ON COMMUTING TRACES OF GENERALIZED BIDERIVATIONS OF PRIME RINGS

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Abstract. In this paper, we prove some theorems on symmetric generalized biderivations of a ring, which extend a result of Vukman [9, Theorem 1] and a result of Bresar [3, Theorem 4.1].

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1. Introduction

Throughout the paper all ring will be associative. We shall denote by $Z(R)$ the centre of ring R and by C the extended centroid of R, which is the centre of the two sided Martindale quotients ring Q (we refer the reader [3] for more details). A ring R is said to be prime (resp. semiprime) if $aRb = (0)$ implies that either $a = 0$ or $b = 0$ (resp. $aRa = (0)$ implies that $a = 0$). We shall write for any pair of elements $x, y \in R$ the commutator $xy - yx$ and $x \circ y$ stands for the skew commutator $xy + yx$. We make extensive use of the basic commutator identities (i) $[x, yz] = [x, y]z + y[x, z]$ and (ii) $[xy, z] = [x, z]y + x[y, z]$. An additive mapping $d : R \longrightarrow R$ is called a derivation if $d(xy) = d(x)y + xd(y)$, for all $x, y \in R$. A derivation d is inner if there exists an element $a \in R$ such that $d(x) = [a, x]$ for all $x \in R$. A mapping $D : R \times R \longrightarrow R$ is said to be symmetric if $D(x, y) = D(y, x)$, for all $x, y \in R$. A mapping $f : R \longrightarrow R$ defined by $f(x) = D(x,x)$, where $D : R \times R \longrightarrow R$ is a symmetric mapping, is called the trace of D. It is obvious that in the case $D : R \times R \longrightarrow R$ is a symmetric mapping which is also biadditive (i.e. additive in both arguments). The trace f of D satisfies the relation $f(x+y) = f(x) + f(y) + 2D(x,y)$, for all $x, y \in R$. A biadditive symmetric mapping $D : R \times R \longrightarrow R$ is called a symmetric biderivation if $D(xy,z) = D(x,z)y + xD(y,z)$ for all $x, y, z \in R$. Obviously, in this case the relation $D(x,yz) = D(x,y)z + yD(x,z)$ is also satisfied for all $x, y, z \in R$.

Typical examples are mapping of the form $(x,y) \mapsto \lambda[x,y]$ where $\lambda \in C$. We shall call such maps inner biderivations. In [6] it was shown that every biderivation D of a noncommutative prime ring R is of the form $D(x,y) = \lambda[x,y]$ for some $\lambda \in C$. Further Bresar extended this result for semiprime rings. Some results on biderivations can be found in[2], [6] and [8].

G. Maksa [8] introduced the concept of a symmetric biderivation (see also [9], where an example can be found). It was shown in [8] that symmetric biderivations are related to general solution of some functional equations. Some results on symmetric biderivation in prime and semiprime rings can be found in [5], [11] and [12]. The notion of additive commuting mappings is closely connected with the notion of biderivations. Every commuting additive mapping $f : R \longrightarrow R$ gives rise to a biderivation on R. Namely linearizing $[x, f(x)] = 0$ for all $x, y \in R$ $(x,y) \mapsto [f(x), y]$ is a biderivation (moreover, all derivations appearing are inner).

The notion of generalized symmetric biderivations was introduced by Nurcan [1]. More precisely, a generalized symmetric biderivation is defined as follows: Let R be a ring and $D : R \times R \longrightarrow R$ be a biadditive map. A biadditive mapping $\Delta : R \times R \longrightarrow R$ is said to be generalized biderivation if for every $x \in R$, the map $y \mapsto \Delta(x,y)$ is a generalized derivation of R associated with function $y \mapsto D(x,y)$ as well as if for every $y \in R$, the map $x \mapsto \Delta(x,y)$ is a generalized derivation of R associated with function $x \mapsto D(x,y)$ for all $x, y \in R$. It also satisfies $\Delta(x,yz) = \Delta(x,y)z + yD(x,z)$ and $\Delta(xy,z) = \Delta(x,z)y + xD(y,z)$ for all $x, y, z \in R$. For example consider a biderivation Δ of R and biadditive a function $\alpha : R \times R \longrightarrow R$ such that $\alpha(x,yz) = \alpha(x,y)z$ and $\alpha(xy,z) = \alpha(x,z)y$ for all $x, y, z \in R$. Then $\Delta + \alpha$ is a generalized Δ-biderivation of R.

An additive mapping $h : R \longrightarrow R$ is called left (resp. right) multiplier of R if $h(xy) = h(x)y$ (resp. $h(xy) = xh(y)$) for all $x, y \in R$. A biadditive mapping $D : R \times R \longrightarrow R$ is said to be a left (resp. right) bi-multiplier of R if $D(x,yz) = D(x,y)z$ (resp. $D(xz,y) = xD(z,y)$) for all $x, y, z \in R$.

In this paper, we prove some theorems on symmetric generalized biderivations of a ring which extend a result of Vukman [9, Theorem 1] and a result of Bresar [3, Theorem 4.1].

2. Generalized biderivations on prime rings

The result proved in this section generalizes Theorem 1 in [11]. More precisely, we consider the case when the ring R is prime and replace symmetric biderivations with symmetric generalized biderivations.

In [11], Vukman proved the following result: Let R be a noncommutative prime ring of characteristic different from two and $D : R \times R \longrightarrow R$ be a symmetric biderivation with trace f. If f is commuting on R, then $d = 0$. Vukman [10, Theorem 2] further generalized the result by proving that let R be a noncommutative prime ring of characteristic different from two. Suppose there exists a symmetric biderivation $D : R \times R \longrightarrow R$ with trace f such that the mapping $x \mapsto [f(x), x]$ is centralizing on R. In this case $D = 0$.

Theorem 2.1. *Let R be a prime ring of characteristic different from two and I be a nonzero left ideal of R. If Δ is a symmetric generalized biderivation with associated biderivation D such that $[\Delta(x, x), x] = 0$ for all $x \in I$, then either R is commutative or Δ acts as a left bimultiplier on I.*

Proof. Suppose that

$$[\Delta(x, x), x] = 0, \quad \text{for all } x \in I. \tag{2.1}$$

Linearization of (2.1) yields that

$$\begin{aligned}[][\Delta(x, x), x] + [\Delta(x, x), y] + [\Delta(x, y), x] + [\Delta(x, y), y] + [\Delta(y, x), x] \\ +[\Delta(y, x), y] + [\Delta(y, y), x] + [\Delta(y, y), y] = 0, \quad \text{for all } x, y \in I.\end{aligned} \tag{2.2}$$

Since Δ is symmetric and using (2.1), we obtain

$$\begin{aligned}[][\Delta(x, x), y] + 2[\Delta(x, y), x] + 2[\Delta(x, y), y] + [\Delta(y, y), x] = 0, \\ \text{for all } x, y \in I.\end{aligned} \tag{2.3}$$

Substituting $-y$ for y in (2.3), we have

$$\begin{aligned}-[\Delta(x, x), y] - 2[\Delta(x, y), x] + 2[\Delta(x, y), y] + [\Delta(y, y), x] = 0, \\ \text{for all } x, y \in I.\end{aligned} \tag{2.4}$$

Adding (2.3) and (2.4) and using char $R \neq 2$, we find

$$2[\Delta(x, y), y] + [\Delta(y, y), x] = 0, \quad \text{for all } x, y \in I. \tag{2.5}$$

Replace x by xz in (2.5) to get

$$\begin{aligned}2\Delta(x, y)[z, y] + 2[\Delta(x, y), y]z + 2x[D(z, y), y] + 2[x, y]D(z, y) \\ +[\Delta(y, y), x]z + x[\Delta(y, y), z] = 0, \quad \text{for all } x, y, z \in I.\end{aligned} \tag{2.6}$$

In view of (2.5), (2.6) gives that

$$\begin{aligned}\Delta(x, y)[z, y] + 2x[D(z, y), y] + 2[x, y]D(z, y) + x[\Delta(y, y), z] = 0, \\ \text{for all } x, y, z \in I.\end{aligned} \tag{2.7}$$

Substitute y for z to obtain

$$2x[D(y, y), y] + 2[x, y]D(y, y) = 0, \quad \text{for all } x, y \in I. \tag{2.8}$$

Since char R not two, we have

$$x[D(y,y),y]+[x,y]D(y,y)=0, \quad \text{for all } x,y\in I. \tag{2.9}$$

Substitute rx for x in (2.9) and using (2.9), we obtain

$$[r,y]xD(y,y)=0, \quad \text{for all } x,y\in I, \quad \text{for all } r\in R. \tag{2.10}$$

Replace r by rs in (2.10), we find

$$[r,y]RxD(y,y)=0, \quad \text{for all } x,y\in I, \ \text{for all } r\in R. \tag{2.11}$$

Primeness of R yields that either $[y,r]=0$ or $xD(y,y)=0$ for all $x,y\in I$. If $[y,r]=0$ for all $y\in I$ and $r\in R$, then I is contained in $Z(R)$. Since I is a central ideal of R, we have R is commutative by [10]. On the other hand, we have $xD(y,y)=0$ for all $x,y\in I$. Linearization in y yields that $xD(y,z)+xD(z,y)=0$ for all $x,y,z\in I$. Since D is symmetric and using char $R\neq 2$, we get $xD(y,z)=0$ for all $x,y,z\in I$, i.e. Δ acts as a left bimultiplier on I.

Corollary 2.1. *Let R be a prime ring of characteristic different from two and I be a nonzero left ideal of R. If Δ is a symmetric generalized biderivation with associated biderivation D such that $\Delta(x,y)\mp[x,y]\in Z(R)$ for all $x,y\in I$, then either R is commutative or Δ acts as a left bimultiplier on I.*

Corollary 2.2. *Let R be a prime ring of characteristic different from two and I be a nonzero left ideal of R. If Δ is a symmetric generalized biderivation with associated biderivation D such that $\Delta(x,y)\mp x\circ y\in Z(R)$ for all $x,y\in I$, then either R is commutative or Δ acts as a left bimultiplier on I.*

Theorem 2.2. *Let R be a prime ring of characteristic different from two and I be a nonzero left ideal of R. If Δ is a symmetric generalized biderivation with associated biderivation D such that $\Delta(x,x)\circ x=0$ for all $x\in I$, then either R is commutative or Δ acts as a left bimultiplier on I.*

Proof. By assumption, we have

$$\Delta(x,x)\circ x=0 \quad \text{for all } x\in I. \tag{2.12}$$

Linearization of (2.12) yields that

$$\begin{aligned}&\Delta(x,x)x+\Delta(y,y)x+2\Delta(x,y)x+\Delta(x,x)y+\Delta(y,y)y\\&\quad+2\Delta(x,y)y+x\Delta(x,x)+x\Delta(y,y)+2x\Delta(x,y)+y\Delta(x,x)\\&\quad+y\Delta(y,y)+2y\Delta(x,y)=0 \quad \text{for all } x,y\in I.\end{aligned} \tag{2.13}$$

In view of (2.12), (2.13), gives that

$$\begin{aligned}&\Delta(y,y)x+2\Delta(x,y)x+\Delta(x,x)y+2\Delta(x,y)y+x\Delta(y,y)\\&\quad+2x\Delta(x,y)+y\Delta(x,x)+2y\Delta(x,y)=0 \quad \text{for all } x,y\in I.\end{aligned} \tag{2.14}$$

Substituting $-y$ for y in (2.14), we have

$$\begin{aligned}\Delta(y,y)x - 2\Delta(x,y)x - \Delta(x,x)y + 2\Delta(x,y)y + x\Delta(y,y)\\ -2x\Delta(x,y) - y\Delta(x,x) + 2y\Delta(x,y) = 0 \quad \text{for all } x,y \in I.\end{aligned} \tag{2.15}$$

Adding (2.14) and (2.15) and using the fact that char$R \neq 2$, we get

$$\Delta(y,y)x + 2\Delta(x,y)y + x\Delta(y,y) + 2y\Delta(x,y) = 0, \quad \text{for all } x,y \in I. \tag{2.16}$$

Replacing x by xu in (2.16), we have

$$\begin{aligned}\Delta(y,y)xu + 2\Delta(x,y)uy + 2xD(u,y)y + xu\Delta(y,y)\\ +2y\Delta(x,y)u + 2yxD(u,y) = 0 \quad \text{for all } x,y \in I.\end{aligned} \tag{2.17}$$

Right multiplying (2.16) by u and then subtracting from (2.17), we obtain

$$\begin{aligned}2\Delta(u,y)[u,y] + 2x\Delta(u,y)y + x[u,\Delta(y,y)] + 2yxD(u,y) = 0,\\ \text{for all } x,y,u \in I.\end{aligned} \tag{2.18}$$

Substituting u by y in (2.18), we get

$$2x\Delta(y,y)y + x[y,\Delta(y,y)] + 2yxD(y,y) = 0 \quad \text{for all } x,y \in I. \tag{2.19}$$

Replacing rx for x in (2.19) and using it, we obtain

$$\begin{aligned}2rx\Delta(y,y)y + rx[y,\Delta(y,y)] + 2yrxD(y,y) = 0,\\ \text{for all } x,y \in I \quad \text{and} \quad \text{for all } r \in R.\end{aligned} \tag{2.20}$$

Left multiplying (2.19) by r and then subtracting from (2.20), we get

$$2[y,r]xD(y,y) = 0, \quad \text{for all } x,y \in I \quad \text{and} \quad \text{for all } r \in R. \tag{2.21}$$

This implies that $2[y,r]RxD(y,y) = 0$ for all $x,y \in I$ and for all $r \in R$. Since char$R \neq 2$ we have $[y,r]RxD(y,y) = 0$ for all $x,y \in I$ and for all $r \in R$. Primeness of R yields that either $[y,r] = 0$ or $xD(y,y) = 0$ for all $x,y \in I$ and for all $r \in R$. Arguing in the similar manner as in the proof of Theorem 2.1, we get the result.

Theorem 2.3. *Let R be a $2,3$ and 5-torsion free semiprime ring, I an additive subgroup of R such that $x^2 \in I$ for all $x \in I$ and $\Delta : R \times R \to R$ be a symmetric generalized biderivation associated with biderivation D with the trace f of Δ. If f is centralizing on I, then f is commuting on I.*

Proof. Let $x \in I$ and take $t = [f(x),x]$, where $f(x) = \Delta(x,x)$. Then $t \in Z(R)$. By our hypothesis, we have

$$[f(x),x] \in Z(R) \quad \text{for all } x \in I. \tag{2.22}$$

Replacing x by $x+y$ in (2.22), we have

$$\begin{aligned}(2.23)\qquad &[f(x),x]+[f(x),y]+[f(y),x]+[f(y),y]+[\Delta(x,y),x]+[\Delta(y,x),x]\\ &+[\Delta(y,x),y]+[\Delta(x,y),y]\in Z(R)\quad \text{for all}\ \ x,y\in I.\end{aligned}$$

Putting $x=-x$ in (2.23) and using (2.22), we get

$$(2.24)\qquad [f(x),y]+2[\Delta(x,y),x]\in Z(R)\quad \text{for all}\ \ x,y\in I.$$

Substituting x^2 for y in (2.24), we have

$$(2.25)\qquad [f(x),x^2]+[\Delta(x,x)x+xD(x,x),x]\in Z(R)\quad \text{for all}\ \ x\in I.$$

We have $[f(x),x^2]=[f(x),x]x+x[f(x),x]=2tx$.
Since $[\Delta(x,x^2),x]=2tx+2x[D(x,x),x]$, the last expression reduces to

$$(2.26)\qquad 2(x[D(x,x),x]+2tx)\in Z(R)\quad \text{for all}\ \ x\in I.$$

Since R is 2-torsion free, we get $x[D(x,x),x]+2tx\in Z(R$.
Let $z=x[D(x,x),x]+2tx\in Z(R)$. This implies that

$$(z-2tx)=x[D(x,x),x].$$

Replacing x by x^2 in our hypothesis, we can write

$$\begin{aligned}[f(x^2),x^2]&=[\Delta(x^2,x^2),x^2]=[\Delta(x^2,x)x+xD(x^2,x),x^2]\\ &=[\Delta(x^2,x),x^2]x+x[D(x^2,x),x^2]x\\ &=[\Delta(x,x)x+xD(x,x),x^2]x+x[D(x,x)x+xD(x,x),x^2]\\ &=[\Delta(x,x),x^2]x^2+x[D(x,x),x^2]x+x[D(x,x),x^2]x+x^2[D(x,x),x^2]\\ &=[\Delta(x,x),x]x^3+x[\Delta(x,x),x]x^2+2x^2[D(x,x),x]x\\ &+2x[D(x,x),x]x^2+x^3[D(x,x),x]+x^2[D(x,x),x]x\\ &=2tx^3+2x(z-2tx)x+2(z-2tx)x^2+x^2(z-2tx)+x(z-2tx)x\\ &=-10tx^3+6zx^2\end{aligned}$$

This implies that $-10tx^3+6zx^2\in Z(R)$. Commuting both sides with $f(x)$, we get $[f(x),-10tx^3+6zx^2]=0$, i.e.,

$$\begin{aligned}&-10t[f(x),x^3]+6z[f(x),x^2]\\ &\quad=-10t[f(x),x]x^2-10tx[f(x),x^2]+6z[f(x),x]x+6zx[f(x),x]\\ &\quad=-10t^2x^2-10tx[f(x),x]x-10tx^2[f(x),x]+12ztx\\ &\quad=-30t^2x^2+12ztx=0.\end{aligned}$$

Again commuting with $f(x)$, we have

$$\begin{aligned}-30t^2[f(x),x^2]+12zt[f(x),x]&=-30t^2[f(x),x]x-30t^2x[f(x),x]+12zt^2\\ &=-60t^3x+12zt^2=0.\end{aligned}$$

Repeating the same argument, we finally arrive at $-60t^4 = 0$. Since R is $2, 3$ and 5 torsion free, we get $t^4 = 0$. Since the center of a semiprime ring contains no nonzero nilpotent elements, we conclude that $t = 0$. This completes the proof.

3. Cocommuting biderivations

In this section, we consider the case in which the mappings $\mu, \phi : R \longrightarrow R$ satisfy $\mu(x)x + x\phi(x) = 0$ for all $x \in R$. Bresar [3, Theorem 4.1] proved that if R is a prime ring, I a nonzero left ideal of R and α and β are nozero derivations of R satisying $\alpha(x)x - x\beta(x) \in Z(R)$ for all $x \in I$, then R is commutative. Argac [1, Theorem 3.5] proved a result for generalized derivation of R.

We extend the aforementioned results by proving the following theorem for a biderivation of R.

Theorem 3.1. *Let R be a prime ring of characteristic not two, I a nonzero left ideal of R and D, G be symmetric biderivations of R with trace f and g respectively. If $D(x,x)x+xG(x,x) = 0$ for all $x \in I$, then either R is commutative or G acts as a left bimultiplier on I. Moreover, in the last case either $D = 0$ or $I[I, I] = 0$.*

Proof. By hypothesis, we have

$$f(x)x + xg(x) = 0 \text{ for all } x \in I, \tag{3.1}$$

where $f(x) = D(x,x)$ and $g(x) = G(x,x)$. Linearization of (3.1) yields that

$$\begin{aligned} &f(y)x + f(x)y + 2D(x,y)x + 2D(x,y)y + xg(y) + yg(x) \\ &\qquad +2xG(x,y) + 2yG(x,y) = 0, \text{ for all } x, y \in I. \end{aligned} \tag{3.2}$$

Substituting $-y$ for y in (3.2), we get

$$\begin{aligned} &f(y)x - f(x)y - 2D(x,y)x + 2D(x,y)y + xg(y) - yg(x) \\ &\qquad -2xG(x,y) + 2yG(x,y) = 0, \text{ for all } x, y \in I. \end{aligned} \tag{3.3}$$

Adding (3.1) and (3.2), we obtain

$$2f(y)x + 4D(x,y)y + 2xg(y) + 4yG(x,y) = 0, \text{ for all } x, y \in I. \tag{3.4}$$

Since char R is not two, we have

$$f(y)x + 2D(x,y)y + xg(y) + 2yG(x,y) = 0, \text{ for all } x, y \in I. \tag{3.5}$$

Replacing x by xz in (3.5), we obtain

$$\begin{aligned} &f(y)xz + 2D(x,y)zy + 2xD(z,y)y + xzg(y) + 2yG(x,y)z + 2yxG(z,y) = 0, \\ &\qquad \text{for all } x, y, z \in I. \end{aligned} \tag{3.6}$$

Comparing (3.5) and (3.6), we obtain

$$\begin{aligned} -2D(x,y)yz - xg(y)z + 2D(x,y)zy + 2xD(z,y)y + xzg(y) \\ +2yxG(z,y) = 0, \text{ for all } x,y,z \in I. \end{aligned} \tag{3.7}$$

This implies that

$$\begin{aligned} 2D(x,y)[z,y] + x[z,g(y)] + 2xD(z,y)y + 2yxG(z,y) = 0, \\ \text{for all } x,y,z \in I. \end{aligned} \tag{3.8}$$

Substituting rx for x in (3.8), we get

$$\begin{aligned} 2rD(x,y)[z,y] + 2D(r,y)x[z,y] + rx[z,g(y)] \\ +2rxD(z,y)y + 2yrxG(z,y) = 0, \\ \text{for all } x,y,z \in I, \text{ for all } r \in R. \end{aligned} \tag{3.9}$$

Comparing (3.8) and (3.9), we get

$$\begin{aligned} 2D(r,y)x[z,y] + 2yrxG(z,y) - 2ryxG(z,y) = 0, \\ \text{for all } x,y,z \in I, \text{ for all } r \in R. \end{aligned} \tag{3.10}$$

Since R is of characteristic not two, we obtain

$$D(r,y)x[z,y] + [y,r]xG(z,y) = 0, \text{ for all } x,y,z \in I, \text{ for all } r \in R. \tag{3.11}$$

Replacing y by z in (3.12), we obtain

$$[z,r]xg(z) = 0, \text{ for all } x,z \in I, \text{ for all } r \in R \tag{3.12}$$

Substituting rx for x in (3.12), we get

$$[z,r]Rxg(z) = 0, \text{ for all } x,z \in I, \text{ for all } r \in R \tag{3.13}$$

Primeness of R yields that either $[z,r] = 0$ or $xg(z) = 0$. If $[z,r] = 0$ for all $z \in I$ and $r \in R$, then R is commutative by [10]. Suppose $xg(z) = 0$ for all $x,z \in I$. Linearization in z yields that

$$0 = xG(z,y) + xG(y,z) = 2xG(y,z)$$

and using R is not of characteristic two, we get

$$xG(y,z) = 0 \text{ for all } x,y,z \in I.$$

This implies that

$$G(x,yz) = G(x,y)z.$$

Hence G acts as left multiplier. Since $xG(y,z) = 0$ for all $x,y,z \in I$ and using (3.11), we arrive at

$$D(r,y)x[z,y] = 0, \text{ for all } x,y,z \in I, \ r \in R. \tag{3.14}$$

Replace r by rs in (3.14) to get

$$D(r,y)Rx[z,y]=0,\ \text{for all}\ \ x,y,z\in I,\ r\in R. \tag{3.15}$$

Primeness of R implies that either $D(r,y)=0$ or $x[z,y]=0$ for all $x,y,z\in I$. Later yields that $I[I,I]=0$ as $D\neq 0$.

Proceeding on the same parallel lines, we can prove the following:

Theorem 3.2. *Let R be a prime ring of characteristic not two, I a nonzero right ideal of R and D, G are symmetric biderivations of R with trace f and g respectively. If $D(x,x)x+xG(x,x)=0$ for all $x\in I$, then then either R is commutative or D acts as a left bimultiplier on I. Moreover in the last case either $G=0$ or $I[I,I]=0$.*

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