ON L-FUZZY 2-ABSORBING IDEALS

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Abstract. Let L be a complete lattice. In this paper we introduce various definitions of L-fuzzy 2-absorbing ideals of a commutative ring R and give some basic results concerning these classes of ideals.

Keywords: L-fuzzy 2-absorbing ideal, L-fuzzy strongly 2-absorbing ideal, L-fuzzy weakly completely 2-absorbing ideal, L-fuzzy K-2-absorbing ideal, L-fuzzy prime ideal.

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1. Introduction

Zadeh in [14] introduced the notion of a fuzzy subset μ of a non-empty set X as a function from X to [0; 1]. Goguen in [2] generalized the notion of a fuzzy subset of X to that of an L-fuzzy subset, namely a function from X to a lattice L.

Later Rosenfeld considered the fuzzification of algebraic structures [12]. Liu [5], introduced and examined the notion of a fuzzy ideal of a ring. Since then several authors have obtained interesting results on L-fuzzy ideals of R and L-fuzzy modules. See [8] for a comprehensive survey of the literature on these developments.

L-fuzzy prime ideals play an important role in fuzzy commutative ring theory. Let R be a commutative ring. Of course for a non-constant fuzzy ideal ξ : R → L, ξ is called an L-fuzzy prime ideal of R if for any L-fuzzy points x_r, y_s ∈ F(R), x_r y_s ∈ ξ implies that either x_r ∈ ξ or y_s ∈ ξ. A particular attention was paid to the fuzzy prime ideals and prime fuzzy ideals (see for example [3], [6], [9], [10], [11], [13], [15], [16] and the papers cited there). In this paper, we introduce some generalizations of L-fuzzy prime ideals, namely L-fuzzy 2-absorbing ideals.
2. Preliminaries

Throughout this paper $R$ is a commutative ring with a nonzero identity and $L$ stands for a complete lattice with least element 0 and greatest element 1. Given a nonempty set $X$, an $L$-fuzzy subset $\mu$ is a function from $X$ to $L$. We denote by $F(X)$ the set of all $L$-fuzzy subsets of $X$. For $\mu, \nu \in F(X)$ we say $\mu \subseteq \nu$ if and only if $\mu(x) \leq \nu(x)$, for all $x \in X$. Also, $\mu \subset \nu$ if and only if $\mu \subseteq \nu$ and $\mu \neq \nu$.

Let $\mu \in F(X)$ and $t \in L$. Then the set $\mu_t = \{x \in X \mid \mu(x) \geq t\}$ is called the $t$-level subset of $X$ with respect to $\mu$. By an $L$-fuzzy point $x_r$ of $X$, $x \in X$; $r \in L \setminus \{0\}$, we mean $x_r \in F(X)$ defined by

$$x_r(y) = \begin{cases} r, & \text{if } y = x; \\ 0, & \text{otherwise}. \end{cases}$$

If $x_r$ is an $L$-fuzzy point of $X$ and $x_r \subseteq \mu \in F(X)$, we write $x_r \in \mu$. For $A \subseteq X$ the characteristic function of $A$, $\chi_A \in F(X)$, is defined by

$$\chi_A(x) = \begin{cases} 1, & \text{if } x \in A; \\ 0, & \text{otherwise}. \end{cases}$$

We recall two following basic definitions given in [8].

**Definition 2.1** Let $\xi \in F(R)$. Then $\xi$ is called an $L$-fuzzy ideal of $R$ if for all $x, y \in R$,

(i) $\xi(x - y) \geq \xi(x) \land \xi(y)$,

(ii) $\xi(xy) \geq \xi(x) \lor \xi(y)$.

(iii) $\xi(0) = 1$.

We denote by $I(R)$ the set of all $L$-fuzzy ideals of $R$.

The following are two basic operations which will be used to define $L$-fuzzy prime ideals and their generalizations.

**Definition 2.2** Let $\xi, \mu \in F(R)$. Define the composition $\xi o \mu$ and the product $\xi \mu$ (both are $L$-fuzzy subsets of $R$) respectively as follows: For all $w \in R$,

$$\xi o \mu(w) = \sup \{\xi(r) \land \mu(s) \mid r, s \in R, w = rs\},$$

$$\xi \mu(w) = \sup \left\{ \inf \{\xi(r_i) \land \mu(s_i)\} \mid r_i, s_i \in R, n \in \mathbb{N}, w = \sum_{i=1}^{n} r_i s_i \right\},$$

where as usual the supremum of an empty set is taken to be 0.

Notice that $\xi o \mu$ is the case $n = 1$ in the definition of $\xi \mu$. Thus $\xi o \mu \subseteq \xi \mu$.

**Definition 2.3** ([10]) Let $A \in F(R)$. Then the $L$-fuzzy ideal of $R$ generated by $A$, denoted by $\langle A \rangle$, is defined to be the intersection of all $L$-fuzzy ideals of $R$ containing $A$. 
Lemma 2.4 Let $R$ be a commutative ring with identity and let $x_r$ and $y_s$ be two $L$-fuzzy points of $R$. Then

(i) $x_r y_s = (xy)_{\inf\{r,s\}}$. 

(ii) $\langle x_r \rangle \langle y_s \rangle = \langle x_r y_s \rangle$. 

Proof. [10].

Definition 2.5 ([6]) For a non-constant $\xi \in I(R)$, $\xi$ is called an $L$-fuzzy prime ideal of $R$ if for any $L$-fuzzy ideals $\mu, \nu \in I(R)$, $\mu \nu \subseteq \xi$ implies that either $\mu \subseteq \xi$ or $\nu \subseteq \xi$.

Definition 2.6 ([11]) For a non-constant $\xi \in I(R)$, $\xi$ is called an $L$-fuzzy completely prime ideal of $R$ if for any $L$-fuzzy points $x_r, y_s \in F(R)$, $x_r y_s \in \xi$ implies that either $x_r \in \xi$ or $y_s \in \xi$.

Definition 2.7 ([11]) A fuzzy ideal $\xi$ of $R$ is said to be a fuzzy weakly completely prime ideal if $\xi$ is a non-constant function and for all $x, y \in R$, $\xi(xy) = \max\{\xi(x), \xi(y)\}$.

Definition 2.8 ([4]) Let $\xi$ be a non-constant $L$-fuzzy ideal of $R$. $\xi$ is said to be an $L$-fuzzy $K$-prime ideal if $\xi(xy) = \xi(0)$ implies either $\xi(x) = \xi(0)$ or $\xi(y) = \xi(0)$ for any $x, y \in R$.

These different definitions of prime are not independent. In fact, in the commutative case, Definition 2.5 is equivalent to Definition 2.6. Moreover, Definition 2.6 $\Rightarrow$ Definition 2.7 $\Rightarrow$ Definition 2.8 ([3]).

Definition 2.9 ([1]) Let $R$ be a commutative ring with identity. A proper ideal $I$ of $R$ is said to be 2-absorbing provided that whenever $a, b, c \in R$ with $abc \in I$ then either $ab \in I$ or $ac \in I$ or $bc \in I$. $R$ is called a 2-absorbing ring if and only if its zero ideal is 2-absorbing.

3. $L$-fuzzy weakly completely 2-absorbing ideals and $L$-fuzzy $K$-2-absorbing ideals

In this section we first give two definitions for fuzzy 2-absorbing ideals of $R$, and then prove some fundamental properties between these classes of ideals and fuzzy quotient rings.

Definition 3.1 Let $\eta$ be an $L$-fuzzy ideal of $R$.

- $\eta$ is called an $L$-fuzzy weakly completely 2-absorbing ideal of $R$ if for all $x, y, z \in R$, $\eta(xyz) = \eta(xy)$ or $\eta(xyz) = \eta(xz)$ or $\eta(xyz) = \eta(yz)$.

- $\eta$ is called an $L$-fuzzy $K$-2-absorbing ideal of $R$ if $\eta(xyz) = \eta(0)$ implies that $\eta(xy) = \eta(0)$ or $\eta(xz) = \eta(0)$ or $\eta(yz) = \eta(0)$.
Let $\eta$ be a non-constant $L$-fuzzy ideal of $R$. It is easy to see that $\eta$ is an $L$-fuzzy weakly completely 2-absorbing ideal of $R$ if and only if, for every $x, y, z \in R$, 

$$
\eta(xyz) = \max\{\eta(xy), \eta(xz), \eta(yz)\}.
$$

Note also that every $L$-fuzzy weakly completely 2-absorbing ideal is $L$-fuzzy $K$-2-absorbing. But the following example shows that the converse is not necessarily true.

Example 3.2 Let $R = \mathbb{Z}$, the ring of integers. Define the fuzzy ideal $\eta$ of $\mathbb{Z}$ by

$$
(3.1) \quad \eta(x) = \begin{cases} 
1, & \text{if } x = 0; \\
\frac{1}{2}, & \text{if } x \in 8\mathbb{Z} - \{0\}; \\
\frac{1}{3}, & \text{if } x \in \mathbb{Z} - 8\mathbb{Z}.
\end{cases}
$$

Then $\eta$ is a fuzzy $K$-2-absorbing ideal of $\mathbb{Z}$. But since

$$
\eta(40) = \frac{1}{2} > \frac{1}{3} = \max\{\eta(20), \eta(4), \eta(20)\},
$$

$\eta$ is not a fuzzy weakly completely 2-absorbing ideal.

Proposition 3.3

(1) Every $L$-fuzzy weakly completely prime ideal of $R$ is $L$-fuzzy weakly completely 2-absorbing.

(2) Every $L$-fuzzy $K$-prime ideal of $R$ is $L$-fuzzy $K$-2-absorbing.

Proof. (1) Let $\mu$ be an $L$-fuzzy weakly completely prime ideal of $R$. Then, for every $x, y, z \in R$, $\mu(xyz) = \mu(x)$ or $\mu(xyz) = \mu(y)$ or $\mu(xyz) = \mu(z)$. Assume that $\mu(xyz) = \mu(x)$. Then from $\mu(xyz) \geq \mu(xy) \geq \mu(x)$ we get $\mu(xyz) = \mu(xy)$. In a similar way we can show that if $\mu(xyz) = \mu(y)$ or $\mu(xyz) = \mu(z)$, then $\mu(xyz) = \mu(yz)$ or $\mu(xyz) = \mu(xz)$. Hence $\mu$ is $L$-fuzzy weakly completely 2-absorbing.

(2) The proof is similar to that of (1).

Theorem 3.4 Let $\eta$ be an $L$-fuzzy ideal of $R$. The following statements are equivalent:

(i) $\eta$ is an $L$-fuzzy weakly completely 2-absorbing ideal of $R$.

(ii) For every $t \in L$, the $t$-level subset $\eta_t$ of $\eta$ is a 2-absorbing ideal of $R$.

Proof. (i)$\Rightarrow$(ii) Assume that $\eta$ is $L$-fuzzy weakly completely 2-absorbing and let $x, y, z \in R$ be such that $xyz \in \eta_t$ for some $t \in L$. Then $\max\{\eta(xy), \eta(xz), \eta(yz)\} = \eta(xyz) \geq t$. Hence $\eta(xy) \geq t$ or $\eta(xz) \geq t$ or $\eta(yz) \geq t$, that is $xy \in \eta_t$ or $xz \in \eta_t$ or $yz \in \eta_t$. Therefore that is $\eta_t$ is 2-absorbing in $R$.

(ii)$\Rightarrow$(i) Assume that $\eta_t$ is a 2-absorbing ideal of $R$ for every $t \in L$. For $x, y, z \in R$ set $\eta(xyz) = t$. Then $xyz \in \eta_t$ and $\eta_t$ 2-absorbing gives $xy \in \eta_t$ or $xz \in \eta_t$ or $yz \in \eta_t$. Thus $\eta(xy) \geq t$ or $\eta(xy) \geq t$ or $\eta(yz) \geq t$, that is
Let $\eta(xyz) = \max\{\eta(xy), \eta(xz), \eta(yz)\}$. Also since $\eta$ is a $L$-fuzzy ideal of $R$, we have $\eta(xyz) \geq \max\{\eta(xy), \eta(xz), \eta(yz)\}$.

Hence $\eta(xyz) = \max\{\eta(xy), \eta(xz), \eta(yz)\}$, that is $\eta$ is an $L$-fuzzy weakly completely $2$-absorbing ideal.

Here, we recall the definition of an $L$-fuzzy quotient ring of $R$ induced by an $L$-fuzzy ideal of $R$. Let $X$ be a set and $\mu$ an $L$-fuzzy relation on $X$. Then $\mu$ is called a fuzzy equivalence relation if (i) $\mu(x, x) = 1$ for all $x \in X$; (ii) $\mu(x, y) = \mu(y, x)$ for all $x, y \in X$; and (iii) for all $x, y \in X$, $\mu(x, y) \geq \sup_{z \in X} \min\{\mu(x, z), \mu(z, y)\}$. If $\mu$ is an $L$-fuzzy equivalence relation on $X$, then, for each $a \in X$, we shall denote $\mu[a](x) = \mu(a, x)$ for every $x \in X$. We shall say that $\mu[a]$ is the fuzzy class corresponding to $a$. In this case the set $X/\mu = \{\mu[a] : a \in X\}$ is called the fuzzy quotient set. Now let $\nu$ be an $L$-fuzzy ideal of $R$. It is proved in [4] that the relation $\mu$ on $R$ defined by $\mu(x, y) = \nu(x - y)$ is a fuzzy equivalence relation. Note that, for $x, y \in R$, $\mu[x] = \mu[y]$ if and only if $\nu(x - y) = 1$.

Define summation and multiplication of $L$-fuzzy classes as follows:

$$\mu[x] + \mu[y] = \mu[x + y] \quad \text{and} \quad \mu[x]\mu[y] = \mu[xy].$$

Then $R/\mu$ is an $L$-fuzzy ring with these operations. We will write $R/\nu$ for $R/\mu$ and call it $L$-fuzzy quotient ring of $R$ induced by the $L$-fuzzy ideal $\nu$.

**Theorem 3.5** Let $\eta$ be a non-constant $L$-fuzzy ideal of $R$. Then $\eta$ is an $L$-fuzzy $K$-$2$-absorbing ideal of $R$ if and only if $R/\eta$ is a $2$-absorbing ring.

**Proof.** Assume first that $\eta$ is an $L$-fuzzy $K$-$2$-absorbing ideal of $R$ and let $\mu[x], \mu[y], \mu[z] \in R/\eta$ be such that $\mu[x]\mu[y]\mu[z] = \mu[0]$. Since $\mu[x]\mu[y]\mu[z] = \mu[xyz]$, we have $\eta(xyz) = \eta(xyz - 0) = 1 = \eta(0)$. As $\eta$ is considered to be $L$-fuzzy $K$-$2$-absorbing, $\eta(xy) = \eta(0) = 1$ or $\eta(xz) = \eta(0) = 1$ or $\eta(yz) = \eta(0) = 1$. This implies that $\mu[x]\mu[y] = \mu[0]$ or $\mu[x]\mu[z] = \mu[0]$ or $\mu[y]\mu[z] = \mu[0]$, that is $R/\eta$ is a $2$-absorbing ring.

Conversely, assume that $R/\eta$ is a $2$-absorbing ring and let $\eta(xyz) = \eta(0) = 1$ for $x, y, z \in R$. Then we have $\mu[x]\mu[y]\mu[z] = \mu[xyz] = \mu[0]$. Since $R/\eta$ is $2$-absorbing, $\mu[x]\mu[y] = \mu[0]$ or $\mu[x]\mu[z] = \mu[0]$ or $\mu[y]\mu[z] = \mu[0]$. Hence $\eta(xy) = \eta(0)$ or $\eta(xz) = \eta(0)$ or $\eta(yz) = \eta(0)$, that is $\eta$ is $L$-fuzzy $K$-$2$-absorbing.

**Corollary 3.6** If $\eta$ is an $L$-fuzzy weakly completely $2$-absorbing ideal of $R$, then $R/\eta$ is a $2$-absorbing ring.

### 4. $L$-fuzzy $2$-absorbing ideals and $L$-fuzzy strongly $2$-absorbing ideals

**Definition 4.1** Let $\eta$ be an $L$-fuzzy ideal of $R$. $\eta$ is called an $L$-fuzzy $2$-absorbing ideal of $R$ if for any $L$-fuzzy points $x_r, y_s, z_t \in F(R)$ ($x, y, z \in R$ and $r, s, t \in L$), $x_r y_s z_t \in \eta$ implies that either $x_r y_s \in \eta$ or $x_t z_t \in \eta$ or $y_s z_t \in \eta$.

**Lemma 4.2** Every $L$-fuzzy prime ideal of $R$ is $L$-fuzzy $2$-absorbing.
Proof. The proof is straightforward.

Proposition 4.3 The intersection of every pair of distinct \( L \)-fuzzy prime ideals of \( R \) is an \( L \)-fuzzy 2-absorbing ideal of \( R \).

Proof. Let \( \eta \) and \( \mu \) be two distinct \( L \)-fuzzy prime ideals of \( R \). Assume that \( x_r, y_s, z_t \in F(R) \) are \( L \)-fuzzy points such that \( x_r y_s z_t \in \eta \cap \mu \) but \( x_r y_s \notin \eta \cap \mu \) and \( x_r z_t \notin \eta \cap \mu \). Then we have the following cases:

Case 1. \( x_r y_s \notin \eta \) and \( x_r z_t \notin \eta \). As \( \eta \) is an \( L \)-fuzzy prime ideal of \( R \) we will have \( z_t \notin \eta \). Therefore \( x_r z_t \notin \eta \) which is a contradiction.

Case 2. \( x_r y_s \notin \eta \) and \( x_r z_t \notin \mu \). In this case from \( x_r y_s z_t \in \eta \cap \mu \) we get \( z_t \in \eta \) and \( y_s \in \mu \). Hence \( y_s z_t \in \eta \cap \mu \).

Case 3. \( x_r y_s \notin \mu \) and \( x_r z_t \notin \eta \). By a similar argument as in the Case 2 we may show that \( y_s z_t \in \eta \cap \mu \).

Case 4. \( x_r y_s \notin \mu \) and \( x_r z_t \notin \mu \). A similar argument as in the Case 1 leads us to a contradiction.

Therefore \( \eta \cap \mu \) is \( L \)-fuzzy 2-absorbing.

Theorem 4.4 Let \( \eta \) be an \( L \)-fuzzy 2-absorbing ideal of \( R \). Then, for every \( t \in L \) with \( \eta_t \neq R \), \( \eta_t \) is a 2-absorbing ideal of \( R \).

Proof. Assume that \( x, y, z \in R \) are such that \( x y z \in \eta_t \). Then \( \eta(x y z) \geq t \). Then we have \( x_t y_t z_t = (x y z)_t \in \eta \). Since \( \eta \) is an \( L \)-fuzzy 2-absorbing ideal of \( R \), we get \( (x y)_t = x_t y_t \in \eta \) or \( (x z)_t = x_t z_t \in \eta \) or \( (y z)_t = y_t z_t \in \eta \). If \( a_t \in \eta \) for some \( a \in R \), then \( \eta(a) \geq t \). So \( a \in \eta_t \). Therefore \( x y \in \eta_t \) or \( x z \in \eta_t \) or \( y z \in \eta_t \). Hence \( \eta_t \) is a 2-absorbing ideal of \( R \).

Corollary 4.5 If \( \eta \) is an \( L \)-fuzzy 2-absorbing ideal of \( R \), then \( \eta_\ast = \{ x \in R \mid \eta(x) = \eta(0) \} \) is a 2-absorbing ideal of \( R \).

Proof. Since \( \eta \) is a non-constant \( L \)-fuzzy ideal of \( R \), \( \eta_\ast \neq R \). Now the result follows from Theorem 4.4.

Definition 4.6 Let \( \alpha \in L \setminus \{1\} \). Then \( \alpha \) is called a 2-absorbing element of \( L \) if \( r \land s \land t \leq \alpha \) implies that \( r \land s \leq \alpha \) or \( r \land t \leq \alpha \) or \( s \land t \leq \alpha \) for all \( r, s, t \in L \).

Proposition 4.7 Let \( \eta \) be an \( L \)-fuzzy 2-absorbing ideal of \( R \). Then \( \alpha = \eta(1) \) is a 2-absorbing element of \( L \).

Proof. Assume that \( x, y, z \in L \) with \( x \land y \land z \leq \alpha \). Consider the three \( L \)-fuzzy points \( 1_x, 1_y, 1_z \) of \( R \). Then we have \( 1_x 1_y 1_z = 1_x \land y \land z \in \eta \). Since \( \eta \) is 2-absorbing we get \( 1_x y = 1_x 1_y \in \eta \) or \( 1_x z = 1_x 1_z \in \eta \) or \( 1_y z = 1_y 1_z \in \eta \). Hence \( x \land y \leq \eta(1) = \alpha \) or \( x \land z \leq \eta(1) = \alpha \) or \( y \land z \leq \eta(1) = \alpha \), that is \( \alpha \) is 2-absorbing in \( L \).

Theorem 4.8 Let \( A \) be an 2-absorbing ideal of \( R \) and \( \alpha \) a 2-absorbing element of \( L \). If \( \eta \) is the \( L \)-fuzzy subset of \( R \) defined by

\[
\eta(x) = \begin{cases} 
1, & \text{if } x \in A; \\
\alpha, & \text{otherwise.}
\end{cases}
\]

for all \( x \in R \), then \( \eta \) is an \( L \)-fuzzy 2-absorbing ideal of \( R \).
By Lemma 4.2, every nonconstant ideal $IJK \subseteq \mathbb{Z}$ is a 2-absorbing ideal of $\mathbb{Z}$. Let $L$ be a nonconstant ideal of $\mathbb{Z}$ and so $xy \notin A$. Similarly, $xz \notin A$ and $yz \notin A$. As $A$ is assumed to be 2-absorbing, we get $xyz \notin A$. So that $\eta(xyz) = \alpha$. Also from $(xyz)_{r \land s \land t} = x_ry_ysz_t \in \eta$ we have $r \land s \land t \leq \eta(xyz) = \alpha$. Hence $r \land s \leq \alpha$ or $r \land t \leq \alpha$ or $s \land t \leq \alpha$, since $\alpha$ is a 2-absorbing element, which is a contradiction. Therefore $x_ry_s \in \eta$ or $x_sz_t \in \eta$ or $y_sz_t \in \eta$, that is $\eta$ is 2-absorbing.

Example 4.9 By Lemma 4.2, every $L$-fuzzy prime ideal of $R$ is $L$-fuzzy 2-absorbing, but the converse does not necessarily hold. For example, consider the case where $R = \mathbb{Z}$. Let $p$ and $q$ be a pair of distinct prime numbers, and set $A = pq\mathbb{Z}$. It is not difficult to show that $A$ is a 2-absorbing ideal of $\mathbb{Z}$. Now define $\eta : \mathbb{Z} \rightarrow [0, 1]$ by

$$\eta(x) = \begin{cases} 1, & \text{if } pq|x; \\ 0, & \text{otherwise.} \end{cases}$$

Then $\eta$ is a fuzzy 2-absorbing ideal of $R$ by Theorem 4.8. Moreover $\eta_0 = A$ is a 2-absorbing ideal of $\mathbb{Z}$ that is not a prime ideal. Hence $\eta$ is not a fuzzy prime ideal of $R$.

Definition 4.10 Let $\eta$ be an $L$-fuzzy ideal of $R$. $\eta$ is said to be an $L$-fuzzy strongly 2-absorbing ideal of $R$ provided that it is non-constant and whenever $\lambda, \mu, \nu \in I(R)$ with $\lambda \mu \nu \subseteq \eta$, then $\lambda \mu \subseteq \eta$ or $\lambda \nu \subseteq \eta$ or $\mu \nu \subseteq \eta$.

Theorem 4.11

(1) Every $L$-fuzzy prime ideal of $R$ is $L$-fuzzy strongly 2-absorbing.

(2) Every $L$-fuzzy strongly 2-absorbing ideal of $R$ is $L$-fuzzy 2-absorbing.

Proof. (1) Straightforward.

(2) Suppose that $\eta$ is an $L$-fuzzy strongly 2-absorbing ideal of $R$. Assume that $x_r, y_s, z_t \in \eta$ for some $L$-fuzzy points $x_r, y_s, z_t \in R$. Then, by Lemma 2.4, we have $\langle x_r \rangle \langle y_s \rangle \langle z_t \rangle \subseteq \eta$. Since $\eta$ is $L$-fuzzy strongly 2-absorbing, we have $\langle x_r y_s \rangle = \langle x_r \rangle \langle y_s \rangle \subseteq \eta$ or $\langle x_r z_t \rangle = \langle x_r \rangle \langle z_t \rangle \subseteq \eta$ or $\langle y_s z_t \rangle = \langle y_s \rangle \langle z_t \rangle \subseteq \eta$. Therefore $x_r y_s \in \eta$ or $x_r z_t \in \eta$ or $y_s z_t \in \eta$, that is $\eta$ is an $L$-fuzzy 2-absorbing ideal of $R$.

Let $A$ be an ideal of the commutative ring $R$. It is proved in [1] that $A$ is a 2-absorbing ideal of $R$ if and only if whenever $I, J, K$ are ideals of $R$ with $IJK \subseteq A$, then $IJ \subseteq A$ or $IK \subseteq A$ or $JK \subseteq A$. It is also well known that a nonconstant ideal $\xi \in I(R)$ is an $L$-fuzzy prime ideal of $R$ if and only if for any two $L$-fuzzy ideals $\mu$ and $\nu$ of $R$, $\mu \nu \subseteq \xi$ implies that either $\mu \subseteq \eta$ or $\nu \subseteq \eta$. Let $I$ be a nonconstant $L$-fuzzy ideal of $R$. We proved in Theorem 4.11 that every $L$-fuzzy strongly 2-absorbing ideal of $R$ is $L$-fuzzy 2-absorbing, but I was unable to prove or disprove the converse.
Question 1. Is every $L$-fuzzy 2-absorbing ideal of $R$ of the form 4.1, where $A$ is a 2-absorbing ideal of $R$ and $\alpha$ is a 2-absorbing element of $L$?

Question 2. Is every $L$-fuzzy 2-absorbing ideal of $R$ $L$-fuzzy strongly 2-absorbing?

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