HYPERCYCLIC AND SUPERCYCLIC OPERATORS SATISFYING GENERALIZED A-WEYL'S THEOREM

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Abstract. A Banach space operator T satisfies generalized a-Weyl's theorem if the complement of its upper semi B-Weyl spectrum in its approximate point spectrum is the set of eigenvalues of T which are isolated in the approximate spectrum of T. In this note we characterize hypecyclic and supercyclic operators satisfying generalized a-Weyl's theorem.

Keywords: Weyl's and a-Weyl's theorems;generalized Weyl's and generalized a-Weyl's theorems; Browder's and a-Browder's theorems; generalized Browder's and generalized a-Browder's; hypercyclicity; supercyclicity.

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1. Introduction

The open problem in operator theory known as the invariant subspace problem assert that: any Hilbert space operator acting on an infinitely-dimensional, separable, complex Hilbert space operator has proper invariant subspaces. In [23] the author gave a negative answer to this problem in the case of a Banach space operators. As for Hilbert space operators, this is also an open problem. In [22], it is proved that this problem has a negative answer if and only if there is some hypercyclic operator T on the Hilbert space H such that any nonzero x in H is a hypercyclic vector of T. Hence interest in hypercyclic operators arises from the invariant subspace problem. Recall that an operator T on the Hilbert space H is hyponormal if $T^*T \geq TT^*$. In [13], it is proved that hyponormal operators satisfy generalized Weyl's theorem. However, in [19, Corollary 4.5], it is proved that hyponormal operator are not hypercyclic. Hence it is naturel to ask about the relation between hypercyclic/supercyclic operators and Weyl type theorems. In this direction and using Herrero's results [16] about spectral properties satisfied by hypercyclic/supercyclic operators, Duggal in [14] gave necessary and sufficient conditions for hypercyclic/supercyclic Banach spaces operator to satisfy a-Weyl's theorem. Hypercyclic/supercyclic operators which satisfy Weyl type theorems are also recently studied in [12], [25].

In this note, we prove that if T is a hypercyclic or supecyclic Banach space operator, then T and T^* satisfy generalized a-Browder's theorem. More, we give necessary and/ or sufficient conditions for hypercyclic and supercyclic Banach space operators to satisfies generalized a-Weyl's theorem.

2. Notation and terminology

Let X an infinite complex separable Banach space and $T \in \mathcal{L}(X)$ be the algebra of all bounded linear operator on X. The T-orbit of a vector space $x \in X$ is the set

$$O(x,T) = \{T^n x \ ; \ n \in \mathbb{N}\}.$$

The operator T is said to be *hypercyclic* if there is some vector $x \in X$ such that $O(x,T)$ is dense in X. Similarly, T is said to be *supercyclic* if there exists a vector $x \in X$ such that the set

$$\mathbb{C}O(x,T) = \{\lambda T^n x \ ; \ n \in \mathbb{N}, \ \lambda \in \mathbb{C}\}$$

is dense in X. The sets of all hypercyclic operators and supercyclic operators will be denoted by $HC(X)$ and $SC(X)$ respectively.

Recall that $T \in \mathcal{L}(X)$ is said to be bounded below, if T is injective and the range $R(T)$ of T is closed. Denote the approximate point spectrum of T by

$$\sigma_a(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is not bounded below}\}.$$

Let

$$\sigma_s(T) = \{\lambda \in \mathbb{C} : T - \lambda I \text{ is not surjective}\}$$

denote the surjectivity spectrum of T. Clearly, $\sigma(T) = \sigma_a(T) \cup \sigma_s(T)$, the spectrum of T. Denote by $\alpha(T)$ the dimension of the kernel $\ker T$, and by $\beta(T)$ the codimension of the range $R(T)$. An operator $T \in \mathcal{L}(X)$ is said to be upper semi-Fredholm (respectively, lower semi-Fredholm) operators if $R(T)$ is closed and $\alpha(T)$ is finite (respectively, $\beta(T)$ is finite). In this case the index of T is defined by $ind(T) = \alpha(T) - \beta(T)$. An operator T is said to be a Fredholm operator if it is both upper semi-Fredholm and lower semi-Fredholm. $SF_+(X)$ and $SF_-(X)$ will stand for the set of all upper semi-Fredholm operators and by the set of all lower semi-Fredholm operators, respectively. This two classes of operators generate the upper semi-Fredholm spectrum and lower semi-Fredholm spectrum defined by

$$\sigma_{SF_+}(T) = \{\lambda \in \mathbb{C} : T - \lambda \notin SF_+(X)\}$$

and

$$\sigma_{SF_-}(T) = \{\lambda \in \mathbb{C} : T - \lambda \notin SF_-(X)\}.$$

In addition the set

$$SF_+^-(X) = \{T \in SF_+(X) : ind(T) \leq 0\}$$

generates the following Weyl essential approximate point spectrum

$$\sigma_{SF_+^-}(T) = \{\lambda \in \mathbb{C} : T - \lambda \notin SF_+^-(X)\}.$$

Recall that the ascent $a(T)$ of an operator T, is defined by $a(T) = \inf\{n \in \mathbb{N} : \ker T^n = \ker T^{n+1}\}$ and the descent $d(T) = \inf\{n \in \mathbb{N} : T^n X = T^{n+1} X\}$, with $\inf \emptyset = \infty$. It is well known that if $a(T)$ and $d(T)$ are both finite, then $a(T) = d(T)$ (see [17] for more details).

The Weyl essential approximate point spectrum and the Browder essential approximate point spectrum of $T \in \mathcal{L}(X)$ are the sets

$$\sigma_{aw}(T) = \{\lambda \in \sigma_a(T) : T - \lambda I \ \text{ is not upper semi-Fredholm or } \ 0 < ind(T - \lambda I)\}$$

and

$$\sigma_{ab}(T) = \{\lambda \in \sigma_a(T) : \lambda \in \sigma_{aw}(T) \ \text{ or } \ a(T - \lambda I) = \infty\}.$$

It is clear that

$$\sigma_{SF_+}(T) \subseteq \sigma_{aw}(T) \subseteq \sigma_{ab}(T) \subseteq \sigma_a(T).$$

An operator $T \in \mathcal{L}(X)$ is said to be Weyl operator if it is Fredholm operator of index zero. The Weyl spectrum $\sigma_w(T)$ is defined by

$$\sigma_w(T) = \{\lambda \in \mathbb{C} : T - \lambda I \ \text{ is not Weyl operator}\}.$$

For $T \in \mathcal{L}(X)$ and a nonnegative integer n define T_n to be the restriction of T to $R(T^n)$ viewed as a map from $R(T^n)$ into $R(T^n)$. If for some integer n the range space $R(T^n)$ is closed and T_n is semi-Fredholm operator, upper or lower, in the usual sense, then T is called semi B-Fredholm operator. In this case the index of T is defined as the index of the semi-Fredholm operator T_n.

Similarly, we define the upper semi B-Fredholm, lower semi B-Fredholm and B-Fredholm operator of T by:

$$\sigma_{SBF_+}(T) = \{\lambda \in \sigma(T) : T - \lambda \text{ is not upper semi B-Fredholm }\},$$

$$\sigma_{SBF_-}(T) = \{\lambda \in \sigma(T) : T - \lambda \text{ is not lower semi B-Fredholm }\}$$

and

$$\sigma_{BF}(T) = \{\lambda \in \sigma(T) : T - \lambda \text{ is not B-Fredholm }\},$$

respectively. If T is B-Fredholm operator of index zero, then T is said to be a B-Weyl operator and the B-Weyl spectrum is defined by:

$$\sigma_{bw}(T) = \{\lambda \in \mathbb{C} : T - \lambda \text{ is not B-Weyl operator}\}.$$

A complex number $\lambda \in \sigma(T)$ is a pole of resolvent of $T \in \mathcal{L}(X)$ if $T - \lambda I$ has finite ascent and descent. Apparently, if $\lambda \in \sigma(T)$ is a pole of the resolvent of $T \in \mathcal{L}(X)$, then $\lambda \in iso\sigma(T)$, the set of isolated points of $\sigma(T)$. To introduce the definitions of the main notions studied in this work, some notations are needed. Let $T \in \mathcal{L}(X)$ and denote by $\Pi(T) = \{\lambda \in \mathbb{C} : a(T - \lambda I) = d(T - \lambda I) < \infty\}$ (respectively, $\Pi_0(T) = \{\lambda \in \Pi(T) : \alpha(T - \lambda I) < \infty\}$) the set of poles of T (respectively, the set of poles of finite rank of T). Similarly denote by $\Pi^a(T) = \{\lambda \in \sigma_a(T) : a(T - \lambda I) = d < \infty \text{ and } (T - \lambda I)^{d+1}E \text{ is closed}\}$ (respectively, $\Pi_0^a(T) = \{\lambda \in \Pi^a(T) : \alpha(T - \lambda I) < \infty\}$) the set of left poles of T (respectively, the set of left poles of finite rank of T). In the sequel let $I(T) = iso\sigma(T) \setminus \Pi(T)$ and $I^a(T) = iso\sigma_a(T) \setminus \Pi^a(T)$. In addition, given $T \in \mathcal{L}(X)$, let $E(T) = \{\lambda \in iso\sigma(T) : 0 < \alpha(T - \lambda I)\}$ (respectively, $E_0(T) = \{\lambda \in E(T) : \alpha(T - \lambda I) < \infty\}$) the set of eigenvalues of T which are isolated in the spectrum of T (respectively, the set of eigenvalues of finite multiplicity isolated in $\sigma(T)$). Now we introduce the main notions that has been studied in [5], [9], [10].

Definition 2.1 Let $T \in \mathcal{L}(X)$. Then it will be said that

(i) Browder's theorem holds for T, if $\sigma_w(T) = \sigma(T) \setminus \Pi_0(T)$,

(ii) Generalized Browder's theorem holds for T, if $\sigma_{bw}(T) = \sigma(T) \setminus \Pi(T)$,

(iii) a-Browder's theorem holds for T, if $\sigma_{aw}(T) = \sigma_a(T) \setminus \Pi_0^a(T)$,

(iv) Generalized a-Browder's theorem holds for T, if $\sigma_{SBF_+^-}(T) = \sigma_a(T) \setminus \Pi^a(T)$.

Definition 2.2 Let $T \in \mathcal{L}(X)$. Then it will be said that

(i) Weyl's theorem holds for T, if $\sigma_w(T) = \sigma(T) \setminus E_0(T)$,

(ii) Generalized Weyl's theorem holds for T, if $\sigma_{bw}(T) = \sigma(T) \setminus E(T)$,

(iii) Property (w) holds for T, if $\sigma_{aw}(T) = \sigma_a(T) \setminus E_0(T)$,

(iv) Property (gw) holds for T, if $\sigma_{SBF_+^-}(T) = \sigma_a(T) \setminus E(T)$.

Recall that an operator $T \in \mathcal{L}(X)$ is said to have the single valued extension property at $\lambda_0 \in \mathbb{C}$ (abbreviated SVEP at λ_0), if for every open disc $\mathbb{D}$ centered at λ_0, the only analytic function $f : \mathbb{D} \to \mathcal{X}$ which satisfies the equation $(T - \lambda I)f(\lambda) = 0$ for all $\lambda \in \mathbb{D}$ is the function $f \equiv 0$. An operator $T \in L(\mathcal{X})$ is said to have SVEP if T has SVEP at every $\lambda \in \mathbb{C}$, see [1], [21] for more details about the SVEP.

3. Hypercyclic/Supercyclic operators satisfying generalized a-Weyls's theorem

In the sequel, we will use the following lemma.

Lemma 3.1 *Let $T \in \mathcal{L}(X)$. If $T \in HC(\mathrm{X}) \cup SC(\mathrm{X})$, then the SVEP holds for T^*.*

Proof. if $T \in SC(\mathrm{X})$, then from [7]

$$\sigma_p(\mathrm{T}^*) = \emptyset \text{ or } \sigma_p(\mathrm{T}^*) = \{\alpha\}, \text{ where } \alpha \in \mathbb{C}\backslash\{0\}.$$

Hence T^* satisfies the SVEP. ∎

Recall that an operator $T \in \mathcal{L}(X)$ is Drazin invertible if and only if it has a finite ascent and descent, which is also equivalent to the fact $T = T_0 \oplus T_1$, where T_0 is a nilpotent operator and T_1 is an invertible operator, see [20, Proposition A]. The Drazin spectrum is given by

$$\sigma_D(T) = \{\lambda \in \mathbb{C} \text{ such that } T - \lambda \text{ is not Drazin invertible }\}.$$

We observe that

$$\sigma_D(T) = \sigma(T) \setminus \Pi(T).$$

An operator $T \in \mathcal{L}(X)$ is called left Drazin invertible if $a(T) < \infty$ and $R(T^{a(T)+1})$ is closed [10, Definition 2.4]. The left Drazin spectrum is given by

$$\sigma_{ld}(T) = \{\lambda \in \mathbb{C} \text{ such that } T - \lambda \text{ is not lefty Drazin invertible }\}.$$

Similarly, we have that $\sigma_{ld}(T) = \sigma_a(T) \setminus \Pi^a(T)$. In [2] the authors proved that:

T satisfies generalized Browder's theorem if and only if $\sigma_{bw}(T) = \sigma_D(T)$

and

T satisfy generalized a-Browder's theorem if and only if $\sigma_{SBF_+^-}(T) = \sigma_{ld}(T)$.

Using this results, we will prove the following theorem.

Theorem 3.2 *Let $T \in \mathcal{L}(X)$. If $T \in HC(\mathrm{X}) \cup SC(\mathrm{X})$, then T and T^* satisfy generalized a-Browder's theorem.*

Proof. By Lemma 3.1, the SVEP holds for T^*. Hence from [4, Theorem 3.2], T satisfies generalized a-Browder's theorem. Now, we shows that generalized a-Browder's theorem holds for T^*. We have that

$$\sigma_{SBF_+^-}(T^*) \subset \sigma_{ld}(T^*).$$

To prove that T^* also satisfies a-Browder's theorem it suffice to prove that

$$\sigma_{SBF_+^-}(T^*) \supset \sigma_{ld}(T^*).$$

For this let $\lambda \notin \sigma_{SBF_+^-}(T^*)$, then $T^* - \lambda$ is upper semi B-Fredholm and $ind(T^* - \lambda) > 0$. Since T^* has SVEP, then

$$d(T^* - \lambda) < \infty.$$

By [1, Theorem 3.17], we have that $ind(T^* - \lambda) \leq 0$. Hence $ind(T^* - \lambda) = 0$ and $T^* - \lambda$ is B-Fredholm. Since $d(T^* - \lambda) < \infty$, then by [1, Theorem 3.3] $a(T^* - \lambda) = d(T^* - \lambda)$ and hence$\lambda \notin \sigma_{ld}(T^*)$. This complete the result since $HC(\mathrm{X}) \subset SC(\mathrm{X})$. ∎

For $T \in \mathcal{L}(X)$ denote by $H(\sigma(T))$ the set of all analytic function on a neighbourhood of $\sigma(T)$. In the following corollary, we prove that the conclusion of Theorem 3.2 holds for $f(T)$ whenever $T \in HC(\mathrm{X}) \cup SC(\mathrm{X})$.

Corollary 3.3 *Let $T \in \mathcal{L}(X)$. If $T \in HC(\mathrm{X}) \cup SC(\mathrm{X})$, then $f(T)$ satisfies generalized a-Browder's theorem for every $f \in H(\sigma(T))$.*

Proof. Since from Lemma 3.1 T^* satisfies the SVEP, then by [21] $f(T^*)$ also satisfies the SVEP for every $f \in H(\sigma(T))$. Hence from [4, Theorem 3.2], $f(T)$ also satisfies generalized a-Browder's theorem. ∎

In what follow, we will prove that the adjoint of hypercyclic or supercyclic Banach space operator satisfy generalized Weyl's theorem.

Theorem 3.4 *Let $T \in \mathcal{L}(X)$.*

(i) *If $T \in HC(\mathrm{X}) \cup SC(\mathrm{X})$, then T^* satisfies generalized Weyl's theorem.*

(ii) *If $T \in HC(\mathrm{X}) \cup SC(\mathrm{X})$ and $E(T) \subseteq E(T^*)$, then T satisfies generalized a-Weyl's theorem.*

Proof. (i) If $T \in SC(\mathrm{X})$, then we can conclude that $E(T^*) = \Pi(T^*)$. Indeed, if $T \in SC(\mathrm{X})$ then either $E(T^*) = \emptyset$ or $E(T^*) = \{\alpha\}$ for non zero α such that $\alpha \notin \sigma_b(T^*)$. If $E(T^*) = \emptyset$, then $E(T^*) = \Pi(T^*) = \emptyset$. If $E(T^*) = \{\alpha\}$ with $\alpha \notin \sigma_b(T^*)$, then $\alpha \in \Pi(T^*)$ and hence $E(T^*) = \Pi(T^*)$. Since, by Theorem 3.2, T^* satisfies generalized a-Browder's theorem, then T^* satisfies Browder's theorem. It then follows from [6, Corollaray 2.1] that T^* satisfies generalized Weyl's theorem.

(ii) Assume that $E(T) \subseteq E(T^*)$. From Theorem 3.2, we conclude that T satisfies generalized a-Browder's theorem, and hence generalized Browder's theorem that is $\sigma(T)/\sigma_{bw}(T) = \Pi(T)$, hence

$$\sigma(T)/\sigma_{bw}(T) = \Pi(T) \subseteq E(T).$$

From the proof of (i), T^* satisfies the generalized Weyl's theorem, hence by [6, Corollary 2.1] we have that $E(T^*) = \Pi(T^*)$. Finally,

$$\begin{aligned} \sigma(T)/\sigma_{bw}(T) &= \Pi(T) \\ &\subseteq E(T) \\ &\subseteq E(T^*) \\ &= \Pi(T^*) \\ &= \Pi(T). \end{aligned}$$

Hence $\sigma(T)/\sigma_{bw}(T) = E(T)$ that is T satisfies generalized Weyl's theorem. Now, since T^* satisfies the SVEP, then $\sigma(T) = \sigma_a(T)$, $\Pi(T) = \Pi^a(T)$ and then $\sigma_{bw}(T) = \sigma_{SBF_{+^-}}(T)$ and from the fact that T satisfies generalized Weyl's theorem, we conclude from [6, Corollary 2.1] that $E(T) = \Pi(T)$, and hence $E(T) = \Pi^a(T)$.

Finally,

$$\sigma(T)/\sigma_{bw}(T) = E(T) = \Pi(T) = \sigma_a(T)/\sigma_{SBF_+^-}(T) = \Pi^a(T).$$

That is, T satisfies generalized a-Weyl's theorem. ∎

In the following, we will characterize generalized a-Weyl's theorem for hypercyclic and supercyclic Banach space operator in term of isolated eigenvalues in its spectrum.

Theorem 3.5 *For $T \in \mathcal{L}(X)$. We have the following assertions:*

(i) *Let $T \in HC(X)$. Then T satisfies generalized a-Weyl's theorem if and only if $E(T) = \emptyset$.*

(ii) *Let $T \in SC(X)$. Then T satisfies generalized a-Weyl's theorem if and only if there is an $\alpha \in \mathbb{C} \backslash \sigma_b(T)$ such that $E(T) \subset \{\alpha\}$.*

Proof. (i) If $T \in HC(X)$, then $E(T^*) = \emptyset$. Since by Theorem 3.4 T^* satisfies generalized Weyl's theorem, then from [6, Corollary 2.1], $E(T^*) = \Pi(T^*)$. If also T satisfies generalized a-Weyl's theorem, then it satisfies generalized Weyl's theorem and again we have $\Pi(T) = E(T) = \emptyset$. So $E(T^*) = \Pi(T^*) = \Pi(T) = E(T) = \emptyset$. For the converse, Assume that $E(T) = \emptyset$. From Theorem 3.2 T satisfies generalized a-Browder's theorem, hence we have $\sigma_a(T)/\sigma_{SBF_+^-}(T) = \Pi^a(T) = \emptyset = E(T) = E^a(T)$. That is T satisfies generalized a-Weyl's theorem.

(ii) If $T \in SC(X)$, then $E(T^*) = \emptyset$ or $E(T^*) = \{\alpha\}$ for a non zero $\alpha \in \mathbb{C}$ such that $\alpha \notin \sigma_b(T)$, that is $E(T^*) \subset \{\alpha\}$. If $E(T^*) = \emptyset$, we conclude as in the proof of (i). If $E(T^*) = \{\alpha\}$, then again $E(T^*) = \Pi(T^*) = \Pi(T) = E(T) = \{\alpha\}$, this since T and T^* satisfy generalized Weyl's theorem. For the converse, assume that there is an $\alpha \in \mathbb{C} \backslash \sigma_b(T)$ such that $E(T) \subset \{\alpha\}$. If $E(T^*) = \emptyset$, then as in the proof of i) T satisfies generalized a-Weyl' theorem. If $E(T^*) = \{\alpha\}$, then $E(T) \subset E(T^*) = \{\alpha\}$. By Theorem 3.4, T satisfies generalized a-Weyl's theorem. ∎

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