REVERSE MAGIC STRENGTH OF FESTOON TREES

S. Sharief Basha
K. Madhusudhan Reddy

Applied Algebra Division
School of Advanced Sciences
VIT University
Vellore 632 014, Tamilnadu
India
e-mails: Shariefbasha.s@vit.ac.in
drkmsreddy@yahoo.com

Abstract. In this paper, we prove that the reverse super edge-magic strength of some different festoon trees.

Keywords. graph labeling, reverse super edge-magic labeling, Festoon trees.

1. Introduction

Consider a family of finite number of stars. Arrange them in an array and join one centre of a star to that of the next one. The tree so obtained is called a $f$estoon tree. In ([9]), we defined a reverse magic labeling of a graph $G(V, E)$ as a bijection $f : V \cup E \to \{1, 2, 3, ..., v + \epsilon\}$ such that for all edges xy, $f(xy) - \{f(x) + f(y)\}$ are the same where v and ϵ denote the order and size of the graph G. A graph is said to be A reverse edge-magic labeling f is called reverse super edge-magic if $f(V) = \{1, 2, 3, ..., \}$ and $f(E) = \{v + 1, v + 2, v + 3, ..., v + \epsilon\}$. A graph G is called reverse super edge-magic if there exists a reverse super edge-magic labeling of G. In ([5]), Hugand introduced the concept of reverse super edge-magic strength of a graph. A reverse edge-magic labeling of a graph $G(V, E)$ is a bijection $f : V \cup E \to \{1, 2, 3, ..., v + \epsilon\}$ such that for all edges xy, $f(xy) - \{f(x) + f(y)\}$ is a constant which is denoted by $c(f)$. The reverse edge-magic strength of a graph $G, rsm(G)$, is defined as the minimum of all $c(f)$ where the minimum is taken over all reverse edge-magic labelings f of G. A reverse magic labeling of a graph $G(V, E)$ is called reverse super edge-magic labeling of G if $f(V) = \{1, 2, ..., v\}$ and $f(E) = \{v + 1, v + 2, ..., v + \epsilon\}$.

In ([5]), the following results have been proved.

1. $\text{rsm}(P_{2n}) = n - 1$, $\text{rsm}(P_{2n+1}) = n$
2. $\text{rsm}(K_{1,n}) = n - 1$
3. $\text{rsm}(B_{n,n}) = n$
4. $\text{rsm}(P_n^2) = n - 2$
5. $\text{rsm}(C_{2n+1}) = n$
6. $\text{rsm}(< K_{1,n} : 2 >) = 2n$
7. $\text{rsm}[(2n + 1)P_2] = n$

Note 1. Let f be a reverse super edge-magic labeling of G with the constant $c(f)$. Then, adding all the constants obtained at each edge, we get

$$\varepsilon\ c(f)\ = \sum_{e\in E} f(e) - \sum_{v\in V} d(v)f(v)$$

In this paper, we determined the reverse super edge-magic strength of some different festoon trees.

2. Reverse super edge-magic strength of Festoon trees

In this paper, we obtain the reverse super edge-magic strength of festoon tree $T_\alpha, n+\alpha$ vertices of the stars $K_{1,n}, n_i \geq 1,\ 1 \leq i \leq \alpha,\ \sum_{i=1}^{\alpha} n_i = n$.

Theorem 2.1. $rsm(T_\alpha) = n + \alpha - n_1 - n_3 - \ldots - n_{\alpha-1} - \frac{\alpha}{2} - 1$.

Proof. We prove this theorem by assigning reverse super edge-magic labeling to T_α, α is even.

Let u_i be the center and v_i be the set of pendent vertices of i^{th} star. Each v_i has n_i vertices, $i = 1, 2, ..., \alpha$. Thus $n_1 + n_2 + n_3 + \cdots + n_\alpha = n$, $v = n + \alpha$, $\varepsilon = n+\alpha-1$. Then the following labeling f is a reverse super edge-magic labeling of T_α.

$$\begin{aligned}
f(u_{2i}) &= n_1 + n_3 + n_5 plus n_{2i-1} + i, \quad i = 1, 2, 3, ..., \frac{\alpha}{2} \\
f(u_{2i-1}) &= f(u_\alpha) + n_2 + n_4 + n_6 + \cdots + n_{2i-2} + i, \quad i = 1, 2, 3, ..., \frac{\alpha}{2} \\
f(v_{2i-1}) &= \{n_1 + n_3 + \cdots + n_{2i-3} + i, ..., n_1 + n_3 + \cdots + n_{2i-1} + i - 1\}, \\
&\qquad i = 1, 2, 3, ..., \frac{\alpha}{2} \\
f(v_{2i}) &= \{f(u_1) + n_2 + n_4 + \ldots + n_{2i-2} + i, ..., f(u_1) + n_2 + n_4 \\
&\qquad + \cdots + n_{2i} + i - 1\}, \quad i = 1, 2, 3, ..., \frac{\alpha}{2}
\end{aligned}$$

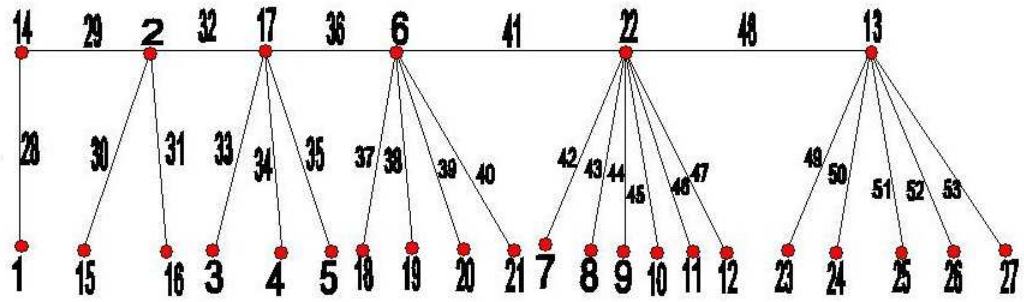

Figure 1:

This labeling of vertices gives $f(x)+f(y)$ for all edges $xy \in E$ to vary from $f(u_\alpha)+2$ to $f(u_\alpha)+n+\alpha$. Thus, for each edge $e=xy$, if $f(x)+f(y)=f(u_\alpha)+i$, $i=2,3,...,n+\alpha$, then $f(e)=n+\alpha+i$. By Note 1, if f is a reverse super edge-magic labeling of T_α with constant $c(f)$.

$$\varepsilon\ c(f)=\sum_{e\in E} f(e)-\sum_{v\in V} d(v)f(v)$$

$$\begin{aligned}
(n+\alpha-1)\ c(f) &= \sum_{e\in E} f(e)-\Big\{\sum_{i=1,v\in V_i}^{\alpha} f(v)+\sum_{i=1}^{\alpha} d(u_i)f(u_i)\Big\}\\
&= \sum_{e\in E} f(e)-\Big\{\sum_{i=1,v\in V_i}^{\alpha} f(v)+(n_1+1)f(u_1)\\
&\quad +(n_2+2)f(u_2)+\ldots+(n_{\alpha-1}+2)f(u_{\alpha-1})+(n_\alpha+1)f(u_\alpha)\Big\}\\
&= \sum_{e\in E} f(e)-\Big\{\Big[\sum_{i=1,v\in V_i}^{\alpha} f(v)+f(u_1)+f(u_2)+\ldots+f(u_\alpha)\Big]\\
&\quad +n_1f(u_1)+(n_2+1)f(u_2)+\ldots+(n_{\alpha-1}+1)f(u_{\alpha-1})+n_\alpha f(u_\alpha)\Big\}\\
&= \sum_{e\in E} f(e)-\Big\{\sum_{i=1_i}^{n+\alpha-1} i+n_1[f(u_\alpha)+1]+(n_2+1)(n_1+1)\\
&\quad +(n_3+1)[f(u_\alpha)+n_2+2]+(n_4+1)[n_1+n_3+2]\\
&\quad +\ldots+(n_{\alpha-2}+1)\big[n_1+n_3+\ldots+n_{\alpha-3}+\big(\frac{\alpha}{2}-1\big)\big]\\
&\quad +(n_{\alpha-1}+1)\big[f(u_\alpha)+n_2+n_4+\ldots+n_{\alpha-2}+\frac{\alpha}{2}\big]\\
&\quad +n_\alpha\big[n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big]\Big\}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(n+\alpha-1)[n+\alpha+1+2n+2\alpha-1]}{2}\\
&\quad - \Big\{ \frac{(n+\alpha-1)(n+\alpha)}{2} + f(u_\alpha)\big[n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha-2}{2}\big]\\
&\quad + n_1 + (n_2+1)(n_1+1) + (n_3+1)(n_2+2) + (n_4+1)(n_1+n_3+2)\\
&\quad + (n_5+1)(n_2+n_4+3) + \ldots + (n_{\alpha-2}+1)\big[n_1+n_3+\ldots+n_{\alpha-3}+\big(\frac{\alpha}{2}-1\big)\big]\\
&\quad + (n_{\alpha-1}+1)\big[n_2+n_4+\ldots+n_{\alpha-2}+\frac{\alpha}{2}\big] + n_\alpha\big[n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big]\Big\}\\
&= \frac{(n+\alpha-1)[3n+3\alpha]}{2} - \frac{(n+\alpha-1)(n+\alpha)}{2}\\
&\quad - \Big\{\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big)\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha-2}{2}\big)\\
&\quad + n_1 + (n_3+1)(n_2+2) + (n_5+1)(n_2+n_4+3) + \ldots\\
&\quad + (n_{\alpha-1}+1)\big[n_2+n_4+\ldots+n_{\alpha-2}+\frac{\alpha}{2}\big]\\
&\quad + (n_2+1)(n_1+1) + (n_4+1)(n_1+n_3+2) + \ldots\\
&\quad + (n_{\alpha-2}+1)\big[n_1+n_3+\ldots+n_{\alpha-3}+\big(\frac{\alpha}{2}-1\big)\big]\\
&\quad + n_\alpha\big[n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big]\Big\}\\
&= (n+\alpha-1)[n+\alpha] - \Big\{\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big)\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha-2}{2}\big)\\
&\quad + n_1(n_1+n_2+n_3+\ldots+n_\alpha) + n_3(n_1+n_2+n_3+\ldots+n_\alpha) + \ldots\\
&\quad + \frac{\alpha}{2}(n_1+n_2+n_3+\ldots+n_\alpha) + \big(\frac{\alpha}{2}+\frac{\alpha}{2}-1\big)(n_1+n_3+\ldots+n_{\alpha-1})\\
&\quad + \big(2+3+4+\ldots+\frac{\alpha}{2}\big) + \big(1+2+3+\ldots+\frac{\alpha}{2}-1\big)\Big\}\\
&= (n+\alpha-1)[n+\alpha]\\
&\quad - \Big\{\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big)(n_1+n_3+\ldots+n_{\alpha-1}+\alpha-1)\\
&\quad + \frac{\alpha}{2}\big(\frac{\alpha}{2}-1\big) + \frac{\alpha}{2}(\alpha-1) + \frac{\frac{\alpha}{2}(\frac{\alpha}{2}+1)}{2} + \frac{\frac{\alpha}{2}(\frac{\alpha}{2}-1)}{2} - 1\Big\}\\
&= (n+\alpha-1)[n+\alpha] - \Big\{\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big) + (n+\alpha-1)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
(n+\alpha-1)c(f) &= (n+\alpha-1)(n+\alpha)\\
&\quad - \Big\{\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big) + (n+\alpha-1)\Big\}\\
c(f) &= n+\alpha-\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big)\\
\therefore rsm(T_\alpha) &= n+\alpha-\big(n_1+n_3+\ldots+n_{\alpha-1}+\frac{\alpha}{2}\big)
\end{aligned}
$$

∎

Note 2. $rsm(T_\alpha)$ can be increased if the number of vertices in a star at any odd position is less that the no. of vertices in a star at any even position.

For example, we have $rsm(T_6) = 17$ in Fig. 2.

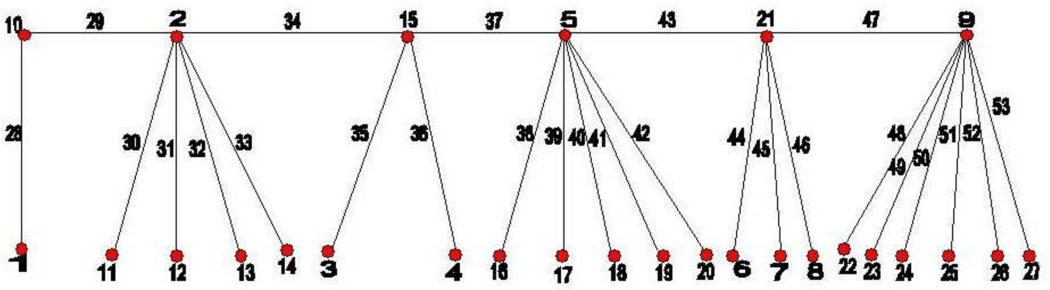

Figure 2: $\alpha = 6,\ n_1 = 1,\ n_2 = 4,\ n_3 = 2,\ n_4 = 5,\ n_5 = 3,\ n_6 = 6,\ rsm(T_6) = 17$

Note 3. $rsm(T_\alpha)$ can also be reduced if the number of vertices in the star at any odd position is greater than the number of vertices in the star at any even position. For example, we have $rsm(T_6) = 8$ in Fig. 3

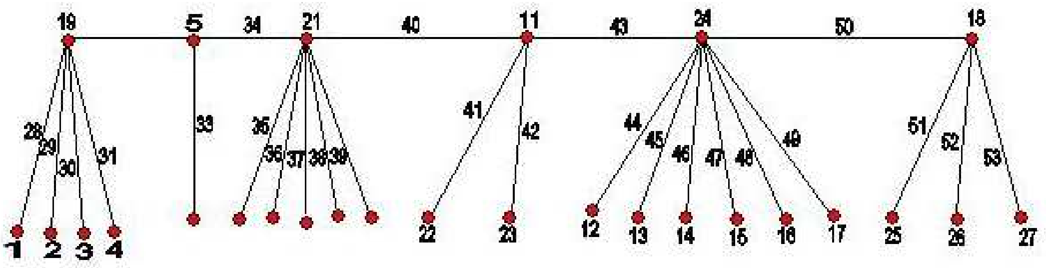

Figure 3: $\alpha = 6,\ n_1 = 4,\ n_2 = 1,\ n_3 = 5,\ n_4 = 2,\ n_5 = 6,\ n_6 = 3,\ rsm(T_6) = 8$

Theorem 2.2. $rsm(T_\alpha) = n + \alpha - n_2 - n_4 - \ldots - n_{\alpha-1} - \left(\dfrac{\alpha+3}{2}\right)$

Proof. We prove this theorem by assigning reverse super edge-magic labeling to T_α, α is odd.

Let u_i be the center and v_i be the set of pendent vertices of the i^{th} star.

Each v_i has n_i vertices, $i = 1, 2, ..., \alpha$. Thus

$$n_1 + n_2 + n_3 + \cdots + n_\alpha = n,\ v = n + \alpha,\ \varepsilon = n + \alpha - 1.$$

Then, the following labeling f is a reverse super edge-magic labeling of T_α.

$$
\begin{aligned}
f(u_1) &= 1 \\
f(u_{2i}) &= f(u_\alpha) + n_1 + n_3 + n_5 + \cdots + n_{2i-1} + i, && i = 1, 2, 3, \ldots \frac{(\alpha-1)}{2} \\
f(u_{2i-1}) &= f(u_1) + n_2 + n_4 + n_6 + \cdots + n_{2i-2} + i - 1, && i = 2, 3, \ldots \frac{(\alpha-1)}{2} \\
f(v_{2i-1}) &= \{f(u_\alpha) + n_1 + n_3 + \cdots + n_{2i-3} + i, \ldots, \\
&\quad f(u_\alpha) + n_1 + n_3 + \cdots + n_{2i-1} + i - 1\}, && i = 1, 2, 3, \ldots, \frac{\alpha+1}{2} \\
f(v_{2i}) &= \{f(u_1) + n_2 + n_4 + \cdots + n_{2i-2} + i, \ldots, \\
&\quad f(u_1) + n_2 + n_4 + \cdots + n_{2i} + i - 1\}, && i = 1, 2, 3, \ldots, \frac{\alpha-1}{2}.
\end{aligned}
$$

This labeling of vertices gives $f(x) + f(y)$ for all edges $xy \in E$ to vary from $f(u_\alpha) + 2$ to $f(u_\alpha) + n + \alpha$. Thus for each edge $e = xy$ if $f(x) + f(y) = f(u_\alpha) + i$, $i = 2, 3, \ldots, n + \alpha$ then $f(e) = n + \alpha - 1$.

For example, the reverse super edge-magic labeling of $T_\alpha, \alpha = 7$ is shown in Fig. 4.

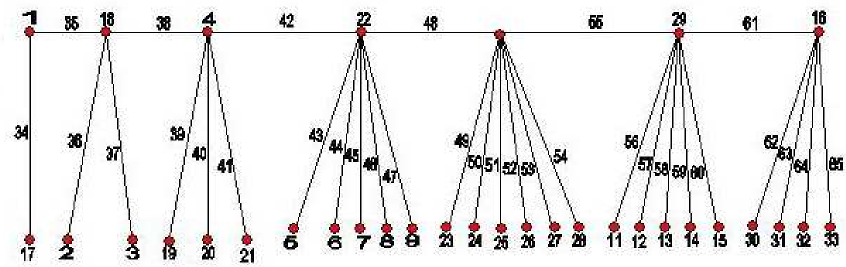

Figure 4: α=7, n_1=1, $n_2 = 2$, n_3=3, n_4=5, n_5=6, n_6=5, $rsm(T_7)$=16

By Note 1, if f is a reverse super edge-magic labeling of T_α with constant $c(f)$.

$$
\begin{aligned}
\varepsilon\, c(f) &= \sum_{e \in E} f(e) - \sum_{v \in V} d(v) f(v) \\
(n + \alpha - 1)\, c(f) &= \sum_{e \in E} f(e) - \left\{ \sum_{i=1, v \in V_i}^{\alpha} f(v) + \sum_{i=1}^{\alpha} d(u_i) f(u_i) \right\} \\
&= \sum_{e \in E} f(e) - \Big\{ \sum_{i=1, v \in V_i}^{\alpha} f(v) + (n_1 + 1) f(u_1) + (n_2 + 2) f(u_2) + \ldots \\
&\quad + (n_{\alpha-1} + 2) f(u_{\alpha-1}) + (n_\alpha + 1) f(u_\alpha) \Big\} \\
&= \sum_{e \in E} f(e) - \Big\{ \Big[\sum_{i=1, v \in V_i}^{\alpha} f(v) + f(u_1) + f(u_2) + \ldots + f(u_\alpha) \Big] \\
&\quad + n_1 f(u_1) + (n_2 + 1) f(u_2) + \cdots + (n_{\alpha-1} + 1) f(u_{\alpha-1}) + n_\alpha f(u_\alpha) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
&= \sum_{e\in E} f(e) - \Big\{ \sum_{i=1_i}^{n+\alpha-1} i + n_1 f(u_1) + (n_2+1)[f(u_\alpha)+n_2+1] \\
&\quad + (n_3+1)[f(u_\alpha)+n_2+1] \\
&\quad + (n_4+1)[f(u_\alpha)+n_1+n_3+2] + (n_5+1)[f(u_1)+n_2+n_4+2] + \cdots \\
&\quad + (n_{\alpha-1}+1)\big[f(u_\alpha)+n_1+n_3+\ldots+n_{\alpha-2}+\big(\frac{\alpha-1}{2}\big)\big] \\
&\quad + n_\alpha\big[f(u_1)+n_2+n_4+\ldots+n_{\alpha-1}+\frac{\alpha-1}{2}\big]\Big\} \\
&= \sum_{e\in E} f(e) - \Big\{\frac{(n+\alpha-1)(n+\alpha)}{2} + f(u_1)\big[n_1+n_3+\ldots+n_\alpha+\frac{\alpha-3}{2}\big] \\
&\quad + f(u_\alpha)\big[n_2+n_2+\ldots+n_{\alpha-1}+\frac{\alpha-1}{2}\big] \\
&\quad + (n_2+1)(n_1+1) + (n_3+1)(n_2+1) \\
&\quad + (n_4+1)(n_1+n_3+2) + (n_5+1)(n_2+n_4+2) + \ldots \\
&\quad + (n_{\alpha-1}+1)\big[n_1+n_3+\ldots+n_{\alpha-2}+\big(\frac{\alpha-1}{2}\big)\big] \\
&\quad + n_\alpha\big[n_2+n_4+\ldots+n_{\alpha-1}+\frac{\alpha-1}{2}\big]\Big\} \\
&= \{(n+\alpha+1)+(n+\alpha+2)+\ldots+(2n+2\alpha-1)\} \\
&\quad - \Big\{\frac{(n+\alpha-1)(n+\alpha)}{2} + \big(n_1+n_3+\ldots+n_\alpha+\frac{(\alpha-3)}{2}\big) \\
&\quad + \big(n_2+n_4+\ldots+n_{\alpha-1}+\frac{\alpha+1}{2}\big)\big(n_2+n_4+\ldots+n_{\alpha-1}+\frac{\alpha-1}{2}\big) \\
&\quad + (n_2+1)(n_1+1) + (n_4+1)(n_1+n_3+2) + \ldots \\
&\quad + (n_{\alpha-1}+1)\big[n_1+n_3+\ldots+n_{\alpha-2}+\frac{\alpha-1}{2}\big] \\
&\quad + (n_3+1)(n_2+1) + (n_5+1)(n_2+n_4+2) + \ldots \\
&\quad + n_\alpha\big[n_2+n_4+\ldots+n_{\alpha-1}+\frac{(\alpha-1)}{2}\big]\Big\} \\
&= \frac{n+\alpha-1}{2}[n+\alpha+1+2n+2\alpha-1] \\
&\quad - \Big\{\frac{(n+\alpha-1)(n+\alpha)}{2} - \big(n_1+n_3+\ldots+n_\alpha+\frac{(\alpha-3)}{2}\big) \\
&\quad + (n_2+n_4+\ldots+n_{\alpha-1})(n_2+n_4+\ldots+n_{\alpha-1}+\alpha) \\
&\quad + \big(\frac{\alpha+1}{2}\big)\big(\frac{\alpha-1}{2}\big) + \big(\frac{\alpha-1}{2}\big)[n_1+n_2+\ldots+n_\alpha] \\
&\quad + \big(1+2+3+\ldots+\frac{\alpha-1}{2}\big) + \big(1+2+3+\ldots+\frac{\alpha-3}{2}\big)\Big\} \\
&= \frac{n+\alpha-1}{2}[3n+3\alpha-n-\alpha] - \Big\{\big(n_1+n_3+\ldots+n_\alpha+\frac{(\alpha-3)}{2}\big) \\
&\quad + (n_2+n_4+\ldots+n_{\alpha-1})(n+\alpha) + \big(\frac{\alpha+1}{2}\big)(n+\alpha) + \big(\frac{\alpha+1}{2}\big)\big(\frac{\alpha-1}{2}\big) \\
&\quad + \alpha\big(\frac{\alpha+1}{2}\big) + \big(\frac{(\frac{\alpha-1}{2})(\frac{\alpha+1}{2})}{2} + \frac{(\frac{\alpha-3}{2})(\frac{\alpha-1}{2})}{2}\big)\Big\}
\end{aligned}
$$

$$= \frac{n+\alpha-1}{2}[2n+2\alpha] - \Big\{\big(n_1+n_3+\ldots+n_\alpha+\frac{(\alpha-3)}{2}\big)$$
$$+ (n_2+n_4+\ldots+n_{\alpha-1})(n+\alpha)$$
$$+ (\frac{\alpha^2-1}{2}) + \alpha(\frac{\alpha-1}{2}) + \frac{\alpha^2-1}{8} + \frac{\alpha^2-4\alpha+3}{2} + (\frac{\alpha+1}{2})(n+\alpha)\Big\}$$
$$= (n+\alpha)\big[n+\alpha-1-n_2-n_4-\ldots-n_{\alpha-1}-\frac{\alpha+1}{2}\big]$$
$$- \Big\{\big(n_1+n_3+\ldots+n_\alpha+\frac{(\alpha-3)}{2}\big)+$$
$$+ \frac{1}{2}[2\alpha^2-2-4\alpha^2+4\alpha^2+\alpha^2-1+\alpha^2-4\alpha+3]\Big\}$$
$$= (n+\alpha)\big[n+\alpha-1-n_2-n_4-\ldots-n_{\alpha-1}-\frac{\alpha+1}{2}\big]$$
$$- \big[n_1+n_3+\ldots+n_\alpha+\frac{(\alpha-3)}{2}\big]$$
$$= (n+\alpha-1)\big[n+\alpha-1-n_2-n_4-\ldots-n_{\alpha-1}-(\frac{\alpha+1}{2})\big]$$
$$+ \big[n+\alpha-1-n_2-n_4-\ldots-n_{\alpha-1}-(\frac{\alpha+1}{2})\big]$$
$$- \big[n_1+n_3+\ldots+n_\alpha+\frac{(\alpha-3)}{2}\big]$$

$$(n+\alpha-1)c(f) = (n+\alpha-1)\big[n+\alpha-1-n_2-n_4-\ldots-n_{\alpha-1}-(\frac{\alpha+1}{2})\big]$$
$$c(f) = n+\alpha-n_2-n_4-\ldots-n_{\alpha-1}-(\frac{\alpha+3}{2})$$
$$rsm[T_\alpha] = n+\alpha-n_2-n_4-\ldots-n_{\alpha-1}-(\frac{\alpha+3}{2})$$

■

Note 4. $rsm(T_\alpha)$ is odd, will also be increased if the number of vertices in the star at any even position is less than the number of vertices in the star at any odd position as shown in Fig. 5.

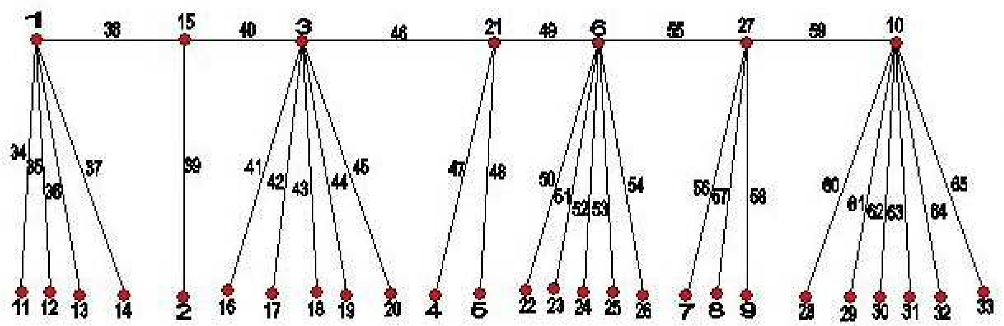

Figure 5: α=7, n_1=4, n_2=1, n_3=5, n_4=2, n_5=5, n_6=3, n_7=6, $rsm(T_7)$=22

Note 5. $rsm(T_\alpha)$ is odd, will also be decreased if the number of vertices in the star at any even position is greater than the number of vertices in the star at any odd position as shown in Fig. 6.

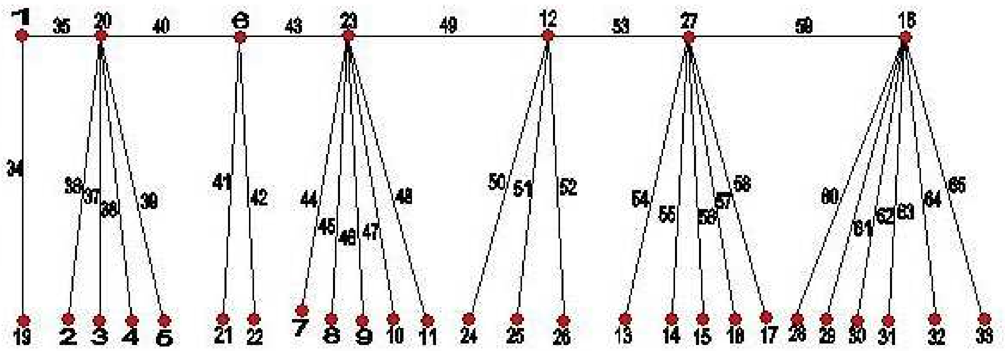

Figure 6: α=7, n_1=1, n_2=4, n_3=2, n_4=5, n_5=3, n_6=5, n_7=6, $rsm(T_6)$=14

3. Observation

Theorem 3.1. $rsm\left(\langle K_{1,1}, K_{1,2}, K_{1,r}\rangle\right) = r - 2$.

Proof. This is obtained by joining centres u, v, w of $K_{1,1}, K_{1,2}$, and $K_{1,r}$ to a new X. The pendent vertex of the star $K_{1,1}$ is denoted by u_1; the pendent vertices of $K_{1,2}$ are denoted by v_1, v_2 and the pendent vertices of $K_{1,r}$ are denoted by $w_1, w_2, ..., w_r$. The reverse super edge-magic labeling f of the graph is given below: $f(u_1) = 1$, $f(v) = 2$, $f(w_1) = 3$, $f(w) = 4$, $f(u) = 5$, $f(v_1) = 6$, $f(x) = 7$, $f(v_2) = 8$ and $f(w_i) = i + 7$, $i = 2, 3, 4, ..., r$, $f(wu_1) = r + 8$, $f(ww_1) = r + 9$, $f(vv_1) = r + 10$, $f(xv) = r + 11$, $f(vv_2) = r + 12$, $f(xw) = r + 13$, $f(xu) = r + 14$, $f(ww_i) = f(w_r) + 3r + 1 + i$, $i = 2, 3, 4, ...$ Thus, $f(xu) - \{f(x) + f(u)\} = r + 14 - \{7 + 5\} = r + 2$. ∎

This observation is illustrated in Fig. 7 for $r = 4$.

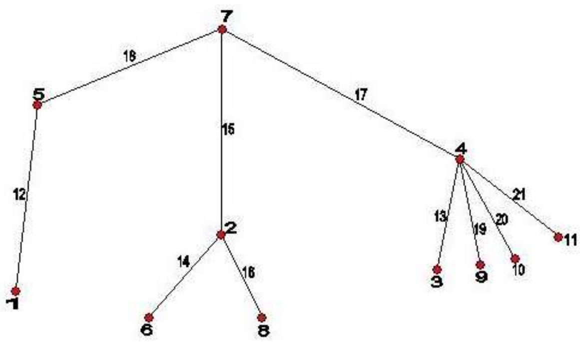

Figure 7:

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Accepted: 21.04.2014