SEMIGROUP COMPACTIFICATION IN AN INTUITIONISTIC FUZZY CONVERGENCE TOPOLOGICAL SPACE

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Abstract. The purpose of this paper is to discuss the problem of intuitionistic fuzzy lim semigroup compactification on intuitionistic fuzzy convergence topological semigroup and prove that the set of all intuitionistic fuzzy lim semigroup compactification of an intuitionistic fuzzy convergence topological semigroup is an upper complete semi Lattice.

Keywords: intuitionistic fuzzy convergence topological space, intuitionistic fuzzy convergence topological semigroup,intuitionistic fuzzy Bohr lim compactification, intuitionistic fuzzy lim semigroup compactification.

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1. Introduction

The concept of fuzzy set was introduced by Zadeh [15]. Since then the concept has invaded nearly all branches of Mathematics. Chang [3] introduced and developed the theory of fuzzy topological spaces and since then the various notions in classical topology have been extended to fuzzy topological spaces. Fuzzy sets have applications in many fields such as information [12] and control [11]. Atanassov [1] generalized fuzzy sets to intuitionistic fuzzy sets. Coker [5] introduced the notions of an intuitionistic fuzzy topological space, intuitionistic fuzzy continuous function and some related concepts. K. Geetha [7] discussed F-Semigroup compactification and Lowen et al [8] defined the notion of a fuzzy convergence space. The main objective of this paper is to discuss the problem of intuitionistic fuzzy lim semigroup compactification on intuitionistic fuzzy convergence topological semigroup and prove that the set of all intuitionistic fuzzy lim semigroup compactification of an intuitionistic fuzzy convergence topological semigroup is an upper complete semi Lattice.

2. Preliminaries

Definition 2.1. [1] Let X be a non empty fixed set and I be the closed interval $[0, 1]$. An intuitionistic fuzzy set (IFS) A is an object of the following form $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$ where the function $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\gamma_A(x)$) for each element $x \in X$ to the set A respectively and $0 \leq \mu_A(x)+\gamma_A(x) \leq 1$ for each x $\in$ X. Obviously, every fuzzy set A on a nonempty set X is an IFS of the following form $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$. For the sake of simplicity, we shall use the symbol $A = \langle x, \mu_A(x), \gamma_A(x)\rangle$ for the intuitionistic fuzzy set $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in X\}$.

Definition 2.2. [5] An intuitionistic fuzzy topology (IFT) in Coker's sense on a non empty set X is a family τ of IFSs in X satisfying the following axioms.

(T_1) $0_\sim$,$1_\sim \in \tau$

(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(T_3) $\cup G_i \in \tau$ for arbitrary family $\{G_i/i \in I\} \subseteq \tau$

Definition 2.3. [4] Let X be a non empty set and $x \in X$ a fixed element in X. If $r \in I_0$, $s \in I_1$ are fixed real numbers such that $r + s \leq 1$, then the intuitionistic fuzzy set $x_{r,s} = \langle x, x_r, 1 - x_{1-s}\rangle$ is said to be an intuitionistic fuzzy point (IFP) in X, where r denotes the degree of membership of $x_{r,s}$ and $x \in X$ the support of $x_{r,s}$.

The IFP $x_{r,s}$ is contained in the IFS A ($x_{r,s} \in A$) if and only if $r < \mu_A(x)$ and $s > \gamma_A(x)$.

Definition 2.4. [13] Let S be a non empty set and $\circ$ be a binary operation on S. The algebraic system $(S, \circ)$ is said to be a semigroup if the operation $\circ$ is associative.

Definition 2.5. [13] Let $\langle S, *\rangle$ be a semigroup and $T \subseteq S$. If the set T is closed under the operation $*$, then $\langle T, *\rangle$ is said to be a subsemigroup of $\langle S, *\rangle$.

Definition 2.6. [13] Let $(S, \circ)$ and $(Y, *)$ be two algebraic systems of the same type in the sense that both $\circ$ and $\star$ are binary operations. A function $g : X \to Y$ is said to be a homomorphism from $(S, \circ)$ and $(Y, *)$ if for any $x_1, x_2 \in X$, $g(x_1 \circ x_2) = g(x_1) \star g(x_2)$.

Definition 2.7. [13] Let g be a homomorphism from $(X, \circ)$ to $(Y, *)$. If $g : X \to Y$ is one to one and onto then g is said to be an isomorphism.

Definition 2.8. [6] A semigroup is a non empty set S together with an associative multiplication $(x, y) \to xy$ from $S \times S$ into S. If S has a Hausdroff topology such that $(x, y) \to xy$ is continuous with the product topology on $S \times S$, then S is said to be topological semigroup.

Definition 2.9. [9] If S is a topological semigroup, then a Bohr compactification of S is a pair (β, B) where B is a compact topological semigroup, $\beta : S \to B$ is a continuous homomorphism of S into a compact topological semigroup B and if $g : S \to T$ is a continuous homomorphism of S into a compact topological semigroup T, then there exists a unique continuous homomorphism $f : B \to T$, such that $f \circ \beta = g$.

Definition 2.10. [8] Given a set X, the pair $(X, \lim)$ is said to be a fuzzy convergence space, where $\lim : \mathbb{F}(X) \to I^X$, provided:

(i) $\lim \mathfrak{F} = \inf_{\mathfrak{G} \in P_m(\mathfrak{F})} \lim \mathfrak{G}, \mathfrak{F} \in \mathbb{F}(X)$

(ii) $\lim \mathfrak{F} \leq c(\mathfrak{F}), \mathfrak{F} \in \mathbb{F}_P(X)$,

(iii) $\lim \mathfrak{G} \leq \lim \mathfrak{F}$ when $\mathfrak{F} \subseteq \mathfrak{G}, \mathfrak{F}, \mathfrak{G} \in \mathbb{F}_P(X)$,

(iv) $\lim \alpha \dot{1}_x \geq \alpha 1_x$ for $0 < \alpha \leq 1$ and $x \in X$.

3. Intuitionistic fuzzy convergence topological semigroup

Note 3.1.

(i) ζ^X is the set of all intuitionistic fuzzy sets on X.

(ii) $(\zeta^X, \subseteq \cap, \cup)$ is an intuitionistic fuzzy lattice with minimum element $0_\sim$ and maximum element $1_\sim$.

Definition 3.1. A nonempty collection $\mathfrak{F}$ of elements in the lattice $(\zeta^X, \subseteq \cap, \cup)$ is said to be an intuitionistic fuzzy filter on X if

(i) $0_\sim \notin \mathfrak{F}$.

(ii) $A, B \in \mathfrak{F}$ implies $A \cap B \in \mathfrak{F}$.

(iii) $C \in \mathfrak{F}$ when $C \supseteq A$ and $A \in \mathfrak{F}$.

Definition 3.2. An intuitionistic fuzzy filter $\mathfrak{F}$ is said to be an intuitionistic fuzzy ultra filter if there is no other intuitionistic fuzzy filter finer than $\mathfrak{F}$.

Definition 3.3. An intuitionistic fuzzy filter $\mathfrak{F}$ is said to be an intuitionistic fuzzy prime filter (or) prime intuitionistic fuzzy filter if $A \cup B \in \mathfrak{F}$, then $A \in \mathfrak{F}$ or $B \in \mathfrak{F}$.

Note 3.2. Minimal prime intuitionistic fuzzy filter is the smallest prime intuitionistic fuzzy filter.

Note 3.3.

(i) $U(X)$: Set of all intuitionistic fuzzy ultrafilters on X.

(ii) $\mathbb{IF}(X)$: Set of all intuitionistic fuzzy filters on X.

(iii) $\mathbb{IF}_p(X)$: Set of all intuitionistic fuzzy prime filters on X.

(iv) $\mathbb{IF}_m(\mathfrak{F})$: Set of all minimal prime intuitionistic fuzzy filters finer than $\mathfrak{F}$.

Definition 3.4. A base for an intuitionistic fuzzy filter is a nonempty subset $\mathfrak{B}$ of ζ^X obeying

(i) $0_\sim \notin \mathfrak{B}$,

(ii) $A, B \in \mathfrak{B}$ implies $A \cap B \supseteq C$ for some $C \in \mathfrak{B}$.

The intuitionistic fuzzy filter generated by $\mathfrak{B}$ is denoted by

$$[\mathfrak{B}] = \{B \in \zeta^X / B \supseteq A \text{ and } A \in \mathfrak{B}\}.$$

Definition 3.5. Let X be a nonempty set and $A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \in X\}$ be an intuitionistic fuzzy set on X. The characteristic set of $\mathfrak{F} \in \mathbb{IF}(X)$ is defined by

$$c(\mathfrak{F}) = \bigcup_{A \in \mathfrak{F}} A.$$

Definition 3.6. Let $x_{r,s}$ be an intuitionistic fuzzy point. An intuitionistic fuzzy characteristic function of $x_{r,s}$ is denoted by

$$1_{x_{r,s}} = \langle x, \mu_{x_{r,s}}, \gamma_{x_{r,s}}\rangle.$$

Definition 3.7. An intuitionistic fuzzy set $\alpha 1_{x_{r,s}}$ is of the form

$$\alpha 1_{x_{r,s}} = \langle y, \alpha\mu_{x_{r,s}}(y), \alpha\gamma_{x_{r,s}}(y)\rangle,$$

where $r + s \leq 1$ and $\alpha \in (0, 1)$.

Note 3.4. The collection of all intuitionistic fuzzy sets of the form $\alpha 1_{x_{r,s}}$ is denoted by $\mathfrak{C}$.

Definition 3.8. An intuitionistic fuzzy filter generated by $\mathfrak{C}$ is denoted and defined by $[\mathfrak{C}] = \{B \in \zeta^X / B \supseteq A, A \in \mathfrak{C}\}$, however $[\mathfrak{C}]$ is written as $\alpha \dot{1}_{x_{r,s}}$.

Definition 3.9. Let X be a non empty set and the ordered pair $(X, \lim)$ is said to be an intuitionistic fuzzy convergence space if the intuitionistic fuzzy function $\lim : \mathbb{IF}(X) \to \zeta^X$ satisfies the following properties.

(i) For every $\mathfrak{F} \in \mathbb{IF}(X)$, $\lim \mathfrak{F} = \bigcap_{\mathfrak{G} \in \mathbb{IF}_m(\mathfrak{F})} \lim \mathfrak{G}$.

(ii) For every $\mathfrak{F} \in \mathbb{IF}_p(X)$, $\lim \mathfrak{F} \subseteq c(\mathfrak{F})$.

(iii) If $\mathfrak{F} \subseteq \mathfrak{G}$, then $\lim \mathfrak{G} \subseteq \lim \mathfrak{F}$ for every $\mathfrak{F}, \mathfrak{G} \in \mathbb{IF}_p(X)$.

(iv) For every $\alpha \in (0,1)$ and $\alpha 1_{x_{r,s}} \in \mathfrak{E}$, $\lim(\alpha \dot{1}_{x_{r,s}}) \supseteq \alpha 1_{x_{r,s}}$.

Definition 3.10. Let $(X, \lim)$ be an intuitionistic fuzzy convergence space. Then the operator int : $\zeta^X \to \zeta^X$ is defined by $\text{int}(\lim \mathfrak{F}) = \cup\{\lim \mathfrak{G} / \lim \mathfrak{G} \subseteq \lim \mathfrak{F}$, for every $\mathfrak{G} \in U(X)$ and $\mathfrak{F} \in \mathbb{IF}(X)\}$.

Definition 3.11. Let $(X, \lim)$ be an intuitionistic fuzzy convergence space and $T_{\lim} = \{\lim \mathfrak{F} / \text{int}(\lim \mathfrak{F}) = \lim(\mathfrak{F})$, for every $\mathfrak{F} \in \mathbb{IF}(X)\}$. Then the ordered pair $(X, T_{\lim})$ is said to be an intuitionistic fuzzy convergence topological space and the members of $T_{\lim}$ are said to be an intuitionistic fuzzy lim open sets (in short, IFLOS).

The complement of an intuitionistic fuzzy lim open sets are intuitionistic fuzzy lim closed sets (in short, IFLCS).

Note 3.5. $I_0 = (0,1]$ and $I_1 = [0,1)$.

Definition 3.12. An intuitionistic fuzzy net S is a function $S : \mathfrak{D} \to \mathfrak{I}$ where $\mathfrak{D}$ is a directed set with order relation $\leq$ and $\mathfrak{I}$ the collection of all intuitionistic fuzzy points in X. Thus for each $n \in \mathfrak{D}$, S(n) is an intuitionistic fuzzy point belonging to $\mathfrak{I}$. Let its value in $I_0 \times I_1$ be denoted as an intuitionistic fuzzy pair $\langle r_n, s_n \rangle$ such that $r_n \in I_0$ and $s_n \in I_1$ with $r_n + s_n \leq 1$. Thus, the crisp net is denoted as $V(S) = \{\langle r_n, s_n \rangle / n \in \mathfrak{D}\}$ in [0,1]. If $\{r_n\}$ converges to α and $\{s_n\}$ converges to β where $\alpha \in (0,1]$ and $\beta \in [0,1)$ such that $\alpha + \beta \leq 1$ then V(S) is said to be an intuitionistic fuzzy (α, β) net.

Definition 3.13. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological space, $x_{r,s}$ be an intuitionistic fuzzy point for any $r \in I_0$, $s \in I_1$ with $r + s \leq 1$ and $\lim \mathfrak{P} = \{\langle x, \mu_{\lim \mathfrak{P}}(x), \gamma_{\lim \mathfrak{P}}(x) \rangle : x \in X\}$ be an intuitionistic fuzzy lim closed set in $(X, T_{\lim})$. Then $\lim \mathfrak{P}$ is said to be an intuitionistic fuzzy remoted lim neighborhood (in short., $IFR\lim -nbd$) of $x_{r,s}$ if $x_{r,s} \notin \lim \mathfrak{P}$.

Definition 3.14. An intuitionistic fuzzy point $x_{r,s}$ of an intuitionistic fuzzy convergence topological space $(X, T_{\lim})$ is said to be an intuitionistic fuzzy cluster point of an intuitionistic fuzzy net $S : \mathfrak{D} \to \mathfrak{I}$ if for each intuitionistic fuzzy remoted lim neighbourhood $\lim \mathfrak{P}$ of $x_{r,s}$, $S(n)$ frequently does not belong to $\lim \mathfrak{P}$.

Definition 3.15. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological space. Then $(X, T_{\lim})$ is said to be a T_2 intuitionistic fuzzy convergence topological space if for every pair of distinct intuitionistic fuzzy points $x_{r,s}$ and $y_{l,m}$ with distinct supports x and y there are intuitionistic fuzzy remoted lim neighborhoods $\lim \mathfrak{P} = \{\langle x, \mu_{\lim \mathfrak{P}}(x), \gamma_{\lim \mathfrak{P}}(x) \rangle : x \in X\}$ and $\lim \mathfrak{Q} = \{\langle x, \mu_{\lim \mathfrak{Q}}(x), \gamma_{\lim \mathfrak{Q}}(x) \rangle : x \in X\}$ of $x_{r,s}$ and $y_{l,m}$ respectively so that $\lim \mathfrak{P} \cup \lim \mathfrak{Q} = 1_{\sim}$.

4. Intuitionistic fuzzy convergence topological semigroup

Definition 4.1. Let $(X, T_{\lim})$ and $(Y, S_{\lim})$ be any two intuitionistic fuzzy convergence topological spaces and $f : (X, T_{\lim}) \to (Y, S_{\lim})$ be an intuitionistic fuzzy function. Then f is said to be an intuitionistic fuzzy lim continuous function if for every intuitionistic fuzzy lim open set $\lim \mathfrak{F}$ in $(Y, S_{\lim})$, $f^{-1}(\lim \mathfrak{F})$ is an intuitionistic fuzzy lim open set in $(X, T_{\lim})$.

Definition 4.2. Let $(X, T_{\lim})$ be an T_2 intuitionistic fuzzy convergence topological space. Then $(X, T_{\lim})$ is said to be an intuitionistic fuzzy convergence topological semigroup if for every $x, y \in X$ the intuitionistic fuzzy function $f : (x, y) \to (xy)$ of $(X, T_{\lim}) \times (X, T_{\lim})$ into $(X, T_{\lim})$ is an intuitionistic fuzzy lim continuous function.

Note 4.1. Let $\lim \mathfrak{F} = \{\langle x, \mu_{\lim \mathfrak{F}}(x), \gamma_{\lim \mathfrak{F}}(x)\rangle : x \in X\}$. Then

$$(\lim \mathfrak{F})^{-1} = \{\langle x, \mu^{-1}_{lim\mathfrak{F}}(x), \gamma^{-1}_{\lim \mathfrak{F}}(x)\rangle : x \in X\}.$$

Definition 4.3. An intuitionistic topological group is a set X which carries a group structure, an intuitionistic topology and satisfies the following axioms.

(i) The function $(x, y) \to xy$ of $X \times X$ into X is continuous.

(ii) The function $x \to x^{-1}$ of X into X is continuous.

Definition 4.4. Let (X, T) be an intuitionistic topological space. Then $f : (X, T) \to [0.1]$ is said to be a lower semi intuitionistic continuous function if $f^{-1}((\alpha, 1))$ is intuitionistic open in X for some $\alpha \in I$.

Note 4.2. $i(T_{\lim})$ is the smallest intuitionistic topology for X which makes every member of $T_{\lim}$ a lower semi intuitionistic continuous function.

Remark 4.1. An association between intuitionistic fuzzy convergence topological semigroup and topological semigroup are discussed in Propositions 4.1 and 4.2.

Proposition 4.1. *A topological semigroup X is an intuitionistic fuzzy convergence topological semigroup with an intuitionistic fuzzy convergence topology*

$$T_{\lim} = \{\lim \mathfrak{F} / \lim \mathfrak{F} \text{ is a lower semi continuous function } \}.$$

Proof. Let $f : (x, y) \to (xy)$ of $(X, T_{\lim}) \times (X, T_{\lim})$ into $(X, T_{\lim})$ be an intuitionistic fuzzy function. Let $\lim \mathfrak{F} \in T_{\lim}$. Since $\lim \mathfrak{F}$ is a lower semi intuitionistic continuous function $(\lim \mathfrak{F})^{-1}(\alpha, 1)$ is intuitionistic open in X. Since X is an intuitionistic topological semigroup, $f^{-1}((\lim \mathfrak{F})^{-1}(\alpha, 1)) = (\lim \mathfrak{F} \circ f)^{-1}(\alpha, 1)$ is intuitionistic open in $(X, T_{\lim}) \times (X, T_{\lim})$. Therefore $(\lim \mathfrak{F} \circ f)$ is a lower semi intuitionistic continuous function. Hence, $f^{-1}(\lim \mathfrak{F})$ is an intuitionistic fuzzy lim open set in $(X, T_{\lim}) \times (X, T_{\lim})$. Therefore, f is an intuitionistic fuzzy lim continuous function. Hence, $(X, T_{\lim})$ is an intuitionistic fuzzy convergence topological semigroup.

Proposition 4.2. *If* $(X, T_{\lim})$ *is an intuitionistic fuzzy convergence topological semigroup, then* $(X, i(T_{\lim}))$ *is an intuitionistic topological semigroup.*

Proof. Let $(X, T_{\lim})$ is an intuitionistic fuzzy convergence topological semigroup and $f : (x,y) \to (xy)$ of $(X, i(T_{\lim})) \times (X, i(T_{\lim}))$ into $(X, i(T_{\lim}))$. Suppose that $W \in i(T_{\lim})$ be a subbasic intuitionistic open set in X. Then, there exists $\lim \mathfrak{g} \in T_{\lim}$ such that $W = (\lim \mathfrak{g})^{-1}(\alpha, 1)$ for $\alpha \in I$. Since $(X, T_{\lim})$ is an intuitionistic fuzzy convergence topological semigroup, $\lim \mathfrak{g} \in T_{\lim}$ implies $f^{-1}(\lim \mathfrak{g}) \in T_{\lim} \times T_{\lim}$. That is, $\lim \mathfrak{g} \circ f \in T_{\lim} \times T_{\lim}$. That is, $f^{-1}((\lim \mathfrak{g})^{-1}(\alpha, 1)) \in i(T_{\lim} \times T_{\lim})$. Hence, $f^{-1}(W) \in i(T_{\lim} \times T_{\lim})$. Hence, f is intuitionistic continuous. Therefore, $(X, i(T_{\lim}))$ is a topological semigroup.

Definition 4.5. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological semigroup. Then $(X, T_{\lim})$ is said to be topologically generated provided that the intuitionistic fuzzy convergence topology $T_{\lim}$ is the set of all lower semi continuous function from $(X, i(T_{\lim}))$ into I.

Definition 4.6. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological space. If $S \subset X$, then the characteristic function χ_S is defined as

$$\chi_S(x) = \begin{cases} 1_\sim & x \in S \\ 0_\sim & x \notin S \end{cases}$$

for all $x \in X$.

Note 4.3. Let X be a non empty set and $A \subseteq X$. Then an intuitionistic fuzzy set χ_A^* is of the form $\langle x, \chi_A, 1 - \chi_A \rangle$.

Definition 4.7. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological space. Then the intuitionistic fuzzy lim closure * of A is denoted and defined as $IFcl^*_{\lim}(A) = \cap\{\lim \mathfrak{G} / \lim \mathfrak{G} \supseteq A, \lim \mathfrak{G} \text{ is an intuitionistic fuzzy lim closed set}\}$.

Definition 4.8. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological space and A be an intuitionistic fuzzy set in $(X, T_{\lim})$. Then A is said to be an intuitionistic fuzzy lim dense in $(X, T_{\lim})$ if $IFcl^*_{\lim}(A) = \chi_X^*$.

Definition 4.9. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological space and Y be any subset of X. Then $T_{\lim}/Y = \{\lim \mathfrak{A}/Y / \lim \mathfrak{A} \in T_{\lim}\}$ is an intuitionistic fuzzy convergence topology on Y and is said to be an induced or relative intuitionistic fuzzy convergence topology. The ordered pair $(Y, T_{\lim}/Y)$ is said to be an intuitionistic fuzzy convergence subspace of $(X, T_{\lim})$.

Definition 4.10. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological space and $(Y, S_{\lim})$ be an intuitionistic fuzzy convergence subspace of $(X, T_{\lim})$. Then $(Y, S_{\lim})$ is intuitionistic fuzzy lim dense in $(X, T_{\lim})$ if χ_Y^* is an intuitionistic fuzzy lim dense in $(X, T_{\lim})$.

5. Intuitionistic fuzzy Bohr lim compactification

Definition 5.1. Let (X, T_{lim}) and (Y, S_{lim}) be any two intuitionistic fuzzy convergence topological spaces and $f : (X, T_{\text{lim}}) \to (Y, S_{\text{lim}})$ be an intuitionistic fuzzy function. Then f is said to be an intuitionistic fuzzy lim open if for every intuitionistic fuzzy lim open set $\lim \mathfrak{F}$ in (X, T_{lim}), $f(\lim \mathfrak{F})$ is an intuitionistic fuzzy lim open set in (Y, S_{lim}).

Definition 5.2. Let (X, T_{lim}) and (Y, S_{lim}) be any two intuitionistic fuzzy convergence topological spaces and $f : (X, T_{\text{lim}}) \to (Y, S_{\text{lim}})$ be an intuitionistic fuzzy function. Then f is said to be an intuitionistic fuzzy lim homeomorphism if f is an intuitionistic fuzzy bijective function. That is, f is both an intuitionistic fuzzy lim continuous function and an intuitionistic fuzzy lim open function.

Definition 5.3. Let (X, T_{lim}) and (Y, S_{lim}) be any two intuitionistic fuzzy convergence topological semi groups and $\theta : (X, T_{\text{lim}}) \to (Y, S_{\text{lim}})$ be an intuitionistic fuzzy function. Then θ is said to be an intuitionistic fuzzy lim morphism if

(i) θ is an algebraic homomorphism of X into Y.

(ii) θ is an intuitionistic fuzzy lim continuous function of (X, T_{lim}) into (Y, S_{lim}).

A bijective function θ of (X, T_{lim}) into (Y, S_{lim}) is an intuitionistic fuzzy lim isomorphism if

(i) θ is an algebraic isomorphism of X into Y.

(ii) θ is an intuitionistic fuzzy lim homeomorphism of (X, T_{lim}) into (Y, S_{lim}).

Definition 5.4. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space and $\lim \mathfrak{F}$ be an intuitionistic fuzzy set in (X, T_{lim}). Then $\lim \mathfrak{F}$ is said to be an intuitionitic fuzzy N-lim compact if each intuitionistic fuzzy (α, β) net contained in $\lim \mathfrak{F}$ has at least a cluster point $x_{\alpha,\beta}$ with value $\langle \alpha, \beta \rangle$ where $0 < \alpha \leq 1$, $0 \leq \beta < 1$ and $\alpha + \beta \leq 1$.

Definition 5.5. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological space. If (X, T_{lim}) is topologically generated, then (X, T_{lim}) is said to be an intuitionistic fuzzy N-lim compact.

Definition 5.6. Let (X, T_{lim}) be an intuitionistic fuzzy convergence topological semigroup. The intuitionistic fuzzy Bohr lim compactification of (X, T_{lim}) is a pair (α, Y) such that Y is an intuitionistic fuzzy N-lim compact convergence topological semigroup and $\alpha : X \to Y$ is an intuitionistic fuzzy lim morphism provided whenever $\beta : X \to Z$ is an intuitionistic fuzzy lim morphism of X into an intuitionistic fuzzy N-lim compact convergence topological semigroup Z, then there exists a unique intuitionistic fuzzy lim morphism $f : Y \to Z$ such that $f \circ \alpha = \beta$.

Proposition 5.1. *Let (X, T_{lim}) be a topologically generated intuitionistic fuzzy convergence topological semigroup. Then $(X, i(T_{\text{lim}}))$ is an intuitionistic topological semigroup.*

Proof. The proof is simple by Proposition 4.2.

Definition 5.7. Let (X, T_{lim}) and (Y, S_{lim}) be any two intuitionistic fuzzy convergence topological spaces and $\theta : (X, T_{\text{lim}}) \to (Y, S_{\text{lim}})$ be an intuitionistic fuzzy function. Then (X, T_{lim}) is *dense in (Y, S_{lim}) if θ is an intuitionistic fuzzy lim morphism.

Proposition 5.2. *Let (β, B) be the Bohr compactification of $(X, i(T_{\text{lim}}))$ where B is a N-compact topological semigroup and β a continuous homomorphism of X into B. Let $T_{\text{lim}_B} = \{g|g$ is a lower semi continuous function from B into I such that $g|X \in T_{\text{lim}}\}$. Then (B, T_{lim_B}) is an intuitionistic fuzzy N-lim compact convergence topological semigroup and (X, T_{lim}) is *dense in (B, T_{lim_B}).*

Proof. Let (β, B) be the Bohr compactification of $(X, i(T_{\text{lim}}))$ where B is a compact topological semigroup and β a continuous homomorphism of X into B. Define an intuitionistic fuzzy convergence topology as in the hypothesis. Then, by the Proposition 4.2, and by Definition 5.2, (B, T_{lim_B}) is an intuitionistic fuzzy N-lim compact intuitionistic fuzzy covergence topological semigroup.

Let $g \in T_{lim_B}$. Then there exists $V \in i(T_{\text{lim}_B})$ such that $g^{-1}(\alpha, 1) = V$ for $\alpha \in I$. Since β is a continuous function, $\beta^{-1}(V)$ is open in X. Therefore, $\beta^{-1}(g^{-1}(\alpha, 1))$ is an open set in X. That is, $g \circ \beta$ is a lower semi continuous function. Thus, $\beta^{-1}(g) \in T_{\text{lim}}$. Therefore, β is an intuitionistic fuzzy lim continuous function. Hence, β is an intuitionistic fuzzy lim morphism of (X, T_{lim}) into (B, T_{lim_B}). Hence, (X, T_{lim}) is *dense in (B, T_{lim_B}).

Proposition 5.3. Let (X, T_{lim}) be a topologically generated intuitionistic fuzzy convergence topological semigroup and let (f, Y), (g, Z) be any two intuitionistic fuzzy Bohr lim compactifications of X. Then there exists an intuitionistic fuzzy lim isomorphism $h : Y \to Z$ such that $h \circ f = g$.

Proof. Since (f, Y) is an intuitionistic fuzzy Bohr lim compactification of X, there exists an intuitionistic fuzzy lim morphism $h : (Y, S) \to (Z, R)$ such that $h \circ f = g$. Similarly, there exists a intuitionistic fuzzy lim morphism $\pi : Z \to Y$ such that $\pi \circ g = f$. It is clear that $\pi \circ h = I_Y$ and $h \circ \pi = I_Z$. That is, $\pi \circ h$ and $h \circ \pi$ are identity intuitionistic fuzzy lim morphism of Y and Z respectively. Therefore, h is an intuitionistic fuzzy lim homeomorphism. Hence, h is an intuitionistic fuzzy lim isomorphism of Y onto Z.

6. Intuitionistic fuzzy lim semigroup compactification

Definition 6.1. Let (X, T_{lim}) and (Y, S_{lim}) be any two intuitionistic fuzzy convergence topological spaces and $\theta : (X, T_{\text{lim}}) \to (Y, S_{\text{lim}})$ be an Intuitionistic fuzzy lim morphism. Then θ is said to be an intuitionistic fuzzy dense lim morphism if it has a intuitionistic fuzzy lim dense image set.

Definition 6.2. Let X be an intuitionistic fuzzy convergence topological semigroup. Then intuitionistic fuzzy lim semigroup compactification of (X, T_{lim}) is an ordered pair (α, A) where A is a intuitionistic fuzzy N-lim compact intuitionistic fuzzy convergence topological semigroup and $\alpha : X \to A$ is an intuitionistic fuzzy dense lim morphism.

Definition 6.3. Let (α, A) and (β, B) be any two intuitionistic fuzzy lim semigroup compactifications of an intuitionistic fuzzy convergence topological semigroup X. Then (α, A) and (β, B) are said to be equivalent if there exists an intuitionistic fuzzy lim isomorphism $\nu : A \to B$ such that $\nu \circ \alpha = \beta$.

Definition 6.4. Let (α, A) and (β, B) be any two intuitionistic fuzzy lim semigroup compactifications of an intuitionistic fuzzy convergence topological semigroup X. Then $(\alpha, A) \supseteq (\beta, B)$ if there exists an onto intuitionistic fuzzy lim morphism $\nu : A \to B$ such that $\nu \circ \alpha = \beta$.

Definition 6.5. Let (α, A) and (β, B) be any two intuitionistic fuzzy lim semigroup compactifications of an intuitionistic fuzzy convergence topological semigroup X. Then (α, A) and (β, B) are said to be equivalent if $(\alpha, A) \supseteq (\beta, B)$ and $(\beta, B) \supseteq (\alpha, A)$.

Definition 6.6. Let $(X_i, T_{\lim_i})$ be an intuitionistic fuzzy convergence topological space for each $i \in I$. Let X be the cartesian product of $\{X_i / i \in I\}$ and P_i be the intuitionistic fuzzy projection of the product X into the i th co-ordinate set X_i. Let $\beta = \bigcap_{i \in I_1} \{P_i^{-1}(\lim \mathfrak{F}_i) / \lim \mathfrak{F}_i \in T_{\lim_i}$ where$\mathfrak{F}_i \in \mathbb{IF}_i$ $i \in I_1$, where I_1 is any finite subset of I.

Then β is a basis for an intuitionistic fuzzy convergence topology $T_{\lim}$ on X said to be the product intuitionistic fuzzy convergence topology and the ordered pair $(X, T_{\lim})$ is said to be the product space of an intuitionistic fuzzy convergence topological space $(X_i, T_{\lim_i})$ for $i \in I$.

Proposition 6.1. *Let $(X, T_{\lim})$ be the product space of the intuitionistic fuzzy convergence topological semigroups $(X_i, T_{\lim_i})$ for $i \in I$ and P_i be the intuitionistic fuzzy projection of the product X into the i-th co-ordinate set X_i. Then the following are valid.*

(i) *For each $i \in I$, the intuitionistic fuzzy projection P_i is an intuitionistic fuzzy* lim *continuous function.*

(ii) *The intuitionistic fuzzy function $f : (Y, S_{\lim}) \to (X, T_{\lim})$ is an intuitionistic fuzzy lim continuous function if and only if for every $i \in I$, $P_i \circ f$ is intuitionistic fuzzy* lim *continuous function where $(Y, S_{\lim})$ is an intuitionistic fuzzy convergence topological semigroup.*

Proof. (i) By Definition 6.6, P_i is an intuitionistic fuzzy lim continuous function.

(ii) Let $f : (Y, S_{lim}) \to (X, T_{\lim})$ be an intuitionistic fuzzy lim continuous function. Since each P_i is an intuitionistic fuzzy lim continuous function, the composition $P_i \circ f$ is an intuitionistic fuzzy lim continuous function.

Conversely suppose that $P_i \circ f$ is an intuitionistic fuzzy lim continuous function for every $i \in I$. Then

$$(P_i \circ f)^{-1}(\lim \mathfrak{F}_i) = f^{-1}(P_i^{-1}(\lim \mathfrak{F}_i)) \in S_{\lim} \text{ for every } i \in I.$$

Put $\lim \mathfrak{G} = (P_i^{-1}(\lim \mathfrak{F}_i))$. Now, $f^{-1}(\lim \mathfrak{G}) \in S_{\lim}$. Hence, f is an intuitionistic fuzzy lim continuous function.

Proposition 6.2. *Let $\{X_i/i \in I\}$ be a collection of intuitionistic fuzzy convergence topological semigroups and $X = \prod_{i\in I} X_i$. Then X with co-ordinate wise multiplication is an intuitionistic fuzzy convergence topological semigroup and each intuitionistic fuzzy projection $P_i : X \to X_i$ is an intuitionistic fuzzy lim morphism.*

Proof. Let X be a semigroup under co-ordinate wise multiplication, where the associativity of multiplication follows from that of the semigroups in the collection $\{X_i/i \in I\}$. Since each X_i is an intuitionistic fuzzy convergence topological semigroup for each $x_{r,s}, y_{r,s} \in X_i$, the intuitionistic fuzzy function $g_i : (x_{r,s}, y_{r,s}) \to (x_{r,s}y_{r,s})$ is an intuitionistic fuzzy lim continuous function.

Let $g : (x, y) \to (xy)$ of $(X, T_{\lim}) \times (X, T_{\lim})$ into $(X, T_{\lim})$ be an intuitionistic fuzzy function. Let $x_{r,s}, y_{r,s} \in X$ be any two intuitionistic fuzzy points in X. Then, $g(x_{r,s}, y_{r,s}) = (x_{r,s}y_{r,s}) = g_i(x_{r,s}, y_{r,s})$ and, therefore, $P_i \circ g = g_i$. Since each g_i is an intuitionistic fuzzy lim continuous function, g is also an intuitionistic fuzzy lim continuous function. Hence, $\prod_{i\in I} X_i$ is an intuitionistic fuzzy convergence topological semigroup. Hence, from the definition of multiplication on X each P_i is an intuitionistic fuzzy lim morphism.

Proposition 6.3. *Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological semigroup and A be a subsemigroup of X. Then the subspace $(A, T_{\lim}/A)$ is also an intuitionistic fuzzy convergence topological semigroup.*

Proof. Let $T_{\lim}/A = \{\lim \mathfrak{F}/A = \lim \mathfrak{F} \cap \chi_A^* \text{ where } \lim \mathfrak{F} \in T_{\lim}\}$, $x, y \in A$ and $f : (x, y) \to (xy)$ of $(A, T_{\lim}/A) \times (A, T_{\lim}/A)$ into $(A, T_{\lim}/A)$ be an injective intuitionistic fuzzy function. Consider $\lim \mathfrak{G} \in T_{\lim}/A$. That is, there exists $\lim \mathfrak{F} \in T_{\lim}$ such that $lim\mathfrak{F}/A \in T_{\lim}/A$. Since $(X, T_{\lim})$ is an intuitionistic fuzzy convergence topological semigroup, $f : (X, T_{\lim}) \times (X, T_{\lim}) \to (X, T_{\lim})$ is an intuitionistic fuzzy lim continuous function. Hence, $f^{-1}(\lim \mathfrak{F})$ is an intuitionistic fuzzy lim open in $T_{\lim} \times T_{\lim}$. Now, $\lim \mathfrak{G} = \lim \mathfrak{F} \cap \chi_A \in T_{\lim}/A$. Then $f^{-1}(\lim \mathfrak{G}) = f^{-1}(\lim \mathfrak{F} \cap \chi_A^*) \in T_{\lim}/A \times T_{\lim}/A$, which is an intuitionistic fuzzy lim open set. Hence, f/A is an intuitionistic fuzzy lim continuous function. Hence, $(A, T_{\lim}/A)$ is an intuitionistic fuzzy convergence topological semigroup.

Note 6.1. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological semigroup and Y be a subsemigroup of X such that $Y = \overline{Y} = X - Y$. Then

(i) Y is said to be a closed subsemigroup of X.

(ii) the intuitionistic fuzzy subspace $(Y, T_{\lim}/Y)$ is said to be a closed intuitionistic fuzzy convergence topological semigroup.

Definition 6.7. Let $(X, T_{\lim})$ be an intuitionistic fuzzy convergence topological semigroup. If $(X, T_{\lim})$ is an intuitionistic fuzzy $N - \lim$ compact convergence topological semigroup, then each closed intuitionistic fuzzy convergence topological semigroup is an intuitionistic fuzzy $N - \lim$ compact convergence topological semigroup.

Definition 6.8. Let $(X_i, T_{\lim_i})$ be intuitionistic fuzzy convergence topological spaces for every $i \in I$ and X be the cartesian product of $X_i/i \in J$. If each X_i is an intuitionistic fuzzy $N - \lim$ compact convergence topological semigroup, then X is also an intuitionistic fuzzy $N - \lim$ compact convergence topological semigroup.

Notation 6.1. $T_{\lim}(X)$ denotes the set of all equivalence classes of intuitionistic fuzzy lim semigroup compactification of $(X, T_{\lim})$ where $(X, T_{\lim})$ is an intuitionistic fuzzy convergence topological semigroup.

Definition 6.9. Let $(\zeta^X, \subseteq \cap, \cup)$ be an intuitionistic fuzzy lattice. Then $(\zeta^X, \subseteq \cap, \cup)$ is said to be intuitionistic fuzzy complete if each of its non empty subsets has a least upper bound and a greatest lower bound.

Definition 6.10. Intuitionistic fuzzy upper semi lattice $(\zeta^X, \subseteq \cap, \cup)$ is a partially ordered set that has a least upper bound for any non empty finite subset.

Proposition 6.4. *The partially ordered set $(T(X), \subseteq)$ is an upper complete semi lattice.*

Proof. Let $\{\alpha_i X/i \in I\}$ be a subset of $T_{\lim}$(X). Consider the product $\prod_{i\in I}\{\alpha_i X\}$. Since each $\alpha_i X$ is an intuitionistic fuzzy N-lim compact convergence topological semigroup, by Definition 6.8 and Proposition 6.2, $\prod_{i\in I}\{\alpha_i X\}$ is an intuitionistic fuzzy N-lim compact convergence topological semigroup. Define a function $\theta : X \to \prod_{i\in I} \alpha_i X$ by $\theta((x))_i = \alpha_i(x)$. Also define the intuitionistic fuzzy functions $\pi_i : \prod_{i\in I} \alpha_i X \to \alpha_i X$ and $\alpha_i : X \to \alpha_i X$. Clearly $\pi_i \circ \theta = \alpha_i$. Since each α_i is an intuitionistic fuzzy lim continuous function, for every i, $\theta((xy))_i = \alpha_i(xy) = \alpha_i(x)\alpha_i(y) = (\theta(x))_i(\theta(y))_i$. That is, θ is algebraically a homomorphism. Hence, θ is an intuitionistic fuzzy lim morphism of X into $\prod_{i\in I} \alpha_i X$.

Let θX denote $\overline{\theta(X)}$. Then, by Note 6.1, Proposition 6.3, and by Definition 6.7, θX is an intuitionistic fuzzy N-lim compact convergence topological semigroup. Also, $\theta : X \to \theta X$ is a intuitionistic fuzzy dense lim morphism. Therefore, $(\theta X, \theta)$ is an intuitionistic fuzzy lim semigroup compactification of X.

For each $i \in I$, let $f_i : \theta X \to \alpha_i X$ be the restriction of the projection to θX. Consider $(f_i \circ \theta)x = f_i \circ \theta(x) = (\theta(x))_i = \alpha_i(x)$. That is, $f_i \circ \theta = \alpha_i$. Therefore, $(\theta X, \theta) \supseteq (\alpha_i X, \alpha_i)$ for every $i \in I$.

Suppose that $(\theta_0 X, \theta_0) \supseteq (\alpha_i X, \alpha_i)$ for every i. Now, $g_i : \theta_0 X \to \alpha_i X$ such that $g_i \circ \theta_0 = \alpha_i$. Define $f : \theta_0 X \to \prod_{i\in I} \alpha_i X$ by $(f(y))_i = g_i(y)$. Then $\pi_i \circ f = g_i$ where π_i and g_i are IF lim morphisms. Therefore, f is also an intuitionistic fuzzy lim morphism.

Consider $f(\theta_0(x))_i = g_i \circ \theta_0(x) = \alpha_i(x) = (\theta(x))_i$. Therefore, $f \circ \theta_0 = \theta$. Now, $f(\theta_0 x) = f(\theta_0(x)) = \theta X$. Then $\overline{f(\theta_0 X)} = \theta X$. Therefore, f is a onto intuitionistic fuzzy dense lim morphism and $f \circ \theta_0 = \theta$. Hence, $(\theta X, \theta) \subseteq (\theta_0 X, \theta_0)$ and thus θX is the least upper bound of $\{\alpha_i, X\}$ for each $i \in I$. Hence, the partially ordered set $(T_{\lim}(X), \subseteq)$ is an upper complete semi lattice.

Remark 6.1. From Propositions 6.4., $(\theta, \theta X)$ is an intuitionistic fuzzy lim semigroup compactification of an intuitionistic fuzzy convergence topological semigroup $(X, T_{\lim})$.

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