SOFT FUZZY DISCONNECTEDNESS IN DIMENSION THEORY

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Abstract. In this paper the concept of soft fuzzy B disconnected space is introduced and studied. In particular, soft fuzzy B disconnectedness via dimension theory is established.

Keywords: soft fuzzy B boundary, soft fuzzy B extremally disconnected, soft fuzzy B basically disconnected, soft fuzzy large inductive dimension function.

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1. Introduction

The concept of fuzzy set was introduced by L.A. Zadeh [19]. Fuzzy sets have applications in many fields such as information [12] and control [13]. The theory of fuzzy topological spaces was introduced and developed by C.L. Chang [8], and since then various notions in classical topology have been extended to fuzzy topological spaces. G. Balasubramanian [6], [7] introduced the concepts of fuzzy extremally disconnectedness and fuzzy basically disconnectedness. The concept of soft fuzzy topological space was introduced by Ismail U. Triyaki [15]. J. Tong [16] introduced the concept of B-set in topological space. The concept of fuzzy B-set was introduced by M.K. Uma, E. Roja and G. Balasubramanian [17]. D. Vidhya, E. Roja and M.K. Uma [18] introduced the concept of soft fuzzy B-open set.

J.R. Munkers [10], A.R. Pears [11] and R. Engleking [9] discussed the concept of "dimension" in topological space. A.R. Pears [11] and R. Engleking [9] discussed the concepts of small inductive dimension function, *ind* and large inductive dimension function, *Ind*. D. Adnadjevic [1], [2] introduced the concepts of F-*ind* and F-*Ind* and discussed some of its basic properties in GF-spaces. D. Adnadjevic and A.P. Sostak [3] developed these concepts in Hutton fuzzy topological

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spaces. In 2007, S.S. Benchalli, B.M. Ittanagi and P.G. Patil [4] proved that, if X is a fuzzy topological space such that $IndfX = 0$ then X is a normal fuzzy topological space.

Based on the above concepts, the present, study is on "soft fuzzy disconnectedness in dimension theory". Some basic definitions are given in section 2. Section 3 is divided into two parts: soft fuzzy extremally disconnectedness and soft fuzzy basically disconnectedness. In these two sections, we have introduced the concepts of soft fuzzy extremally disconnectedness and soft fuzzy basically disconnectedness and their basic propositions are also discussed. The main purpose of section 4 is to introduce, a new concept called soft fuzzy large inductive dimension function, $SFInd$ on soft fuzzy topological space. This concept is applied, for proving the soft fuzzy large inductive zero dimension function of soft fuzzy topological space is disconnectedness.

2. Preliminaries

Definition 2.1. [14] Let A and B be any two sets. The *relative complement* of B in A (or of B with respect to A), written as $A - B$, is that the set consisting of all elements of A which are not elements of B, that is,

$$A - B = \{x | x \in A \wedge x \notin B\} = \{x | x \in A \wedge \neg(x \in B)\}.$$

The relative complement of B in A is also called the *difference* of A and B.

Definition 2.2. [14] Let A and B be any two sets. The *symmetric difference* (or *Boolean sum*) of A and B is the set $A+B$ is defined by $A+B = (A-B)\cup(B-A)$ or $x \in A + B \Longleftrightarrow x \in \{x \in A \bar{\vee} x \in B\}$ where $\bar{\vee}$ is the exclusive disjunction.

Definition 2.3. [6] Suppose $(X, T(X))$ be any fuzzy topological space. X is said to be *fuzzy extremally disconnected* if $\lambda \in T(X)$ implies $\bar{\lambda} \in T(X)$.

Definition 2.4. [7] Let (X, T) be a fuzzy topological space. A fuzzy set $\lambda : X \to [0, 1]$ is said to a fuzzy G_δ-set if $\lambda = \wedge_{i=1}^{\infty} \lambda_i$ where each $\lambda_i \in T$. The complement of a G_δ-set is F_σ-set.

Definition 2.5. [7] Let (X, T) be any fuzzy topological space. (X, T) is called *fuzzy basically disconnected* if the closure of every fuzzy open F_σ set is fuzzy open.

Definition 2.6. [15] Let X be a non-empty set. A *soft fuzzy set*(in short, SFS) A have the form $A = (\lambda, M)$ where the function $\lambda : X \to I$ denote the degree of membership and M is the subset of X. The set of all soft fuzzy set will be denoted by $SF(X)$.

Proposition 2.1. [15] If $(\mu_j, N_j) \in SF(X), j \in J$, then the family $\{(\mu_j, N_j) | j \in J\}$ has a meet, ie., g.l.b., in $(SF(X), \sqsubseteq)$ denoted by $\sqcap_{j\in J}(\mu_j, N_j)$ and given by $\sqcap_{j\in J}(\mu_j, N_j) = (\mu, N)$ where $\mu(x) = \wedge_{j\in J}\mu_j(x) \forall x \in X$ and $M = \cap M_j$ for $j \in J$.

Proposition 2.2. [15] *If* $(\mu_j, N_j) \in SF(X)$, $j \in J$, *then the family* $\{(\mu_j, N_j)|j \in J\}$ *has a join, ie., l.u.b., in* $(SF(X), \sqsubseteq)$ *denoted by* $\sqcup_{j\in J}(\mu_j, N_j)$ *and given by* $\sqcup_{j\in J}(\mu_j, N_j) = (\mu, N)$ *where* $\mu(x) = \vee_{j\in J}\mu_j(x)\forall x \in X$ *and* $M = \cup M_j$ *for* $j \in J$.

Definition 2.7. [15] Let X be a non-empty set and the soft fuzzy sets A and C be in the form $A = (\lambda, M)$ and $C = (\mu, N)$. Then

(i) $A \sqsubseteq C$ if and only if $\lambda(x) \le \mu(x)$ for all $x \in X$ and $M \subseteq N$.

(ii) $A = C$ if and only if $\lambda(x) = \mu(x)$ for all $x \in X$ and $M = N$.

(iii) $A \sqcap C = (\lambda, M) \sqcap (\mu, N)$ if and only if $\lambda(x) \wedge \mu(x)$ for all $x \in X$, $M \cap N$.

(iv) $A \sqcup C = (\lambda, M) \sqcup (\mu, N)$ if and only if $\lambda(x) \vee \mu(x)$ for all $x \in X$, $M \cup N$.

Definition 2.8. [15] For $(\mu, N) \in SF(X)$ the soft fuzzy set $(\mu, N)' = (1-\mu, X\backslash N)$ is called the *complement* of (μ, N).

Remark 2.1. $(1 - \mu, X/N) = (1, X) - (\mu, N)$.

Proof. $(1, X)-(\mu, N) = (1, X)\sqcap(\mu, N)' = (1, X)\sqcap(1-\mu, X/N) = (1-\mu, X/N)$.

Definition 2.9. [15] Let X be a set. Let T be family of soft fuzzy subsets of X. Then T is called a soft fuzzy topology on X if T satisfies the following conditions:

(i) $(0, \emptyset)$ and $(1, X) \in T$.

(ii) If $(\mu_j, N_j) \in T, j = 1, 2, ..., n$ then $\sqcap_{j=1}^{n}(\mu_j, N_j) \in T$.

(iii) If $(\mu_j, N_j) \in T, j \in J$ then $\sqcup_{j\in J}(\mu_j, N_j) \in T$.

The pair (X, T) is called a *soft fuzzy topological space*(in short, *SFTS*). The members of T are *soft fuzzy open sets* and its complement is *soft fuzzy closed sets.*

Definition 2.10. [18] Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then (λ, M) is said to be a *soft fuzzy t open set* if $SFint(\lambda, M) = SFint(SFcl(\lambda, M))$

Definition 2.11. [18] Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then (λ, M) is said to be a *soft fuzzy B open set*(in short, *SFBoS*) if $(\lambda, M) = (\mu, N) \sqcap (\gamma, L))$ where (μ, N) is a soft fuzzy open set and (γ, L) is a soft fuzzy t open set. The complement of soft fuzzy B open set is a *soft fuzzy B closed set* (in short, *SFBcS*).

Definition 2.12. [18] Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then the *soft fuzzy B closure* (in short, *SFBcl*) of (λ, M) is defined as follows:

$$SFBcl(\lambda, M) = \sqcap \{(\mu, N)|(\mu, N) \text{ is a soft fuzzy } B \text{ closed set and } (\lambda, M)\sqsubseteq(\mu, N)\}.$$

Definition 2.13. [18] Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then the *soft fuzzy B interior*(in short, $SFBint$) of (λ, M) is defined as follows:

$$SFBint(\lambda, M) = \sqcup \{(\mu, N) | (\mu, N) \text{ is a soft fuzzy } B \text{ open set and } (\mu, N) \sqsubseteq (\lambda, M)\}.$$

Property 2.1. [18] *Let (X, T) be a soft fuzzy topological space. For any two soft fuzzy sets (λ, M) and (μ, N) the following statements are valid.*

(i) $(\lambda, M) \sqsubseteq (\mu, N)$ *implies* $SFBcl(\lambda, M) \sqsubseteq SFBcl(\mu, N)$.

(ii) $SFBcl((\lambda, M) \sqcap (\mu, N)) \sqsubseteq SFBcl(\lambda, M) \sqcap SFBcl(\mu, N)$.

(iii) $SFBcl((\lambda, M) \sqcup (\mu, N)) = SFBcl(\lambda, M) \sqcup SFBcl(\mu, N)$.

Property 2.2. [18] *Let (X, T) be a soft fuzzy topological space. For any soft fuzzy set (λ, M) in X, the following statements are valid.*

(i) $SFBint(\lambda, M) \sqsubseteq (\lambda, M) \sqsubseteq SFBcl(\lambda, M)$.

(ii) $(SFBint(\lambda, M))' = SFBcl(\lambda, M)'$.

(iii) $(SFBcl(\lambda, M))' = SFBint(\lambda, M)'$.

Definition 2.14. [19] Let (X, T) be a soft fuzzy topological space. Let (λ, M) be a soft fuzzy set of X. Then (λ, M) is said to be a *soft fuzzy G_δ* set (in short, SFG_δ) if $(\lambda, M) = \sqcap_{i=1}^{\infty}(\mu_i, N_i)$, where each (μ_i, N_i) is a soft fuzzy open set. The complement of soft fuzzy G_δ set is a *soft fuzzy F_σ* set (in short, SFF_σ).

3. Soft fuzzy B extremally disconnected and soft fuzzy B basically disconnected spaces via soft fuzzy large inductive dimension function

3.1. Soft fuzzy B extremally disconnectedness

Definition 3.15. Let (λ, M) and (μ, N) be any two soft fuzzy sets. Then $(\lambda, M) + (\mu, N)$ is defined by $(\lambda, M) + (\mu, N) = ((\lambda, M) \sqcap (\mu, N)') \sqcup ((\lambda, M)' \sqcap (\mu, N))$.

Definition 3.16. Let (X, T) be a soft fuzzy topological space. Then (X, T) is said to be *soft fuzzy B extremally disconnected space* if the soft fuzzy B closure of every soft fuzzy B open set is a soft fuzzy B open set.

Property 3.3. *Let (X, T) be a soft fuzzy topological space. Then the following conditions are equivalent:*

(i) *(X, T) is a soft fuzzy B extremally disconnected space.*

(ii) *For each soft fuzzy B closed set (λ, M), $SFBint(\lambda, M)$ is soft fuzzy B closed.*

(iii) *For each soft fuzzy B open set (λ, M),*

$$SFBcl(\lambda, M) + SFBcl(SFBcl(\lambda, M))' = (1, X).$$

(iv) *For every pair of soft fuzzy B open sets (λ, M) and (μ, N) with $SFBcl(\lambda, M)+$ $(\mu, N) = (1, X)$, we have $SFBcl(\lambda, M) + SFBcl(\mu, N) = (1, X)$.*

Proof. (i)⇒(ii). Let (λ, M) be any soft fuzzy B closed G_δ set in X. Then $(\lambda, M)'$ is a soft fuzzy B open set. Now,

$$SFBcl(\lambda, M)' = (SFBint(\lambda, M))'.$$

By (i), $SFBcl(\lambda, M)'$ is a soft fuzzy B open set. Then $SFBint(\lambda, M)$ is a soft fuzzy B closed set.

(ii)⇒(iii). Let (λ, M) be any soft fuzzy B open set. Then

$$\begin{aligned} &SFBcl(\lambda, M) + SFBcl(SFBcl(\lambda, M))' \\ &\quad = SFBcl(\lambda, M) + SFBcl(SFBint(\lambda, M)') \end{aligned} \tag{3.1}$$

Since (λ, M) is a soft fuzzy B open set, $(\lambda, M)'$ is a soft fuzzy B closed set. Hence by (ii), $SFBint(\lambda, M)'$ is soft L-fuzzy B closed. Therefore, by (3.1)

$$\begin{aligned} &SFBcl(\lambda, M) + SFBcl(SFBcl(\lambda, M))' \\ &\quad = SFBcl(\lambda, M) + SFBcl(SFBint(\lambda, M)') \\ &\quad = SFBcl(\lambda, M) + SFBint(\lambda, M)' \\ &\quad = SFBcl(\lambda, M) + (SFBcl(\lambda, M))' \\ &\quad = (1, X). \end{aligned}$$

Therefore, $SFBcl(\lambda, M) + SFBcl(SFBcl(\lambda, M))' = (1, X)$.

(iii)⇒(iv). Let (λ, M) and (μ, N) be any two soft fuzzy B open sets such that

$$SFBcl(\lambda, M) + (\mu, N) = (1, X). \tag{3.2}$$

Then by (iii),

$$\begin{aligned} (1, X) &= SFBcl(\lambda, M) + SFBcl(SFBcl(\lambda, M))' \\ &= SFBcl(\lambda, M) + SFBcl(\mu, N), \text{ by (3.2)}. \end{aligned}$$

Therefore, $SFBcl(\lambda, M) + SFBcl(\mu, N) = (1, X)$.

(iv)⇒(i). Let (λ, M) be any soft fuzzy B open F_σ set. Put $(\mu, N) = (SFB$ $cl(\lambda, M))' = (1, X) - SFBcl(\lambda, M)$. Then $SFBcl(\lambda, M) + (\mu, N) = (1, X)$. Therefore by (iv), $SFBcl(\lambda, M) + SFBcl(\mu, N) = (1, X)$. This implies that $SFBcl(\lambda, M)$ is a soft fuzzy B open set and so (X, T) is a soft fuzzy B extremally disconnected space.

Property 3.4. *Let (X, T) be a soft fuzzy topological space. Then (X, T) is soft fuzzy B extremally disconnected if and only if for all soft fuzzy B open set (λ, M) and every soft fuzzy B closed set (μ, N) such that $(\lambda, M) \sqsubseteq (\mu, N)$, $SFBcl(\lambda, M) \sqsubseteq SFBint(\mu, N)$.*

Proof. Let (λ, M) be any soft fuzzy B open set and (μ, N) be any soft fuzzy B closed set with $(\lambda, M) \sqsubseteq (\mu, N)$. By (ii) of Property 3.1, $SFBint(\mu, N)$ is a soft fuzzy B closed set. Therefore, $SFBcl(SFBint(\mu, N)) = SFBint(\mu, N)$. Also, since (λ, M) is a soft fuzzy B open set and $(\lambda, M) \sqsubseteq (\mu, N)$, $SFBcl(\lambda, M) \sqsubseteq SFBint(\mu, N)$. Therefore, $SFBcl(\lambda, M) \sqsubseteq SFBint(\mu, N)$.

Conversely, let (μ, N) be any soft fuzzy B closed set then $SFBint(\mu, N)$ is soft fuzzy B open and $SFBint(\mu, N) \sqsubseteq (\mu, N)$. Therefore by assumption, $SFBcl(SFBint(\mu, N)) \sqsubseteq SFBint(\mu, N)$. This implies that, $SFBint(\mu, N)$ is a soft fuzzy B closed set. Hence by (ii) of Property 3.1, it follows that (X, T) is a soft fuzzy B extremally disconnected space.

Property 3.5. *Let (X, T) be a soft fuzzy B extremally disconnected space. Let $\{(\lambda_i, M_i), (\mu_i, N_i)'/i \in \mathbb{N}\}$ be collection such that every $(\lambda_i, M_i)'s$ be a soft fuzzy B open sets and every $(\mu_i, N_i)'s$ be a soft fuzzy B closed sets and let (λ, M) and (μ, N) be soft fuzzy B clopen sets. If*

$$(\lambda_i, M_i) \sqsubseteq (\lambda, M) \sqsubseteq (\mu_j, N_j) \text{ and } (\lambda_i, M_i) \sqsubseteq (\mu, N) \sqsubseteq (\mu_j, N_j)$$

for all $i, j \in \mathbb{N}$, then there exists a soft fuzzy B clopen set (γ, L) such that $SFBcl$ $(\lambda_i, M_i) \sqsubseteq (\gamma, L) \sqsubseteq SFBint(\mu_j, N_j)$ for all $i, j \in \mathbb{N}$.

Proof. By Property 3.2, $SFBcl(\lambda_i, M_i) \sqsubseteq SFBcl(\lambda, M) \sqcap SFBint(\mu, N) \sqsubseteq SFBint\ (\mu_j, N_j)$ for all $i, j \in \mathbb{N}$. Letting, $(\gamma, L) = SFBcl(\lambda, M) \sqcap SFBint(\mu, N)$ is a soft fuzzy B clopen set satisfying the required conditions.

3.2. Soft fuzzy B basically disconnectedness

Definition 3.17. Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then (λ, M) is said to be a *soft fuzzy G_δ set* (in short, SFG_δ) if $(\lambda, M) = \sqcap_{i=1}^{\infty}(\mu_i, N_i)$, where each (μ_i, N_i) is a soft fuzzy open set. The complement of soft fuzzy G_δ set is a *soft fuzzy F_σ set* (in short, SFF_σ).

Definition 3.18. Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then (λ, M) is said to be a

(i) *soft fuzzy B open F_σ set* if (λ, M) is both soft fuzzy B open and soft fuzzy F_σ set.

(ii) *soft fuzzy B closed G_δ set* if (λ, M) is both soft fuzzy B closed and soft fuzzy G_δ set.

(iii) *soft fuzzy B clopen $G_\delta F_\sigma$ set* if (λ, M) is both soft fuzzy B open F_σ set and soft fuzzy B closed G_δ set.

Definition 3.19. Let (X, T) be a soft fuzzy topological space. Then (X, T) is said to be a *soft fuzzy B basically disconnected space* if the soft fuzzy B closure of every soft fuzzy B open F_σ set is a soft fuzzy B open set.

Property 3.6. *Let (X, T) be a soft fuzzy topological space, the following statements are equivalent:*

(i) (X, T) *is a soft fuzzy B basically disconnected space.*

(ii) *For each soft fuzzy B closed G_δ set (λ, M), $SFBint(\lambda, M)$ is soft fuzzy B closed G_δ set.*

(iii) *For each soft fuzzy B open F_σ set (λ, M),*

$$SFBcl(\lambda, M) + SFBcl(SFBcl(\lambda, M))' = (1, X).$$

(iv) *For every pair of soft fuzzy B open F_σ sets (λ, M) and (μ, N) with SFB $cl(\lambda, M) + (\mu, N) = (1, X)$, we have $SFBcl(\lambda, M) + SFBcl(\mu, N) = (1, X)$.*

Proof. The proof is similar to that of Property 3.1.

Property 3.7. *Let (X, T) be soft fuzzy topological space. Then (X, T) is a soft fuzzy B basically disconnected space if and only if for all soft fuzzy B clopen $G_\delta F_\sigma$ sets (λ, M) and (μ, N) such that $(\lambda, M) \sqsubseteq (\mu, N)$, $SFBcl(\lambda, M) \sqsubseteq SFBint(\mu, N)$.*

Proof. Let (λ, M) and (μ, N) be any two soft fuzzy B clopen $G_\delta F_\sigma$ sets with $(\lambda, M) \sqsubseteq (\mu, N)$. By (ii) of Property 3.4, $SFBint(\mu, N)$ is a soft fuzzy B closed G_δ set. Therefore, $SFBcl(SFBint(\mu, N)) = SFBint(\mu, N)$. Also, since (λ, M) is soft fuzzy B clopen $G_\delta F_\sigma$ set and $(\lambda, M) \sqsubseteq (\mu, N)$, $SFBcl(\lambda, M)$ $\sqsubseteq SFBint(\mu, N)$. Therefore, $SFBcl(\lambda, M) \sqsubseteq SFBint(\mu, N)$.

Conversely, let (μ, N) be any soft fuzzy B clopen $G_\delta F_\sigma$ set then $SFBint(\mu, N)$ is a soft fuzzy B open F_σ set and $SFBint(\mu, N) \sqsubseteq (\mu, N)$. Therefore by assumption, $SFBcl(SFBint(\mu, N)) \sqsubseteq SFBint(\mu, N)$. This implies that $SFBint(\mu, N)$ is a soft fuzzy B closed G_δ set. Hence by (ii) of Property 3.4, it follows that (X, T) is a soft fuzzy B basically disconnected space.

Property 3.8. *Let (X, T) be a soft fuzzy B basically disconnected space. Let $\{(\lambda_i, M_i), (\mu_i, N_i)'/i \in \mathbb{N}\}$ be a collection such that $(\lambda_i, M_i)'s$ and $(\mu_i, N_i)'s$ are soft fuzzy B clopen $G_\delta F_\sigma$ sets and let (λ, M) and (μ, N) be soft fuzzy B clopen $G_\delta F_\sigma$ sets. If*

$$(\lambda_i, M_i) \sqsubseteq (\lambda, M) \sqsubseteq (\mu_j, N_j) \text{ and } (\lambda_i, M_i) \sqsubseteq (\mu, N) \sqsubseteq (\mu_j, N_j)$$

for all $i, j \in \mathbb{N}$, then there exists a soft fuzzy B clopen $G_\delta F_\sigma$ set (γ, L) such that $SFBcl$ $(\lambda_i, M_i) \sqsubseteq (\gamma, L) \sqsubseteq SFBint(\mu_j, N_j)$ for all $i, j \in \mathbb{N}$.

Proof. By Property 3.5, $SFBcl(\lambda_i, M_i) \sqsubseteq SFBcl(\lambda, M) \sqcap SFBint(\mu, N) \sqsubseteq SFBint\ (\mu_j, N_j)$ for all $i, j \in \mathbb{N}$. Therefore, (γ, L)=$SFBcl(\lambda, M)$⊓$SFBint(\mu,\ N)$ is a soft fuzzy B clopen $G_\delta F_\sigma$ set satisfying the required conditions.

3.3. Soft fuzzy dimension theory

Definition 3.20. Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then the *soft fuzzy boundary* of (λ, M), is denoted and defined as

$$SFbd(\lambda, M) = SFcl(\lambda, M) - SFint(\lambda, M).$$

Definition 3.21. Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then the *soft fuzzy B boundary* of (λ, M), is denoted and defined as

$$SFBbd(\lambda, M) = SFBcl(\lambda, M) - SFBint(\lambda, M).$$

Property 3.9. *Let (X, T) be a soft fuzzy topological space. Let (λ, M) be any soft fuzzy set. Then $SFBbd(\lambda, M) = (0, \emptyset)$ iff (λ, M) is both soft fuzzy B open and soft fuzzy B closed.*

Proof. The proof follows from Definition 4.2.

Definition 3.22. Let (X, T) be a soft fuzzy topological space. Then the *soft fuzzy large inductive dimension* of (X, T), denoted by $SFIndX$, is defined as follows.

(i) $SFIndX = -1$ iff $X = \emptyset$.

(ii) For any positive integer n, $SFIndX \sqsubseteq n$ if for each soft fuzzy B closed set (λ, M) and each soft fuzzy B open set (μ, N) in (X, T) such that $(\lambda, M) \sqsubseteq (\mu, N)$ there exists a soft fuzzy B open set (γ, L) in (X, T) such that $(\lambda, M) \sqsubseteq (\gamma, L) \sqsubseteq (\mu, N)$ and $SFIndSFBbd(\gamma, L) \sqsubseteq n - 1$.

(iii) $SFIndX = n$ if $SFIndX \sqsubseteq n$ is true and $SFIndX \sqsubseteq n - 1$ is not true.

(iv) $SFIndX = \infty$ if $SFIndX \sqsubseteq n$ is not true for every n.

Property 3.10. *Let (X, T) be a soft fuzzy topological space. If $SFIndX = 0$ then (X, T) is a soft fuzzy B extremally disconnected space.*

Proof. Let (λ, M) be any soft fuzzy B open set and (μ, N) be any soft fuzzy B closed set such that $(\lambda, M) \sqsubseteq (\mu, N)$. By Property 3.2, $SFBcl(\lambda, M) \sqsubseteq SFBint(\mu, N)$. Since $SFIndX \sqsubseteq 0$ and by Definition 4.3, there exist a soft fuzzy B open set (γ, L) in (X, T) such that $SFBcl(\lambda, M) \sqsubseteq (\gamma, L) \sqsubseteq SFBint(\mu, N)$ and $SFIndSFBbd(\gamma, L) \sqsubseteq 0 - 1$. Therefore, $SFBbd(\gamma, L) = (0, \emptyset)$. Since $SFBbd(\gamma, L) = (0, \emptyset)$ from Property 4.1, (γ, L) is both soft fuzzy B open and soft fuzzy B closed. By Property 3.3, $SFBcl(\lambda, M) \sqsubseteq (\gamma, L) \sqsubseteq SFBint(\mu, N)$. Hence (X, T) is a soft fuzzy B extremally disconnected space.

Property 3.11. *Let (X, T) be a soft fuzzy topological space. If $SFIndX = 0$ then (X, T) is a soft fuzzy B basically disconnected space.*

Proof. The proof is similar to that of Property 4.2.

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