $\Theta_{i1} = l_{ci1}^2 m_{i1} + (l_{i1}^2 + l_{ci2}^2)m_{i2} + (l_{i1}^2 + l_{i2}^2 + l_{ci3}^2)m_{i3} + I_{i1} + I_{i2} + I_{i3}$, $\Theta_{i2} = l_{i1}l_{ci2}m_{i2} + l_{i1}l_{i2}m_{i3}$, $\Theta_{i3} = l_{i2}l_{ci3}m_{i3}$, $\Theta_{i4} = l_{i1}l_{ci3}m_{i3}$, $\Theta_{i5} = l_{ci2}^2 m_{i2} + (l_{i2}^2 + l_{ci3}^2)m_{i3} + I_{i2} + I_{i3}$, $\Theta_{i6} = l_{ci3}^2 m_{i3} + I_{i3}$, $\Theta_{i7} = (l_{ci1}m_{i1} + l_{i1}m_{i2} + l_{i1}m_{i3})g$, $\Theta_{i8} = (l_{ci2}m_{i2} + l_{i2}m_{i3})g$, $\Theta_{i9} = l_{ci3}m_{i3}g$, $\Theta_{i10} = F_{i1}$, $\Theta_{i11} = F_{i2}$, $\Theta_{i12} = F_{i3}$, $Y_{i1} = [\dot{p}_{i1}, \alpha_1, \alpha_2, \alpha_3, \dot{p}_{i2}, \dot{p}_{i3}, \mathrm{sin}_{i1}, \mathrm{sin}_{i12}, \mathrm{sin}_{i123}, p_{i1}, 0, 0]^T$, $Y_{i2} = [0, \beta_1, \beta_2, \beta_3, \dot{p}_{i1} + \dot{p}_{i2}, \dot{p}_{i3}, 0, \mathrm{sin}_{i12}, \mathrm{sin}_{i123}, 0, p_{i2}, 0]^T$, $Y_{i3} = [0, 0, \gamma_1, \gamma_2, 0, \dot{p}_{i1} + \dot{p}_{i2} + \dot{p}_{i3}, 0, 0, \mathrm{sin}_{i123}, 0, 0, p_{i3}]^T$, where $l_{c_{i1}} = \frac{1}{2}l_{i1}$, $l_{c_{i2}} = \frac{1}{2}l_{i2}$, $l_{c_{i3}} = \frac{1}{2}l_{i3}$. $I_{i1} = \frac{1}{3}m_{i1}l_{c_{i1}}^2$, $I_{i2} = \frac{1}{3}m_{i2}l_{c_{i2}}^2$ and $I_{i3} = \frac{1}{3}m_{i3}l_{c_{i3}}^2$ are the moments of inertia. $\alpha_1 = 2\mathrm{cos}_{i2}\dot{p}_{i1} + \mathrm{cos}_{i2}\dot{p}_{i2} - \mathrm{sin}_{i2}\dot{q}_{i2}p_{i1} - \mathrm{sin}_{i2}(\dot{q}_{i1} + \dot{q}_{i2})p_{i2}$, $\alpha_2 = 2\mathrm{cos}_{i3}\dot{p}_{i1} + 2\mathrm{cos}_{i3}\dot{p}_{i2} + \mathrm{cos}_{i3}\dot{p}_{i3} - \mathrm{sin}_{i3}[\dot{q}_{i3}p_{i1} + \dot{q}_{i3}p_{i2} + (\dot{q}_{i1} + \dot{q}_{i2} + \dot{q}_{i3})p_{i3}]$, $\alpha_3 = \mathrm{cos}_{i23}(2\dot{p}_{i1} + \dot{p}_{i2} + \dot{p}_{i3}) - \mathrm{sin}_{i23}[(\dot{q}_{i2} + \dot{q}_{i3})p_{i1} + (\dot{q}_{i1} + \dot{q}_{i2} + \dot{q}_{i3})(p_{i2} + p_{i3})]$, $\beta_1 = \mathrm{cos}_{i2}\dot{p}_{i1} + \mathrm{sin}_{i2}\dot{q}_{i1}p_{i1}$, $\beta_2 = \mathrm{cos}_{i3}(2\dot{p}_{i1} + 2\dot{p}_{i2} + \dot{p}_{i3}) - \mathrm{sin}_{i3}[\dot{q}_{i3}p_{i1} + \dot{p}_{i3}p_{i2} + (\dot{q}_{i1} + \dot{q}_{i2} + \dot{q}_{i3})p_{i3}]$, $\beta_3 = \mathrm{cos}_{i23}\dot{p}_{i1} + \mathrm{sin}_{i23}\dot{q}_{i1}p_{i1}$, $\gamma_1 = \mathrm{cos}_{i3}(\dot{p}_{i1} + \dot{p}_{i2}) + \mathrm{sin}_{i3}(\dot{p}_{i1} + \dot{p}_{i2})(p_{i1} + p_{i2})$, $\gamma_2 = \mathrm{cos}_{i23}\dot{p}_{i1} + \mathrm{sin}_{i23}\dot{q}_{i1}p_{i1}$, where

$\cos_{i1} = \cos q_{i1}$, $\cos_{i12} = \cos(q_{i1}+q_{i2})$, $\cos_{i123} = \cos(q_{i1}+q_{i2}+q_{i3})$, $\cos_{i23} = \cos(q_{i2}+q_{i3})$, $\sin_{i1} = \sin q_{i1}$, $\sin_{i12} = \sin(q_{i1} + q_{i2})$, $\sin_{i123} = \sin(q_{i1} + q_{i2} + q_{i3}), \sin_{i23} = \sin(q_{i2} + q_{i3})$. Obviously, $M_i(q_i)$ is symmetric, positive definite, differentiable, and satisfies Properties 2.1 and 2.2. Assume that system (18) is using distributed the control input (9) and the updated adaptive law (10).

The agents are assumed to be communicated using directed topology graph $\overline{\mathbb{G}}$ is illustrated in Fig. 1(b). Matrix H is represented as shown below

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & -1 & 2 \end{bmatrix}.$$

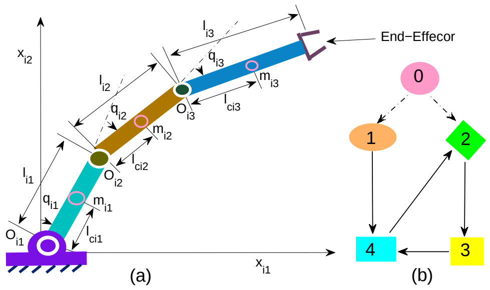

Fig. 1. (a) The link parameters of ith three-link manipulators. (b) The direct communication graph of four nonidentical three-link manipulators followers and one dynamic leader.

In task-space, the ith manipulator position is expressed as:

$$x_i = \begin{bmatrix} -l_{i3}\sin_{i123} - l_{i2}\sin_{i12} - l_{i1}\sin_{i1} \\ l_{i3}\cos_{i123} + l_{i2}\cos_{i12} + l_{i1}\cos_{i1} \end{bmatrix}, \tag{19}$$

where $\sin_{i123} = \sin(q_{i1}+q_{i2}+q_{i3})$, $\sin_{i12} = \sin(q_{i1}+q_{i2})$, $\sin_{i1} = \sin(q_{i1})$, $\cos_{i123} = \cos(q_{i1} + q_{i2} + q_{i3})$, $\cos_{i12} = \cos(q_{i1} + q_{i2})$, $\cos_{i1} = \cos(q_{i1})$.

The Jacobian matrix is shown as:

$$J_i = \begin{bmatrix} -l_{i3}\cos_{i123} - l_{i2}\cos_{i12} - l_{i1}\cos_{i1} & -l_{i3}\cos_{i123} - l_{i2}\cos_{i12} & -l_{i3}\cos_{i123} \\ -l_{i3}\sin_{i123} - l_{i2}\sin_{i12} - l_{i1}\sin_{i1} & -l_{i3}\sin_{i123} - l_{i2}\sin_{i12} & -l_{i3}\sin_{i123} \end{bmatrix} \in \mathbb{R}^{2\times 3}.$$

From (4), the derivative of the manipulator Jacobian matrix with respect to time is as follows:

$$\begin{aligned} \dot{J}_i \;=\; & \begin{bmatrix} l_{i3}\sin_{i123} + l_{i2}\sin_{i12} + l_{i1}\sin_{i1} & l_{i3}\sin_{i123} + l_{i2}\sin_{i12} & l_{i3}\sin_{i123} \\ -l_{i3}\cos_{i123} - l_{i2}\cos_{i12} - l_{i1}\cos_{i1} & -l_{i3}\cos_{i123} - l_{i2}\cos_{i12} & -l_{i3}\cos_{i123} \end{bmatrix} \dot{q}_{i1} \\ & + \begin{bmatrix} l_{i3}\sin_{i123} + l_{i2}\sin_{i12} & l_{i3}\sin_{i123} + l_{i2}\sin_{i12} & l_{i3}\sin_{i123} \\ -l_{i3}\cos_{i123} - l_{i2}\cos_{i12} & -l_{i3}\cos_{i123} - l_{i2}\cos_{i12} & -l_{i3}\cos_{i123} \end{bmatrix} \dot{q}_{i2} \\ & + \begin{bmatrix} l_{i3}\sin_{i123} & l_{i3}\sin_{i123} & l_{i3}\sin_{i123} \\ -l_{i3}\cos_{i123} & -l_{i3}\cos_{i123} & -l_{i3}\cos_{i123} \end{bmatrix} \dot{q}_{i3}. \end{aligned} \tag{20}$$

From (5), we can copulate J_i^+ and $\dot{J}_i^+ = \left[\dot{J}_i^T - J_i^T\left(J_iJ_i^T\right)^{-1}\left(\dot{J}_iJ_i^T + J_i\dot{J}_i^T\right)\right]\left(J_iJ_i^T\right)^{-1}$, clearly, the kinematics of each manipulator are also highly inherent nonlinear.

The leader's position and velocity vectors are chosen as follows

$$x_d(t) = \begin{bmatrix} x_{d1}(t) \\ x_{d2}(t) \end{bmatrix} = \begin{bmatrix} 4.2 + 0.76\sin(0.5t) \\ 3.6 + 0.3\cos(t) \end{bmatrix}, \ \dot{x}_d(t) = \begin{bmatrix} 0.38\cos(0.5t) \\ -0.3\sin(t) \end{bmatrix} \tag{21}$$

The $\phi_i(t)$ for ith manipulator is selected as follows:

$$\phi_i(t) = \begin{bmatrix} \phi_{i1}(t) \\ \phi_{i2}(t) \\ \phi_{i3}(t) \end{bmatrix} = -2\begin{bmatrix} q_{i3}(t) - q_{i2}(t) + 0.5q_{i1}(t) \\ -q_{i3}(t) + q_{i2}(t) - 0.5q_{i1}(t) \\ q_{i3}(t) - q_{i2}(t) + 0.5q_{i1}(t) \end{bmatrix}. \tag{22}$$

Let m_{i1}=0.52 + 0.1i, m_{i2}=0.61 + 0.06i, m_{i3}=0.54 + 0.08i, l_{i1}=3.2 + 0.08i, $l_{i2} = 2.3 + 0.04i$, $l_{i3} = 3.38 + 0.06i$. Let $l_{ci1} = \frac{1}{2}l_{i1}$, $l_{ci2} = \frac{1}{2}l_{i2}$, $l_{ci3} = \frac{1}{2}l_{i3}$, $I_{i1} = \frac{1}{3}m_{i1}l_{ci1}^2$, $I_{i2} = \frac{1}{3}m_{i2}l_{ci2}^2 I_{i3} = \frac{1}{3}m_{i3}l_{ci3}^2$, $F_i = \mathrm{diag}(F_{i1}, F_{i2}, F_{i3})$, $F_{i1} = 5.81 + 0.02i$, $F_{i2} = 5.81 + 0.021i$, $F_{i3} = 5.81 + 0.023i$, $\Gamma_i = 8.5I_3$, $i = 1, ..., 4$, the gravity acceleration be $g = 9.8m/s^2$.

The initial value $q_{ij}, \dot{q}_{ij}, \widehat{\Theta}_{ik}$, $i = 1, ..., 4$, $j = 1, ..., 3$, $k = 1, ..., 12$ can be randomly chosen in $(-\pi, \pi)$.

Let $P = 9.5I_4$, clearly, $Q > 0$, based on Theorem 3.1, $x_i(t) \to x_d(t)$ and $\dot{x}_i(t) \to \dot{x}_d(t)$ as $t \to \infty$, (see Figs. 2, 3). Meanwhile, Figs. 4 and 5 show tracking effect of positions and velocities in task space.

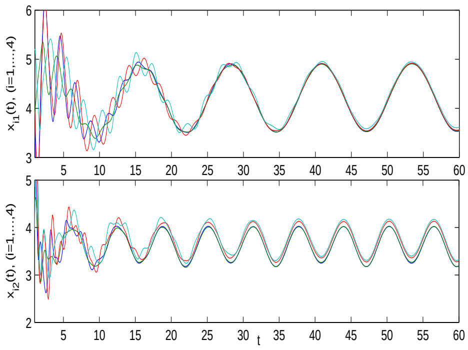

Fig. 2. Desired and actual position task-space trajectories.

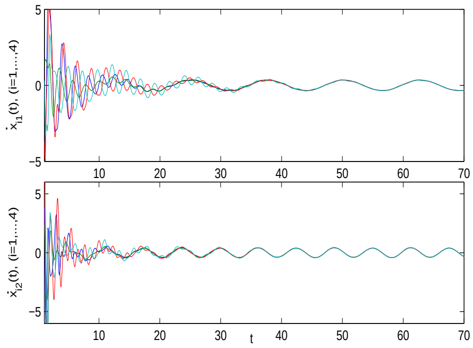

Fig. 3. Desired and actual velocity task-space trajectories.

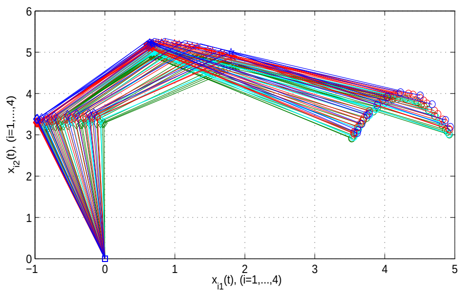

Fig. 4. The position trajectories of four manipulators end-effectors.

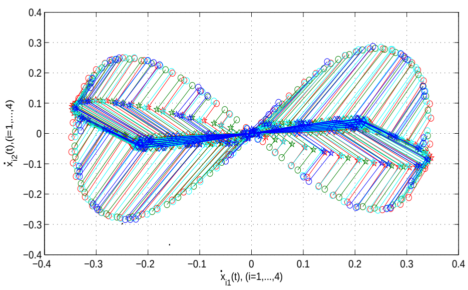

Fig. 5. The velocity trajectories of four manipulators end-effectors.

5. Conclusions

This paper presents two tracking control algorithms of nonidentical networked robot manipulators systems using directed graph theory. While parameters of the considered Lagrange dynamics are uncertain, the proposed adaptive algorithms work to track leader's time-varying positions and velocities vector. Moreover, only a small number of the followers' end-effectors need to obtain position signal of leader. Furthermore, the proposed adaptive algorithms can apply to the tracking control of the nonidentical manipulators with different degree of freedom. The effectiveness of the proposed control algorithms is examined by a simulation example for networked nonidentical manipulator systems with three-links.

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