DYNAMICS IN A COMPETITIVE LOTKA-VOLTERRA PREDATOR-PREY MODEL WITH MULTIPLE DELAYS

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Abstract. In this paper, we investigate a discrete competitive Lotka-Volterra predator-prey model with multiple delays. For general non-autonomous case, sufficient conditions which ensure the permanence and the global stability of the system are derived by applying the differential inequality theory; For periodic case, sufficient conditions which guarantee the existence of an unique globally stable positive periodic solution are established. Some numerical simulations which illustrate our theoretical findings are carried out.

Keywords: competitive Lotka-Volterra predator-prey model; permanence; global attractivity; multiple delays.

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1. Introduction

Since the work of Berryman [1], the research on dynamical behavior of predator-prey models has become one of the dominant themes in both ecology and mathematical ecology. In particular, the stability of the equilibrium, the permanence and extinction of species, the existence of periodic solutions and positive almost periodic solutions, bifurcation and chaos of predator-prey models have been investigated in a number of notable studies [2]-[26]. In many applications, the nature of permanence is of great interest. For example, Fan and Li [27] made a theoretical discussion on the permanence of a cooperative scalar population models with

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delays, Wang and Huang [28] addressed the permanence of a stage-structured predator-prey system with impulsive stocking prey and harvesting predator, Liu et al. [29] analyzed the permanence and global attractivity of an impulsive ratio-dependent predator-prey system in a patchy environment. Zhao and Teng et al. [30] established the permanence criteria for a delayed discrete nonautonomous-species Kolmogorov systems. For more details, we refer to [31]-[40].

In 2008, Qiu and Cao [41] investigated the exponential stability of the following competitive Lotka-Volterra predator-prey system with delays

$$(1.1)\qquad \begin{cases} \dot{x}(t) = x(t)[b_1 - a_{11}x(t-\tau_{11}) - a_{12}y(t-\tau_{12}], \\ \dot{y}(t) = y(t)[b_2 - a_{21}x(t-\tau_{21}) - a_{22}y(t-\tau_{22})], \end{cases}$$

where $x(t)$ and $y(t)$ denotes the density of population at time t, respectively, $b_i, a_{ij},\ i,j = 1,2$, and $\tau_{ij}, i,j = 1,2$ are positive constants. Using the linear matrix inequality(LMI) optimization approach and constructing suitable Lyapunov function, Qiu and Cao [41] obtained a set of easily verifiable sufficient conditions which guarantee the exponential stability of the positive equilibrium of system (1.1). Considering that the environment fluctuates randomly, we think that it is more reasonable to consider varying parameters of predator-prey systems. Inspired by the viewpoint, we modify system (1.1) as follows.

$$(1.2)\qquad \begin{cases} \dot{x}(t) = x(t)[b_1(t) - a_{11}(t)x(t-\tau_{11}(t)) - a_{12}(t)y(t-\tau_{12}(t)], \\ \dot{y}(t) = y(t)[b_2(t) - a_{21}(t)x(t-\tau_{21}(t)) - a_{22}(t)y(t-\tau_{22}(t))]. \end{cases}$$

Here we shall point out that discrete time models governed by difference equations are more appropriate to describe the dynamics relationship among populations than continuous ones when the populations have non-overlapping generations. Moreover, discrete time models can also provide efficient models of continuous ones for numerical simulations [4], [16], [42]. Thus it is reasonable and interesting to investigate discrete time systems governed by difference equations. The principle object of this article is to propose a discrete analogue system (1.2) and explore its dynamics.

Following the ideas of [4], [11], we will discretize the system (1.2). Assume that the average growth rates in system (1.2) change at regular intervals of time, then we can obtain the following modified system:

$$(1.3)\qquad \begin{cases} \dfrac{\dot{x}(t)}{x(t)} = b_1([t]) - a_{11}([t])x([t]-\tau_{11}([t])) - a_{12}([t])y([t]-\tau_{12}([t]), \\ \dfrac{\dot{y}(t)}{y(t)} = b_2([t]) - a_{21}([t])x([t]-\tau_{21}([t])) - a_{22}([t])y([t]-\tau_{22}([t])), \end{cases}$$

where $[t]$ denotes the integer part of $t, t \in (0,+\infty)$ and $t \neq 0,1,2,....$ We integrate (1.3) on any interval of the form $[k, k+1), k = 0,1,2,...,$ and obtain

$$(1.4)\begin{cases} x(t) = x(k)\exp\left\{b_1(k) - a_{11}(k)x(k-\tau_{11}(k)) - a_{12}(k)y(k-\tau_{12}(k)(t-k)\right\}, \\ y(t) = y(k)\exp\left\{b_2(k) - a_{21}(k)x(k-\tau_{21}(k)) - a_{22}(k)y(k-\tau_{22}(k))(t-k)\right\}, \end{cases}$$

where for $k \leq t < k+1, k = 0, 1, 2, ...$ Let $t \to k+1$, then (1.4) takes the following form:

$$(1.5) \quad \begin{cases} x(k+1) = x(k)\exp\{b_1(k) - a_{11}(k)x(k-\tau_{11}(k)) - a_{12}(k)y(k-\tau_{12}(k)\}, \\ y(k+1) = y(k)\exp\{b_2(k) - a_{21}(k)x(k-\tau_{21}(k)) - a_{22}(k)y(k-\tau_{22}(k))\}, \end{cases}$$

which is a discrete time analogue of system (1.2), where $k = 0, 1, 2, ...$ For the point of view of biology, we shall consider (1.5) together with the initial conditions $x(0) \geq 0, y(0) \geq 0$. The principle object of this article is to explore the dynamics of system (1.5). Applying the differential inequality theory and the method of Lyapunov function, we investigate the permanence and the globally asymptotically stability of system (1.5).

Throughout this paper, we assume that

(H1) b_i, a_{ij}, τ_{ij} with $i, j = 1, 2$ are non-negative sequences bounded above and below by positive constants.

Let $\tau = \sup\limits_{1 \leq i,j \leq 2, k \in Z} \{\tau_{ij}(k)\}$. We consider (1.5) together with the following initial conditions

$$(1.6) \quad \begin{aligned} x(\theta) = \varphi(\theta) \geq 0, y(\theta) = \psi(\theta) \geq 0, \theta \in N[-\tau, 0] = \{-\tau, -\tau+1, ..., 0\}, \\ \varphi(0) > 0, \psi(0) > 0. \end{aligned}$$

Obviously, the solutions of (1.5) and (1.6) are well defined for all $k \geq 0$ and satisfy

$$x(k) > 0, y(k) > 0, \text{for } k \in Z.$$

The remainder of the paper is organized as follows: in Section 2, we introduce some basic definitions and lemmas, some sufficient conditions for the permanence of system (1.5) are obtained. In Section 3, some sufficient conditions for the global stability of system (1.5) are established. The existence and stability of system (1.5) are analyzed in Section 4. In Section 5, an example which shows the feasibility of the main results is given. A brief conclusion is drawn in Section 6.

2. Permanence

For convenience, we always use the notations:

$$f^l = \inf_{k \in Z} f(k), \; f^u = \sup_{k \in Z} f(k),$$

where $f(k)$ is a non-negative sequence bounded above and below by positive constants. In order to obtain the main result of this paper, we shall first state the definition of permanence and several lemmas which will be useful in the proof of the main result.

Definition 2.1 [43] We say that system (1.5) is permanent if there are positive constants M_i, m_i $(i = 1, 2)$ such that for each positive solution $(x(k), y(k))$ of system (1.5) satisfies

$$m_1 \leq \lim_{k\to+\infty} \inf x(k) \leq \lim_{k\to+\infty} \sup x(k) \leq M_1,$$

$$m_2 \leq \lim_{k\to+\infty} \inf y(k) \leq \lim_{k\to+\infty} \sup y(k) \leq M_2.$$

Lemma 2.1 [43] *Assume that* $\{x(k)\}$ *satisfies* $x(k) > 0$ *and*

$$x(k+1) \leq x(k) \exp\{a(k) - b(k)x(k)\}$$

for $k \in N$*, where* $a(k)$ *and* $b(k)$ *are non-negative sequences bounded above and below by positive constants. Then*

$$\lim_{k\to+\infty} \sup x(k) \leq \frac{1}{b^l} \exp(a^u - 1).$$

Lemma 2.2 [43] *Assume that* $\{x(k)\}$ *satisfies*

$$x(k+1) \geq x(k) \exp\{a(k) - b(k)x(k)\},\ k \geq N_0,$$

$\lim_{k\to+\infty} \sup x(k) \leq x^*$ *and* $x(N_0) > 0$*, where* $a(k)$ *and* $b(k)$ *are non-negative sequences bounded above and below by positive constants and* $N_0 \in N$*. Then*

$$\lim_{k\to+\infty} \inf x(k) \geq \min\left\{\frac{a^l}{b^u} \exp\{a^l - b^u x^*\}, \frac{a^l}{b^u}\right\}.$$

Now we state our permanence result for system (1.5).

Theorem 2.1 *Let* M_1, M_2, m_1 *and* m_2 *be defined by* (2.4), (2.9), (2.15) *and* (2.20)*, respectively. In addition to condition* (H1)*, assume that the following condition*

$$\text{(H2)}\ b_1^l > a_{12}^u M_2, b_2^l > a_{21}^u M_1$$

holds, then system (1.5) *is permanent, that is, there exist positive constants* m_i, M_i $(i = 1, 2)$ *which are independent of the solution of system* (1.5)*, such that for any positive solution* $(x(k), y(k))$ *of system* (1.5) *with the initial condition* $x(0) \geq 0$*,* $y(0) \geq 0$*, one has*

$$m_1 \leq \lim_{k\to+\infty} \inf x(k) \leq \lim_{k\to+\infty} \sup x(k) \leq M_1,$$

$$m_2 \leq \lim_{k\to+\infty} \inf y(k) \leq \lim_{k\to+\infty} \sup y(k) \leq M_2.$$

Proof. Let $(x_1(k), x_2(k))$ be any positive solution of system (1.5) with the initial condition $(x(0), y(0))$. It follows from the first equation of system (1.5) that

$$\begin{aligned} x(k+1) &= x(k) \exp\{b_1(k) - a_{11}(k)x(k - \tau_{11}(k)) - a_{12}(k)y(k - \tau_{12}(k)\} \\ &\leq x(k) \exp\{b_1(k)\} \leq x(k) \exp\{b_1^u\}. \end{aligned} \tag{2.1}$$

It follows from (2.1) that

$$x(k-\tau_{11}(k)) \geq x(k)\exp\{-b_1^u\tau^u\}. \tag{2.2}$$

Substituting (2.2) into the first equation of system (1.5), it follows that

$$x(k+1) \leq x(k)[b_1^u - a_{11}^l \exp\{-b_1^u\tau^u\}x(k)]. \tag{2.3}$$

It follows from (2.3) and Lemma 2.1 that

$$\lim_{k\to+\infty}\sup x(k) \leq \frac{1}{a_{11}^l}\exp\{b_1^u\tau^u + b_1^u - 1\} := M_1. \tag{2.4}$$

For any positive constant $\varepsilon > 0$, it follows from (2.4) that there exists a $N_1 > 0$ such that for all $k > N_1$

$$x(k) \leq M_1 + \varepsilon. \tag{2.5}$$

By (1.5), we have

$$\begin{aligned} y(k+1) &= y(k)\exp\{b_2(k) - a_{21}(k)x(k-\tau_{21}(k)) - a_{22}(k)y(k-\tau_{22}(k))\} \\ &\leq y(k)\exp\{b_2(k)\} \leq y(k)\exp\{b_2^u\}, \end{aligned} \tag{2.6}$$

which leads to

$$y(k-\tau_{22}(k)) \geq y(k)\exp\{-b_2^u\tau^u\}. \tag{2.7}$$

Substituting (2.7) into the second equation of system (1.5), we have

$$\begin{aligned} y(k+1) &= y(k)\exp\{b_2(k) - a_{21}(k)x(k-\tau_{21}(k)) - a_{22}(k)y(k-\tau_{22}(k))\} \\ &\leq y(k)[b_2^u - a_{22}^l\exp\{-b_2^u\tau^u\}y(k)]. \end{aligned} \tag{2.8}$$

Thus it follows from Lemma 2.1 and (2.8) that

$$\lim_{k\to+\infty}\sup y(k) \leq \frac{1}{a_{22}^l}\exp\{b_2^u\tau^u + b_2^u - 1\} := M_2. \tag{2.9}$$

For any positive constant $\varepsilon > 0$, it follows from (2.9) that there exists a $N_2 > 0$ such that for all $k > N_2$

$$y(k) \leq M_2 + \varepsilon. \tag{2.10}$$

For $k \geq \max\{N_1, N_2\} + \tau^u$, it follows from the first equation of system (1.5) that

$$\begin{aligned} x(k+1) &= x(k)[b_1(k) - a_{11}(k)x(k-\tau_{11}(k)) - a_{12}(k)y(k-\tau_{12}(k))] \\ &\geq x(k)[b_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon)], \end{aligned} \tag{2.11}$$

which leads to

$$x(k-\tau_{11}(k)) \leq x(k)\exp\{-[b_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon)]\tau^u\}. \tag{2.12}$$

Substituting (2.12) into the first equation of system (1.5), it follows that

$$\begin{aligned} x(k+1) &\geq x(k)\{b_1^l - a_{12}^u(M_2+\varepsilon) \\ &\quad -a_{11}^u \exp\{-[b_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon)]\tau^u\}x(k)\}. \end{aligned} \tag{2.13}$$

According to Lemma 2.2, it follows from (2.13) that

$$\lim_{k\to+\infty} \inf x(k) \geq \min\{A_{1\varepsilon}, A_{2\varepsilon}\}, \tag{2.14}$$

where

$$\begin{aligned} A_{1\varepsilon} &= \frac{b_1^l - a_{12}^u(M_2+\varepsilon)}{a_{11}^u \exp\{-[b_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon)]\tau^u\}} \\ &\quad \times \exp\Big\{b_1^l - a_{12}^u(M_2+\varepsilon) - a_{11}^u \exp\{-[b_1^l - a_{11}^u(M_1+\varepsilon) \\ &\quad -a_{12}^u(M_2+\varepsilon)]\tau^u\}\Big\}M_1, \\ A_{2\varepsilon} &= \frac{b_1^l - a_{12}^u(M_2+\varepsilon)}{a_{11}^u \exp\{-[b_1^l - a_{11}^u(M_1+\varepsilon) - a_{12}^u(M_2+\varepsilon)]\tau^u\}}. \end{aligned}$$

Setting $\varepsilon \to 0$ in (2.14), we can get

$$\lim_{k\to+\infty} \inf x(k) \geq \frac{1}{2}\min\{A_1, A_2\} := m_1, \tag{2.15}$$

where

$$\begin{aligned} A_1 &= \frac{b_1^l - a_{12}^u M_2}{a_{11}^u \exp\{-[b_1^l - a_{11}^u M_1 - a_{12}^u M_2]\tau^u\}} \\ &\quad \times \exp\Big\{b_1^l - a_{12}^u M_2 - a_{11}^u \exp\{-[b_1^l - a_{11}^u M_1 - a_{12}^u M_2]\tau^u\}\Big\}M_1, \\ A_2 &= \frac{b_1^l - a_{12}^u M_2}{a_{11}^u \exp\{-[b_1^l - a_{11}^u M_1 - a_{12}^u M_2]\tau^u\}}. \end{aligned}$$

For $k \geq \max\{N_1, N_2\} + \tau^u$, from the second equation of system (1.5), we have

$$\begin{aligned} y(k+1) &= y(k)[b_2(k) - a_{21}(k)x(k-\tau_{21}(k)) - a_{22}y(k-\tau_{22}(k))] \\ &\geq y(t)[b_2^l - a_{21}^u(M_1+\varepsilon) - a_{22}^u(M_2+\varepsilon)], \end{aligned} \tag{2.16}$$

which leads to

$$y(k-\tau_{22}(k)) \leq y(k)\exp\{b_2^l - a_{21}^u(M_1+\varepsilon) - a_{22}^u(M_2+\varepsilon)]\tau^u\}. \tag{2.17}$$

Substituting (2.17) into the second equation of system (1.5), it follows that

$$\begin{aligned} y(k+1) &\geq y(k)\{b_2^l - a_{22}^u \exp\{[b_2^l - a_{21}^u(M_1+\varepsilon) - a_{22}^u(M_2+\varepsilon)]\tau^u\}y(k) \\ &\quad -a_{21}^u(M_1+\varepsilon)\}. \end{aligned} \tag{2.18}$$

By Lemma 2.2 and (2.18), we can get

$$\lim_{k\to+\infty}\inf y(k) \geq \min\{B_{1\varepsilon}, B_{2\varepsilon}\}, \tag{2.19}$$

where

$$\begin{aligned} B_{1\varepsilon} &= \frac{b_2^l - a_{21}^u(M_1+\varepsilon)}{a_{22}^u\exp\{-[b_2^l - a_{21}^u(M_1+\varepsilon) - a_{22}^u(M_2+\varepsilon)]\tau^u\}}\exp\Big\{b_2^l - a_{21}^u(M_1+\varepsilon) \\ &\quad -a_{22}^u\exp\{-[b_2^l - a_{21}^u(M_1+\varepsilon) - a_{22}^u(M_2+\varepsilon)]\tau^u\}\Big\}M_2, \\ B_{2\varepsilon} &= \frac{b_2^u - a_{21}^u(M_1+\varepsilon)}{a_{22}^u\exp\{[b_2^l - a_{21}^l(M_1+\varepsilon) - a_{22}^u(M_2+\varepsilon)]\tau^u\}}. \end{aligned}$$

Setting $\varepsilon \to 0$ in above inequality, it follows that

$$\lim_{k\to+\infty}\inf y(k) \geq \frac{1}{2}\min\{B_1, B_2\} := m_2, \tag{2.20}$$

where

$$\begin{aligned} B_1 &= \frac{b_2^l - a_{21}^u M_1}{a_{22}^u\exp\{-[b_2^l - a_{21}^u M_1 - a_{22}^u M_2]\tau^u\}} \\ &\quad \times\exp\Big\{b_2^l - a_{21}^u M_1 - a_{22}^u\exp\{-[b_2^l - a_{21}^u M_1 - a_{22}^u M_2]\tau^u\}\Big\}M_2, \\ B_2 &= \frac{b_2^u - a_{21}^u M_1}{a_{22}^u\exp\{[b_2^l - a_{21}^l M_1 - a_{22}^u M_2]\tau^u\}}. \end{aligned}$$

By (2.4), (2.9), (2.15) and (2.20), we can conclude that system (1.5) is permanent. The proof of Theorem 2.1 is complete. ∎

Remark 2.1. Under the assumption of Theorem 2.1, the set $[m_1, M_1]\times[m_2, M_2]$ is an invariant set of system (1.5).

3. Global stability

In this section, we formulate the stability property of positive solutions of system (1.5) when all the time delays are zero.

Theorem 3.1 *Let* $\tau_{ij} = 0 (i, j = 1, 2)$. *In addition to* (H1)–(H2), *assume further that* (H3)

$$\begin{aligned} \Theta_1 &= \max\{|1 - a_{11}^u M_1|, |1 - a_{11}^l m_1|\} + a_{12}^u M_2 < 1, \\ \Theta_2 &= \max\{|1 - a_{22}^u M_2|, |1 - a_{22}^l m_3|\} + a_{21}^u M_1 < 1. \end{aligned}$$

Then for any positive solutions $(x(k), y(k))$ *and* $(x^*(k), y^*(k))$ *of system* (1.5), *we have*

$$\lim_{k\to\infty}[x^*(k) - x(k)] = 0, \ \lim_{k\to\infty}[y^*(k) - y(k)] = 0.$$

Proof. Let

$$x(k) = x^*(k)\exp(u(k)), y(k) = y^*(k)\exp(v(k)). \tag{3.1}$$

Then system (1.5) is equivalent to

$$\begin{cases} u(k+1) = u(k) - a_{11}(k)x^*(k)(\exp(u(k)) - 1) \\ \qquad\qquad -a_{12}(k)y^*(k)(\exp(v(k)) - 1) \\ v(k+1) = v(k) - a_{21}(k)x^*(k)(\exp(u(k)) - 1), \\ \qquad\qquad -a_{22}(k)y^*(k)(\exp(v(k)) - 1). \end{cases} \tag{3.2}$$

Then

$$\begin{cases} u(k+1) = u(k) - a_{11}(k)x^*(k)(\exp(\theta_{11}(k)u(k))u(k) \\ \qquad\qquad -a_{12}(k)y^*(k)(\exp(\theta_{12}(k)y(k))y(k) \\ v(k+1) = v(k) - a_{21}(k)x^*(k)(\exp(\theta_{21}(k)u(k))u(k), \\ \qquad\qquad -a_{22}(k)y^*(k)(\exp(\theta_{22}(k)v(k))v(k), \end{cases} \tag{3.3}$$

where $\theta_{ij}(k) \in [0,1](i,j = 1,2)$. To complete the proof, it suffices to show that

$$\lim_{k\to+\infty} u(k) = 0, \lim_{k\to+\infty} v(k) = 0. \tag{3.4}$$

In view of (H3), we can choose $\varepsilon > 0$ small enough such that

$$\Theta_1^\varepsilon = \max\{|1 - a_{11}^u(M_1+\varepsilon)|, |1 - a_{11}^l(m_1-\varepsilon)|\} + a_{12}^u(M_2+\varepsilon) < 1, \tag{3.5}$$

$$\Theta_2^\varepsilon = \max\{|1 - a_{22}^u(M_2+\varepsilon)|, |1 - a_{22}^l(m_2-\varepsilon)|\} + a_{21}^u(M_1+\varepsilon) < 1. \tag{3.6}$$

For above $\varepsilon > 0$, in view of Theorem 2.1 in Section 2, there exists a $k^* \in N$ such that

$$m_1 - \varepsilon \le x^*(k) \le M_1 + \varepsilon, m_2 - \varepsilon \le y^*(k) \le M_2 + \varepsilon \text{ for all } k \ge k^*.$$

Noticing that $\theta_{ij}(k) \in [0,1]$ $(i,j = 1,2)$ implies that $x^*(k)\exp(\theta_{ij}(k)u(k))$ lies between $x^*(k)$ and $x(k)$ and $y^*(k)\exp(\theta_{ij}(k)v(k))$ lies between $y^*(k)$ and $y(k)$. From (3.3), we have

$$u(k+1) \le \max\{|1 - a_{11}^u(M_1+\varepsilon)|, |1 - a_{11}^l(m_1-\varepsilon)|\}|u(k)| + a_{12}^u(M_2+\varepsilon)|v(k)|, \tag{3.7}$$

$$v(k+1) \le \max\{|1 - a_{22}^u(M_2+\varepsilon)|, |1 - a_{22}^l(m_3-\varepsilon)|\}|v(k)| + a_{21}^u(M_1+\varepsilon)|u(k)|. \tag{3.8}$$

Let $\rho = \max\{\Theta_1^\varepsilon, \Theta_2^\varepsilon\}$, then $\rho < 1$. By (3.7) and (3.8), for $k \ge k^*$, we have

$$\max\{|u(k+1)|, |v(k+1)|\} \le \rho\max\{|u(k)|, |v(k)|\},$$

which implies

$$\max\{|u(k)|, |v(k)|\} \leq \rho^{k-k^*} \max\{|u(k^*)|, |v(k^*)|\}.$$

Thus (3.4) holds true and the proof is completed. ∎

4. Existence and stability of periodic solution

In this section, we further assume that $\tau_{ij} = 0 (i, j = 1, 2)$ and the coefficients of system (1.5) satisfy the following condition

(H4) There exists a positive integer ω such that for $k \in N$, $0 < b_i(k+\omega) = b_i(k)$, $0 < a_{ij}(k+\omega) = a_{ij}(k)$ $(i, j = 1, 2)$.

Theorem 4.1 *Assume that* (H1)–(H4) *are satisfied, then system* (1.5) *with all the delays* $\tau_{ij} = 0$ $(i, j = 1, 2)$ *admits at least one positive* ω*-periodic solution which we denote by* $(x^*(k), y^*(k))$.

Proof. As pointed out in Remark 2.1 of Section 2,

$$D^2 \stackrel{\text{def}}{=} [m_1, M_1] \times [m_2, M_2]$$

is an invariant set of system (1.5). Then we define a mapping F on D^2 by

$$F(x_1(0), x_2(0)) = (x_1(\omega), x_2(\omega)), \text{for } (x_1(0), x_2(0)) \in D^2.$$

Clearly, F depends continuously on $(x_1(0), x_2(0))$. Thus F is continuous and maps the compact set D^2 into itself. Therefore , F has a fixed point. It is not difficult to see that the solution $(x_1^*(k), x_2^*(k))$ passing through this fixed point is an ω-periodic solution of the system (1.5). The proof of Theorem 4.1 is complete. ∎

Theorem 4.2 *Assume that* (H1)–(H4) *are satisfied, then system (1.5) with all the delays* $\tau_{ij} = 0(i, j = 1, 2)$ *has a global stable positive* ω*-periodic solution.*

Proof. Under the assumptions (H1)–(H4), it follows from Theorem 4.1 that system (1.5) with all the delays $\tau_{ij} = 0(i, j = 1, 2)$ admits at least one positive ω-periodic solution. In addition, Theorem 3.1 ensures the positive solution to be globally stable. Hence the proof. ∎

5. Numerical example

In this section, we will give an example which shows the feasibility of the main results (Theorem 2.1) of this paper. Let us consider the following discrete system:

$$(5.1)\quad \begin{cases} x(k+1) = x(k)\exp\left\{\left(2 + 0.03\sin\dfrac{k\pi}{2}\right) - \left(0.4 + 0.01\sin\dfrac{k\pi}{2}\right)x(k-1) \right. \\ \qquad\qquad \left. - \left(0.05 + 0.02\cos\dfrac{k\pi}{2}\right)y(k-1)\right\}, \\ y(k+1) = y(k)\exp\left\{\left(3 + 0.05\cos\dfrac{k\pi}{2}\right) - \left(0.5 + 0.2\sin\dfrac{k\pi}{2}\right)x(k-1) \right. \\ \qquad\qquad \left. - \left(0.4 + 0.02\sin\dfrac{k\pi}{2}\right)y(k-1)\right\}. \end{cases}$$

Here $b_1(k) = 2 + 0.03\sin\frac{k\pi}{2}$, $b_2(t) = 3 + 0.05\cos\frac{k\pi}{2}$, $a_{11}(k) = 0.4 + 0.01\sin\frac{k\pi}{2}$, $a_{12}(k) = 0.05 + 0.02\cos\frac{k\pi}{2}$, $a_{21}(k) = 0.5 + 0.2\sin\frac{k\pi}{2}$, $a_{22}(k) = 0.4 + 0.02\sin\frac{k\pi}{2}$, $\tau_{ij}(k) = 1$ $(i, j = 1, 2)$. All the coefficients $b_i(k)$ $(i = 1, 2)$, $a_{ij}(k)$ $(i, j = 1, 2)$, $\tau_{ij}(k)$ $(i, j = 1, 2)$ are functions with respect to k, and it is not difficult to obtain that $b_1^u = 2.03$, $b_1^l = 1.97$, $b_2^u = 3.05$, $b_2^l = 2.95$, $a_{11}^u = 0.401$, $a_{11}^l = 0.39$, $a_{12}^u = 0.07$, $a_{12}^l = 0.03$, $a_{21}^u = 0.7$, $a_{21}^l = 0.3$, $a_{22}^u = 0.42$, $a_{22}^l = 0.38$, $\tau_{ij}^u = \tau_{ij}^l = 1$ $(i, j = 1, 2)$. It is easy to check that all the conditions of Theorem 2.1 are fulfilled. Thus system (5.1) is permanence which is shown in Figure 1.

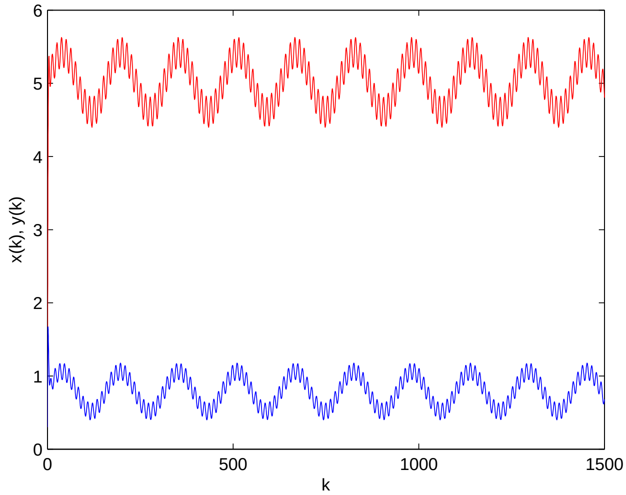

Figure 1: The dynamical behavior of the first the second components of the solution $(x(k), y(k))$ system (5.1), where the red line stands for $x(k)$ and the blue line stands for $y(k)$.

6. Conclusions

In this paper, we have considered the dynamical behavior of a discrete competitive Lotka-Volterra predator-prey model with multiple delays. A set of sufficient conditions which ensure the permanence of the model are obtained. Moreover, we also discuss the global stability of the model with all the delays $\tau_{ij}(k) = 0$ $(i, j = 1, 2)$ and deal with the existence and stability of the system. It is showed that delay has important influence on the permanence of system. Therefore, delay plays an key role in the permanence of the model. When all the delays are zero, we establish some sufficient conditions which guarantee the global stability of the model. Some simulations are presented to illustrate our main theoretical findings.

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