SOFT ISOMORPHISM THEOREMS FOR SOFT HEMIRINGS

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Abstract. In this paper, the concepts of soft strong h-ideals and strong h-idealistic soft hemirings are introduced. Some properties of soft hemirings and strong h-idealistic soft hemirings are given. In particular, we construct a novel soft quotient structure of an idempotent hemiring. By means of a kind of new way, soft isomorphism theorems of soft hemirings are established, which are different from soft isomorphism theorems of soft rings.

Keyword: soft strong h-ideal; strong h-idealistic soft hemiring; soft isomorphism theorems.

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1. Introduction

In 1999, Molodtsov [15] put forward the concept of soft sets as a new mathematical tool for dealing with uncertainties. And then, the research on the soft set theory has been extensively studied by many authors. Recently, some basic operations on soft sets were defined by Maji [13] and Ali [2]. What's more, Çağman [3], [4], [14] applied soft set theory to decision making. We also know that soft sets can also be applied in computer science and information science, which refereed to [13].

It is noted that some soft algebras were also discussed, such as [1], [8], [9]. In 2005, a new definition of soft sets called the parametrization reduction was introduced by Chen[5]. By comparing their definition with the related concept of attributed reduction on rough set theory, the theory of soft sets has been developed.

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The applications of soft set in the ideal theory of BCK/BCI-algebras was investigated by Jun and Park [9], and then Feng [6] started to investigate the structure of soft semirings. It is pointed out that some characterizations of hemirings by soft set theory were investigated by Ma, Zhan and others, which refereed to [10], [11], [12], [17], [18].

In this paper, we construct a novel soft quotient structure of an idempotent hemiring and then soft isomorphism theorems of soft hemirings are established. The remaining part of this paper is organized as follows. In section 2, we first recall some concepts and results on soft sets. In section 3, some properties of soft hemirings will be given. Further, soft isomorphism theorems of soft hemirings are established. In the final section, we give a brief conclusion.

2. Preliminaries

By a hemiring, we mean an additively commutative semiring with zero. By zero of a semiring $(S, +, \cdot)$ we mean an element $0 \in S$ such that $0 \cdot x = x \cdot 0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$. Throughout this paper, S is a hemiring.

A non-empty subset A in S is called a subhemiring of S if A is closed under addition and multiplication. A non-empty subset A in S is called a left (resp. right) ideal of S if A is closed under addition and $SA \subseteq A$ (resp. $AS \subseteq A$). Further, A is called an ideal of S if it is both a left ideal and a right ideal of S.

An ideal I of S is called an h-ideal if $x, z \in S$, $a, b \in I$ and $x + a + z = b + z$ implies $x \in I$. An ideal I of S is called a strong h-ideal if $x, y, z \in S$, $a, b \in I$ and $x + a + z = y + b + z$ implies $x \in y + I$ [9], [16].

The strong h-closure $\widetilde{A}$ in S is defined by

$$y + \widetilde{A} = \{x \in S\}$$

satisfying $x + a_1 + z = y + a_2 + z$ for some $a_1, a_2 \in A, y, z \in S$.

Let ρ be a congruence relation on S, that is, ρ is an equivalence relation on S such that $(a, b) \in \rho$ and $(c, d) \in \rho$ in R implies $(a+c, b+d) \in \rho$ and $(ac, bd) \in \rho$ for all $a, b, c, d \in S$.

Let I be a strong h-ideal of S, $x, y \in S$. We call x is congruent to y mod I, if and only if there exist $a, b \in I$ and $z \in S$ be such that $x + a + z = y + b + z$. It is checked that the relation $x \equiv y (\text{mod } I)$ is a congruence relation on S.

Definition 2.1 [15] A pair $\mathfrak{S} = (F, A)$ is called a soft set over U, where $A \subseteq E$ and $F : A \to \mathscr{P}(U)$ is a set-valued mapping.

For a soft set (F, A), the set $\text{Supp}(F, A) = \{x \in A | F(x) \neq \emptyset\}$ is called a soft support of the soft set (F, A). Thus a null soft set is indeed a soft set with an empty support, and we say that a soft set (F, A) is non-null if $\text{Supp}(F, A) \neq \emptyset$.

Definition 2.2 [6] Let (F, A) be a non-null soft set over S. Then

(1) (F, A) is called a soft hemiring over S if $F(x)$ is a subhemiring of S for all $x \in \mathrm{Supp}(F, A)$;

(2) (F, A) is called an idealistic soft hemiring over S if $F(x)$ is an ideal of S for all $x \in \mathrm{Supp}(F, A)$. The *bi*-idealistic ($k$-idealistic, h-idealistic) soft hemiring are defined similarly.

Definition 2.3 [7] Let (F, A) and (G, B) be two soft sets over a common universe U. The inclusion symbol "$\subseteq$" of (F, A) and (G, B), denoted by $(F, A) \subseteq (G, B)$, is defined as

(1) $A \subseteq B$;

(2) $F(x) \subseteq G(x)$ for all $x \in A$.

Definition 2.4 [2] Let (F, A) and (G, B) be two soft sets over a common universe U.

(1) The *bi*-intersection of (F, A) and (G, B), is defined to the soft set (H, C), where $C = A \cap B$, and $H : C \to \mathscr{P}(U)$ is a mapping given by $H(c) = F(c) \cap G(c)$ for all $c \in C$. This is denoted by $(F, A)\widetilde{\sqcap}(G, B) = (H, C)$.

(2) "$(F, A)\ AND\ (G, B)$", denoted by $(F, A)\widetilde{\wedge}(G, B)$, is defined by $(F, A)\widetilde{\wedge}(G, B) = (H, A \times B)$, where $H(x, y) = F(x) \cap G(y)$ for all $(x, y) \in A \times B$.

(3) The union of (F, A) and (G, B), denoted by $(F, A)\widetilde{\cup}(G, B)$, is defined as the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B, \\ G(e), & \text{if } e \in B - A, \\ F(e) \cup G(e), & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.5 [6] Let (η, A) be a soft hemiring over S. A soft set (γ, I) over S is called a soft ideal of (η, A), denote by $(\gamma, I)\widetilde{\triangleleft}(\eta, A)$, if it satisfies:

(1) $I \subseteq A$;

(2) $\gamma(x)$ is an ideal of $\eta(x)$ for all $x \in \mathrm{Supp}(\gamma, I)$.

Definition 2.6 Let (F, A) be a soft hemiring over S. A soft set (G, B) over S is called a soft strong h-ideal of (F, A), denote by $(G, B)\widetilde{\triangleleft}(F, A)$, if it satisfies:

(1) $B \subseteq A$;

(2) $G(x)$ is a strong h-ideal of $F(x)$ for all $x \in B$.

Theorem 2.7 *Let (F,A) and (G,B) be strong h-ideals of a soft hemiring (H,C) over S. Then the soft set $(F,A)\tilde{\sqcap}(G,B)$ is a soft strong h-ideal of (H,C).*

Proof. Assume that $(F,A)\tilde{\triangleleft}(H,C)$ and $(G,B)\tilde{\triangleleft}(H,C)$. By Definition 2.4 (1), we can write $(F,A)\tilde{\sqcap}(G,B)=(\gamma,I)$, where $I=A\cap B$ and $\gamma(x)=F(x)\cap G(x)$ for all $x\in I$. Obviously, we have $I\subseteq C$. Suppose that the soft set (γ,I) is non-null. If $x\in I$, then $\gamma(x)=F(x)\cap G(x)\neq\emptyset$. Since $(F,A)\tilde{\triangleleft}(H,C)$ and $(G,B)\tilde{\triangleleft}(H,C)$, we deduce that the nonempty sets $F(x)$ and $G(x)$ are both strong h-ideals of $H(x)$. It follows that $\gamma(x)$ is a strong h-ideal of $H(x)$ for all $x\in I$. Therefore $(F,A)\tilde{\sqcap}(G,B)=(\gamma,I)$ is a soft strong h-ideal of (H,C) as required. ∎

Theorem 2.8 *Let (F,A) and (G,B) be strong h-ideals of a soft hemiring (H,C) over S. If A and B are disjoint, then the soft set $(F,A)\tilde{\cup}(G,B)$ is a soft strong h-ideal of (H,C).*

Proof. Assume that $(F,A)\tilde{\triangleleft}(H,C)$ and $(G,B)\tilde{\triangleleft}(H,C)$. According 2.4 (3), we can write $(F,A)\tilde{\cup}(G,B)=(\gamma,I)$, where $I=A\cup B$ and for every $x\in I$,

$$\gamma(x)=\begin{cases}F(x), & \text{if } \ e\in A-B,\\ G(x), & \text{if } \ e\in B-A,\\ F(x)\cup G(x), & \text{if } \ e\in A\cap B.\end{cases}$$

Clearly, we have $I\subseteq C$. Suppose that A and B are disjoint, i.e., $A\cap B=\emptyset$. Then, for every $x\in I$, we know that either $x\in A-B$ or $x\in B-A$. If $x\in A-B$, then $\gamma(x)=F(x)\neq\emptyset$ is a strong h-ideal of $H(x)$ since $(F,A)\tilde{\triangleleft}(H,C)$. Similarly, if $x\in B-A$, then $\gamma(x)=G(x)\neq\emptyset$ is a strong h-ideal of $H(x)$ since $(G,B)\tilde{\triangleleft}(H,C)$. Thus we conclude $\gamma(x)$ is a strong h-ideal of $H(x)$ for all $x\in I$, and so $(F,A)\tilde{\cup}(G,B)$ is a soft strong h-ideal of (H,C). ∎

3. Strong h-idealistic soft hemirings and soft isomorphism theorems

In this section, we define the notion of strong h-idealistic soft hemirings, and then construct a soft quotient structure of an idempotent hemiring. Further, soft isomorphism theorems of soft hemirings are established.

Definition 3.1 Let (F,A) be a soft set over S. Then (F,A) is said to be a strong left(right) h-idealistic soft hemiring over S if and only if $F(x)$ is a strong left(right) h-ideal of S for all $x\in A$. (F,A) is said to be a strong h-idealistic soft hemiring over S if and only if (F,A) is both a strong right h-idealistic soft hemiring over S and a strong left h-idealistic soft hemiring over S.

Example 3.2 Let S and A be the hemirings of all non-negative integers with respect to the usual addition and multiplication of integers. $\forall x\in A$, let $F(x)=\{y\mid y\rho x\Longleftrightarrow y=2xa, a\in A\}$. If $y_1,y_2\in F(x)$, then there exist $a_1,a_2\in A$ such that $y_1=2xa_1, y_2=2xa_2, y_1+y_2=2xa_a+2xa_2=2x(a_1+a_2)$, then $y_1+y_2\in F(x)$.

Let $z_1 \in S, y_1 z_1 = 2xa_1 z_1 = 2x(a_1 z_1) \in F(x)$. Similarly, $z_1 y_1 \in F(x)$. So $F(x)$ is an ideal of S. It is easy to check that $x + a + z = y + b + z$ implies $x \in y + F(x)$ for any $x, y, z \in S$ and $a, b \in F(x)$, then $F(x)$ is a strong h-ideal of S and (F, A) is a strong h-idealistic soft hemiring.

Proposition 3.3 *Let (F, A) be a soft set over S and let $B \subseteq A$. If (F, A) is a strong h-idealistic soft hemiring over S, then so is (F, B) whenever it is non-null.*

Proof. Straightforward. ∎

Theorem 3.4 *Let (F, A) and (G, B) be two strong h-idealistic soft hemirings over S. Then $(F, A)\widetilde{\sqcap}(G, B)$ is a strong h-idealistic soft hemiring over S if it is non-null.*

Proof. By Definition 2.4 (1), we can write $(F, A)\widetilde{\sqcap}(G, B) = (\gamma, I)$, where $I = A \cap B$ and $\gamma(x) = F(x) \cap G(x)$ for all $x \in I$. Suppose that (γ, I) is a non-null soft set over S. If $x \in I$, then $\gamma(x) = F(x) \cap G(x) \neq \emptyset$. Thus the nonempty sets $F(x)$ and $G(x)$ are strong h-ideals of S. It follows that $\gamma(x)$ is a strong h-ideal of S for all $x \in I$. Hence, $(\gamma, I) = (F, A)\widetilde{\sqcap}(G, B)$ is a strong h-idealistic soft hemiring over S. ∎

Theorem 3.5 *Let (F, A) and (G, B) be two strong h-idealistic soft hemirings over S. If A and B are disjoint, then the union $(F, A)\widetilde{\cup}(G, B)$ is a strong h-idealistic soft hemiring over S.*

Proof. According 2.4 (3), we can write $(F, A)\widetilde{\cup}(G, B) = (\gamma, I)$, where $I = A \cup B$ and for every $x \in I$,

$$\gamma(x) = \begin{cases} F(x), & \text{if } \ e \in A - B, \\ G(x), & \text{if } \ e \in B - A, \\ F(x) \cup G(x), & \text{if } \ e \in A \cap B. \end{cases}$$

Suppose that $A \cap B = \emptyset$. Then, for every $x \in I$, we know that either $x \in A - B$ or $x \in B - A$. If $x \in A - B$, then $\gamma(x) = F(x)$ is a strong h-ideal of S since (F, A) is a strong h-idealistic soft hemirings over S. Similarly, if $x \in B - A$, then $\gamma(x) = G(x)$ is a strong h-ideal of S since (G, B) is a strong h-idealistic soft hemirings over S. Thus we conclude that $\gamma(x)$ is a strong h-ideal of S for all $x \in I$, and so $(\gamma, I) = (F, A)\widetilde{\cup}(G, B)$ is a strong h-idealistic soft hemirings over S. ∎

Theorem 3.6 *Let (F, A) and (G, B) be two strong h-idealistic soft hemirings over S. Then $(F, A)\widetilde{\wedge}(G, B)$ is a strong h-idealistic soft hemiring over S if it is non-null.*

Proof. According 2.4 (2), we can write $(F, A)\widetilde{\wedge}(G, B) = (\gamma, C)$, where $C = A \times B$ and $\gamma(x, y) = F(x) \cap G(y)$ for all $(x, y) \in C$. Suppose that (γ, C) is a non-null soft set over S. If $(x, y) \in C$, then $\gamma(x, y) = F(x) \cap G(y) \neq \emptyset$. Since (F, A) and (G, B) are strong h-idealistic soft hemirings over S, we deduce that the nonempty sets $F(x)$ and $G(y)$ are both strong h-ideals of S. Hence, $\gamma(x, y)$ is a strong h-ideal of S for all $(x, y) \in C$, and so we conclude that $(\gamma, C) = (F, A)\widetilde{\wedge}(G, B)$ is a strong h-idealistic soft hemirings over S. ∎

Definition 3.7 Let (F, A) and (G, B) be soft hemirings over two hemirings R and S, respectively. Let $f : R \to S$ and $g : A \to B$ be two mappings. Then the pair (f, g) is called a soft hemiring homomorphism if it satisfies the following conditions:

(1) f is an epimorphism of hemirings.

(2) g is a surjective mapping.

(3) $f(F(x)) = G(g(x))$ for all $x \in A$.

If there exists a soft hemiring homomorphism between (F, A) and (G, B), we say that (F, A) is soft homomorphic to (G, B), which is denoted by $(F, A) \sim (G, B)$. Moreover, if f is an isomorphism of hemirings and g is a bijective mapping, then (f, g) is called a soft hemiring isomorphism. In this case, we say that (F, A) is soft isomorphic to (G, B), which is denoted by $(F, A) \simeq (G, B)$.

Example 3.8 Denote by $\mathbb{Z}$ and Z_n the hemiring of integers and the hemiring of integers module(a positive integer) n, respectively. Let $f : \mathbb{Z} \to Z_n$ be the natural mapping defined by $f(x) = [x]$ for all $x \in \mathbb{Z}$. Evidently, f is an epimorphism of hemirings. Let $\mathbb{Z}^+$ be the set of positive integers and define a mapping $g : \mathbb{Z}^+ \to Z_n$ by $g(x) = [x]$ for all $x \in \mathbb{Z}^+$, then it is easy to see that the mapping g is surjective. Let $(\alpha, \mathbb{Z}^+)$ be a soft set over $\mathbb{Z}$, where $\alpha : \mathbb{Z} \to \mathscr{P}(\mathbb{Z})$ is a set-valued function defined by $\alpha(x) = \{3xk|k \in \mathbb{Z}\}$ for all $x \in \mathbb{Z}^+$. One easily verifies that $\alpha(x) = 3x\mathbb{Z}$ is a subhemiring of $\mathbb{Z}$ for all $x \in \mathbb{Z}^+$. Thus (α, Z^+) is a soft hemiring over $\mathbb{Z}$. Let (β, Z_n) be a soft set over Z_n, where $\beta : Z_n \to \mathscr{P}(Z_n)$ is a set-valued function given by $\beta([x]) = \{[3xk]|k \in \mathbb{Z}\}$ for all $[x] \in Z_n$. Then one can also prove that (β, Z_n) is a soft hemiring over Z_n. Moreover, since $f(\alpha(x)) = f(3x\mathbb{Z}) = \{[3xk]|k \in \mathbb{Z}\}$ and $\beta(g(x)) = \beta([x]) = \{[3xk]|k \in \mathbb{Z}\}$ for all $x \in \mathbb{Z}^+$, we deduce that $f(\alpha(x)) = \beta(g(x))$ for all $x \in \mathbb{Z}^+$. Hence (f, g) is a soft hemiring homomorphism and $(\alpha, \mathbb{Z}^+) \sim (\beta, Z_n)$.

Lemma 3.9 [16] *Let I be a strong h-ideal of S. If $x, y \in S$, then*

[(1) $x \in [y]_I$ *if and only if* $x \in y + I$,

(2) $[x]_I + [y]_I = [x + y]_I$,

(3) $\{ab|a \in [x]_I, b \in [y]_I\} \subseteq [xy]_I$.

Next, S is always an idempotent hemiring, we introduce the concepts of soft quotient structure over an idempotent hemiring.

Lemma 3.10 *Let (F, A) be a strong h-idealistic soft hemiring over S, and $S/F(\alpha) = \{[x]_{F(\alpha)} : x \in S\}$, where $\alpha \in A$. Then, for any $\alpha \in A$, $S/F(\alpha)$ is a hemiring under the binary operation induced by S, which is given by*

$$\begin{aligned} [x]_{F(\alpha)} + [y]_{F(\alpha)} &= [x + y]_{F(\alpha)}, \\ [x]_{F(\alpha)}[y]_{F(\alpha)} &= [xy]_{F(\alpha)} \end{aligned}$$

for all $x, y \in S$.

Proof. Firstly, we show that the above binary operations are well defined. In fact, if $[a]_{F(\alpha)} = [a']_{F(\alpha)}$ and $[b]_{F(\alpha)} = [b']_{F(\alpha)}$ for all $a, a', b, b' \in S, \alpha \in A$. Since (F, A) is a strong h-idealistic soft hemiring over S, then, by Definition 3.1, we know that $F(\alpha)$ is a strong h-ideal of S for all $\alpha \in A$.

By Lemma 3.8,

$$[a]_{F(\alpha)} = a + F(\alpha), \quad [a']_{F(\alpha)} = a' + F(\alpha),$$
$$[b]_{F(\alpha)} = b + F(\alpha), \quad [b']_{F(\alpha)} = b' + F(\alpha),$$

then we have

$$\begin{aligned} [a+b]_{F(\alpha)} &= a + b + F(\alpha) = a + F(\alpha) + b + F(\alpha) \\ &= a' + F(\alpha) + b' + F(\alpha) = a' + b' + F(\alpha) \\ &= [a' + b']_{F(\alpha)}, \\ [ab]_{F(\alpha)} &= ab + F(\alpha) = ab + aF(\alpha) + F(\alpha)b + F(\alpha)^2 \\ &= (a' + F(\alpha))(b' + F(\alpha)) = [a'b']_{F(\alpha)}. \end{aligned}$$

Now, it is easy to check that $S/F(\alpha)$ is a hemiring. ∎

Lemma 3.11 *If A is an ideal of S, then $\widetilde{A}$ is a strong h-ideal of S containing A.*

Proof. Let $a, b \in \widetilde{A}$, then there exist $y_1, y_2 \in S$ such that

$$y_1 + a \in y_1 + \widetilde{A}$$

and

$$y_2 + b \in y_2 + \widetilde{A}$$

satisfying

$$y_1 + a + a_1 + z_1 = y_1 + a_2 + z_1$$

and

$$y_2 + b + b_1 + z_2 = y_2 + b_2 + z_2$$

for some $a_1, a_2, b_1, b_2 \in A, z \in S$. Then we have

$$y_1 + y_2 + a + b + a_1 + b_1 + z_1 + z_2 = y_1 + y_2 + a_2 + b_2 + z_1 + z_2,$$

that is

$$a + b + a_1 + b_1 + z' = a_2 + b_2 + z',$$

where $z' = y_1 + y_2 + z_1 + z_2$.

Since $a_1, a_2, b_1, b_2 \in A$ and A is an ideal, then we have $a_1 + b_1 \in A, a_2 + b_2 \in A$, so $a + b \in 0 + \widetilde{A}$. In a similar way, we have $ra, ar \in \widetilde{A}$ for $r \in S$. Thus $\widetilde{A}$ is a ideal. By the definition of strong h-closure, we know that $\widetilde{A}$ has the strong h-property, so $\widetilde{A}$ is a strong h-ideal. ∎

Theorem 3.12 *Let (F, A) be a strong h-idealistic soft hemiring over S. If (H, B) and (I, C) are soft strong h-ideals of (F, A), then $(P, B) \simeq (Q, B)$ and $(S, C) \simeq (T, C)$, where $P(x) = H(x)/(M \cap N)$, $Q(x) = (H(x) + N)/N$, $S(x) = I(x)/(M \cap N)$, $T(x) = (I(x) + M)/M$, $M = \bigcap_{x\in B} H(x)$ and $N = \bigcap_{x\in C} I(x)$.*

Proof. We first write

$$K = <\widetilde{\bigcup_{x\in B} H(x)}> \text{ and } L = <\widetilde{\bigcup_{x\in C} I(x)}>.$$

By Lemma 3.11, we know that K and L are strong h-ideals of S. Then $M = \bigcap_{x\in B} H(x)$ is a strong h-ideal of S. It is clear that M is also a strong h-ideal of K so that $M \cap N$ is a strong h-ideal of K, and hence, (P, B) is a soft hemiring over $K/(M \cap N)$. Similarly, (Q, B) is a soft hemiring over $(K + N)/N$.

Now, we define $f : K/(M\cap N) \to (K+N)/N$ by $f([k]_{M\cap N}) = [k]_N$ for $k \in K$ and define $g : B \to B$ by $g(x) = x$. By Lemma 3.9, we can check that f is an isomorphism from $K/(M\cap N)$ to $(K+N)/N$. Obviously, g is a bijective mapping and $f(P(x)) = f(H(x)/(M \cap N)) = (H(x) + N)/N = Q(x) = Q(g(x))$. This shows that $(P, B) \simeq (Q, B)$. Similarly, we can prove that $(S, C) \simeq (T, C)$. ∎

Theorem 3.13 *Let (F, A) be a strong h-idealistic soft hemiring over S. If (H, B) and (I, C) are soft strong h-ideals of (F, A) with $B \cap C \neq \emptyset$ and $I(x) \subset H(x)$ for all $x \in B \cap C$, then $(P, B \cap C) \simeq (Q, B \cap C)$, where $P(x) = (F(x)/N)/(M/N)$ and $Q(x) = F(x)/M$ with $M = \bigcap_{x\in B\cap C} H(x)$ and $N = \bigcap_{x\in B\cap C} I(x)$.*

Proof. It can be easily verified that $M = \bigcap_{x\in B\cap C} H(x)$ and $N = \bigcap_{x\in B\cap C} I(x)$ are strong h-ideals of S, and N is a strong h-ideal of M. Now it is easy to see that $(P, B \cap C)$ is a soft hemiring over the hemiring $(S/N)/(M/N)$ and so $(Q, B \cap C)$ is a soft hemiring over S/M.

Define the mapping $f : (S/N)/(M/N) \to S/M$ by $f(\{[r]_N\}_{M/N}) = [r]_M$ for $r \in S$ and define $g : B \cap C \to B \cap C$ by $g(x) = x$. Then by Lemma 3.9, we can check that f is an isomorphism from $(S/N)/(M/N)$ to S/M. Obviously, g is a bijective mapping and $f(P(x)) = f((F(x)/N)/(M/N)) = F(x)/M = Q(g(x))$. Hence, $(P, B \cap C) \simeq (Q, B \cap C)$. ∎

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