APPLICATION OF BIPOLAR FUZZY SOFT SETS IN K-ALGEBRAS

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Abstract. On the basis of the concept of bipolar fuzzy soft sets, a new kind of K-algebra is introduced in this paper. The concepts of bipolar fuzzy soft K-algebras are described and some related properties are investigated. The notion of a generalized bipolar fuzzy soft K-algebra is also introduced and discussed.
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1. Introduction

The notion of a K-algebra $(G, \cdot, \odot, e)$ was first introduced by Dar and Akram [12] in 2003 and published in 2005. A K-algebra is an algebra built on a group $(G, \cdot, e)$ by adjoining an induced binary operation $\odot$ on G which is attached to an abstract K-algebra $(G, \cdot, \odot, e)$. This system is, in general non-commutative and non-associative with a right identity e, if a group is non-commutative. For a given group, the K-algebra is proper if group is not an elementary abelian 2-group. Thus, a K-algebra is abelian and non-abelian purely depends on the base group. In 2004, Dar and Akram further renamed *a K-algebra on a group G* as a $K(G)$-algebra [13] due to its structural basis G. The $K(G)$-algebras have been characterized by their left and right mappings in [13, 14] when group is abelian and non-abelian.

In 1994, Zhang [29] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets [28]. Bipolar fuzzy sets are an extension of fuzzy sets [28] whose membership degree range is $[-1, 1]$. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree $(0, 1]$ of an element indicates that the element somewhat satisfies the property, and the membership degree $[-1, 0)$ of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with bipolar information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several directions. In particular, fuzzy and possibilistic formalisms for bipolar information have been proposed [16], because when we deal with spatial information in image processing or in spatial reasoning applications, this bipolarity also occurs. For instance, when we assess the position of an object in a space, we may have positive information expressed as a set of possible places and negative information expressed as a set of impossible places.

In 1999, Molodtsov [23] initiated the novel concept of soft set theory to deal with uncertainties which can not be handled by traditional mathematical tools. Applications of soft set theory in real life problems are now catching momentum due to the general nature parametrization expressed by a soft set. Maji et al. [22] gave first practical application of soft sets in decision making problems. They also presented the definition of fuzzy soft set [21]. Following the concept of soft sets, several authors have applied this concept to algebraic structures [17-20, 30]. Al-Shehri *et al.*[3] has applied this concept to K-algebras. Akram *et al.* introduced the notions of fuzzy soft K-subalgebras in [6]. Moreover, Akram *et al.* [7] has introduced the notions of intuitionist fuzzy soft K-algebras and studied some of their properties. Bipolar fuzzy sets and soft sets are two different methods for representing uncertainty and vagueness. In this article, we apply these methods in combination to study uncertainty and vagueness in K-algebras. We first introduce the concept of bipolar fuzzy soft K-algebras and investigate some of their properties. Then we introduce the notion of a generalized bipolar fuzzy soft K-algebra and studied some of its related properties. The definitions and terminologies that we used in this paper are standard.

2. Preliminaries

In this section, we review some elementary concepts that are necessary for this paper.

Let $(G, \cdot, e)$ be a group in which each non-identity element is not of order 2. Then a K- *algebra* [12] is a structure $\mathcal{K} = (G, \cdot, \odot, e)$ on a group G in which induced binary operation $\odot : G \times G \to G$ is defined by $\odot(a, b) = a \odot b = a.b^{-1}$ and satisfies the following axioms:

(K1) $(a \odot b) \odot (a \odot c) = (a \odot ((e \odot c) \odot (e \odot b))) \odot a$,

(K2) $a \odot (a \odot b) = (a \odot (e \odot b)) \odot a$,

(K3) $(a \odot a) = e$,

(K4) $(a \odot e) = a$,

(K5) $(e \odot a) = a^{-1}$,

for all a, b, $c \in G$. A K-algebra $\mathcal{K}$ is called abelian if and only if $x \odot (e \odot b) = b \odot (e \odot a)$ for all a, $b \in G$. If a K-algebra $\mathcal{K}$ is abelian, then the axioms (1) and (2) can be written as:

$(\overline{K1})$ $(a \odot b) \odot (a \odot c) = c \odot b$.

$(\overline{K2})$ $a \odot (a \odot b) = b$.

In what follows, we denote a K-algebra by $\mathcal{K}$ unless otherwise specified.

A nonempty subset H of a K-algebra $\mathcal{K}$ is called a *subalgebra* [12] of the K-algebra $\mathcal{K}$ if $a \odot b \in H$, for all a, $b \in H$. Note that every subalgebra of a K-algebra $\mathcal{K}$ contains the identity e of the group $(G, \cdot, e)$. Naturally, the mapping $f : \mathcal{K}_1 \to \mathcal{K}_2$ of K-algebras is called a *homomorphism* [15] if $f(a \odot b) = f(a) \odot f(b)$, for all $a, b \in \mathcal{K}_1$. We refer the reader to the book [10] for further information regarding K-algebras.

Definition 2.1. [29] Let X be a nonempty set. A *bipolar fuzzy set* B in X is an object having the form

$$B = \{(x, \mu^+(x), \mu^-(x)) \mid x \in X\}$$

where $\mu^+ : X \to [0, 1]$ and $\mu^- : X \to [-1, 0]$ are mappings.

We use the positive membership degree $\mu^+(x)$ to denote the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set B, and the negative membership degree $\mu^-(x)$ to denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar fuzzy set B. If $\mu^+(x) \neq 0$ and $\mu^-(x) = 0$, it is the situation that x is regarded as having only positive satisfaction for B. If $\mu^+(x) = 0$ and $\mu^-(x) \neq 0$, it is the situation that x does not satisfy the property of B but somewhat satisfies the counter property of B . It is possible for an element x to be such that $\mu^+(x) \neq 0$ and $\mu^-(x) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

For the sake of simplicity, we shall use the symbol $B = (\mu^+, \mu^-)$ for the bipolar fuzzy set $B = \{(x, \mu^+(x), \mu^-(x)) \mid x \in X\}$.

Definition 2.2. [29] For every two bipolar fuzzy sets $A = (\mu_A^+, \mu_A^-)$ and $B = (\mu_B^+, \mu_B^-)$ in X, we define

- $(A \bigcap B)(x) = (\min(\mu_A^+(x), \mu_B^+(x)), \max(\mu_A^-(x), \mu_B^-(x)))$,
- $(A \bigcup B)(x) = (\max(\mu_A^+(x), \mu_B^+(x)), \min(\mu_A^-(x), \mu_B^-(x)))$.

Definition 2.3. For $x \in X$, define $A_x = \{(y, z) \in X \times X | x = yz\}$. For two bipolar fuzzy subsets $A = (\mu_A^+, \mu_A^-)$ and $B = (\mu_B^+, \mu_B^-)$ of X, the product of two bipolar fuzzy subsets is denoted by $A \circ B$ and is defined as:

$$\mu_A^+ \circ \mu_B^+(x) = \begin{cases} \bigvee_{(s,t)\in A_x} \{\mu_A^+(s) \wedge \mu_B^+(t)\} & \text{if } A_x \neq \emptyset, \\ 0 & \text{if } A_x = \emptyset \end{cases}$$

and

$$\mu_A^- \circ \mu_B^-(x) = \begin{cases} \bigwedge_{(s,t)\in A_x} \{\mu_A^-(s) \vee \mu_B^-(t)\} & \text{if } A_x \neq \emptyset, \\ 0 & \text{if } A_x = \emptyset. \end{cases}$$

Molodtsov [23] defined the notion of soft set in the following way: Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and let A be non-empty subset of E.

Definition 2.4. [23] A pair (Φ, A) is called a *soft set* over U, where Φ is a mapping given by $\Phi : A \to P(U)$.

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A$, $\Phi(\varepsilon)$ may be considered as the set of $\varepsilon-$ approximate elements of the soft set (Φ, A). Clearly, a soft set is not just a subset of U. The concept of bipolar fuzzy soft set, which was originally proposed by [27]. Let $BF(U)$ denote the family of all bipolar fuzzy sets in U.

Definition 2.5. [27] Let U be an initial universe and $A \subseteq E$ be a set of parameters. A pair (ϕ, A) is called an *bipolar fuzzy soft set* over U, where ϕ is a mapping given by $\phi : A \to BF(U)$. A bipolar fuzzy soft set is a parameterized family of bipolar fuzzy subsets of U. For any $\varepsilon \in A$, ϕ_ε is referred to as the set of $a-$ approximate elements of the bipolar fuzzy soft set (ϕ, A), which is actually a bipolar fuzzy set on U and can be

$$\phi_\varepsilon = \{(\mu^+_{\phi_\varepsilon}(x), \mu^-_{\phi_\varepsilon}(x)) \mid x \in U\}$$

where $\mu^+_{\phi_\varepsilon}(x)$ denotes the degree of x keeping the parameter ε, $\mu^-_{\phi_\varepsilon}(x)$ denotes the degree of x keeping the non-parameter ε.

Definition 2.6. [27] Let (ϕ, A) and (ψ, B) be two bipolar fuzzy soft sets over U. We say that (ϕ, A) is a *bipolar fuzzy soft subset* of (ψ, B) and write $(\phi, A) \Subset (\psi, B)$ if

(i) $A \subseteq B$,

(ii) For any $\varepsilon \in A$, $\phi(\varepsilon) \subseteq \psi(\varepsilon)$.

(ϕ, A) and (ψ, B) are said to be bipolar fuzzy soft equal *and write* $(\phi, A) = (\psi, B)$ *if* $(\phi, A) \Subset (\psi, B)$ *and* $(\psi, B) \Subset (\phi, A)$.

Definition 2.7. [27] Let (ϕ, A) and (ψ, B) be two bipolar fuzzy soft sets over U. Then their *extended intersection* is a bipolar fuzzy soft set denoted by (φ, C), where $C = A \cup B$ and

$$\varphi(\varepsilon) = \begin{cases} \phi(\varepsilon) & \text{if } \varepsilon \in A - B, \\ \psi(\varepsilon) & \text{if } \varepsilon \in B - A, \\ \phi(\varepsilon) \cap \psi(\varepsilon) & \text{if } \varepsilon \in A \cap B, \end{cases}$$

for all $\varepsilon \in C$. This is denoted by $(\varphi, C) = (\phi, A)\widetilde{\cap}(\psi, B)$.

Definition 2.8. [27] If (ϕ,A) and (ψ,B) are two bipolar fuzzy soft sets over the same universe U then "(ϕ,A)AND(ψ,B)" is a bipolar fuzzy soft set denoted by $(\phi,A) \wedge (\psi,B)$, and is defined by $(\phi,A) \wedge (\psi,B) = (\varphi,A \times B)$ where, $\varphi(a, b) = \varphi(a) \cap \psi(b)$ for all $(a,b) \in A \times B$. Here $\cap$ is the operation of a bipolar fuzzy intersection.

Definition 2.9. [27] Let (ϕ, A) and (ψ, B) be two bipolar fuzzy soft sets over U. Then their *extended union* denoted by (φ, C), where $C = A \cup B$ and

$$\varphi(\varepsilon) = \begin{cases} \phi(\varepsilon) & \text{if } \varepsilon \in A - B, \\ \psi(\varepsilon) & \text{if } \varepsilon \in B - A, \\ \phi(\varepsilon) \cup \psi(\varepsilon) & \text{if } \varepsilon \in A \cap B, \end{cases}$$

for all $\varepsilon \in C$. This is denoted by $(\varphi, C) = (\phi, A)\widetilde{\cup}(\psi, B)$.

Definition 2.10. [24] Let (ϕ, A) and (ψ, B) be two fuzzy soft sets over a common universe U with $A \cap B \neq \emptyset$, then their *restricted intersection* is a bipolar fuzzy soft set $(\varphi, A \cap B)$ denoted by $(\phi, A) \Cap (\psi, B) = (\varphi, A \cap B)$ where, $\varphi(\varepsilon) = \phi(\varepsilon) \cap \psi(\varepsilon)$ for all $\varepsilon \in A \cap B$.

Definition 2.11. [24] Let (ϕ, A) and (ψ, B) be two bipolar fuzzy soft sets over a common universe U with $A \cap B \neq \emptyset$, then their *restricted union* is denoted by $(\phi, A) \Cup (\psi, B)$ and is defined as $(\phi, A) \Cup (\psi, B) = (\varphi, C)$ where $C = A \cap B$ and for all $\varepsilon \in C$, $\varphi(\varepsilon) = \phi(\varepsilon) \cup \psi(\varepsilon)$.

Definition 2.12. [24] The *extended product* of two bipolar fuzzy soft sets (ϕ, A) and (ψ, B) over U is a fuzzy soft set, denoted by $(\phi \circ \psi, C)$, where $C = A \cup B$ and

$$(\phi \circ \psi)(\varepsilon) = \begin{cases} \phi(\varepsilon) & \text{if } \varepsilon \in A - B, \\ \psi(\varepsilon) & \text{if } \varepsilon \in B - A, \\ \phi(\varepsilon) \circ \psi(\varepsilon) & \text{if } \varepsilon \in A \cap B, \end{cases}$$

for all $\varepsilon \in C$. This is denoted by $(\phi \circ \psi, C) = (\phi, A)\widetilde{\circ}(\psi, B)$.

Definition 2.13. [24] If $A \cap B \neq \emptyset$, then the *restricted product* (φ,C) of two bipolar fuzzy soft sets (ϕ,A) and (ψ,B) over U is defined as the bipolar fuzzy soft set, $(\varphi,A \cap B)$ denoted by $(\phi,A)\, o_R\, (\psi,B)$ where $\varphi(\varepsilon) = \phi(\varepsilon) \circ \psi(\varepsilon)$, for all $\varepsilon \in A \cap B$. Here $\phi(\varepsilon) \circ \psi(\varepsilon)$ is the product of two bipolar fuzzy subsets of U.

3. Bipolar fuzzy soft K-algebras

Definition 3.1. Let (ϕ, A) be a bipolar fuzzy soft set over $\mathcal{K}$. Then (ϕ, A) is said to be a *bipolar fuzzy soft K-subalgebra* over $\mathcal{K}$ if $\phi(x)$ is a bipolar fuzzy K-subalgebra of $\mathcal{K}$ for all $x \in A$, that is, a bipolar fuzzy soft set (ϕ, A) over $\mathcal{K}$ is called a *bipolar fuzzy soft K-subalgebra* of $\mathcal{K}$ if the following conditions are satisfied:

(1) $\mu^+_{\phi_\varepsilon}(x \odot y) \geqslant \min\{\mu^+_{\phi_\varepsilon}(x), \mu^+_{\phi_\varepsilon}(y)\}$,

(2) $\mu^-_{\phi_\varepsilon}(x \odot y) \leqslant \max\{\mu^-_{\phi_\varepsilon}(x), \mu^-_{\phi_\varepsilon}(y)\}$

for all $x, y \in G$.

Definition 3.2. Let (ϕ, A) and (ψ, B) be bipolar fuzzy soft K-subalgebras over $\mathcal{K}$. Then (ϕ, A) is a bipolar fuzzy subalgebra of (ψ, B) if (i) $A \subset B$ and (ii) $\phi(x)$ is a bipolar fuzzy subalgebra of $\psi(x)$ for all $x \in A$.

Example 3.3. Consider the K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where $G = \{e, a, a^2, a^3\}$ is the cyclic group of order 4 and $\odot$ is given by the following Cayley's table:

$\odot$	e	a	a^2	a^3
e	e	a^3	a^2	a
a	a	e	a^3	a^2
a^2	a^2	a	e	a^3
a^3	a^3	a^2	a	e

Let $A = \{e_1, e_2, e_3\}$ and $\phi : A \to \mathcal{P}(G)$ be a set-valued function defined by

$$\phi(e_1) = \left\{\frac{e}{(0.7, -0.2)}, \frac{a}{(0.3, -0.4)}, \frac{a^2}{(0.6, -0.2)}, \frac{a^3}{(0.3, -0.6)}\right\},$$

$$\phi(e_2) = \left\{\frac{e}{(0.5, -0.4)}, \frac{a}{(0.3, -0.3)}, \frac{a^2}{(0.4, -0.3)}, \frac{a^3}{(0.4, -0.5)}\right\},$$

$$\phi(e_3) = \left\{\frac{e}{(0.5, -0.4)}, \frac{a}{(0.5, -0.2)}, \frac{a^2}{(0.5, -0.4)}, \frac{a^3}{(0.2, -0.5)}\right\}.$$

Let $B = \{e_2, e_3\}$ and $\psi : B \to \mathcal{P}(G)$ be a set-valued function defined by

$$\psi(e_2) = \left\{\frac{e}{(0.2, -0.2)}, \frac{a}{(0.2, -0.3)}, \frac{a^2}{(0.1, -0.6)}, \frac{a^3}{(0.3, -0.0)}\right\},$$

$$\psi(e_3) = \left\{\frac{e}{(0.2, -0.2)}, \frac{a}{(0.3, -0.3)}, \frac{a^2}{(0.3, -0.3)}, \frac{a^3}{(0.4, -0.4)}\right\}.$$

Clearly, (ϕ, A) and (ψ, B) are bipolar fuzzy soft sets over $\mathcal{K}$. By routine calculations, it is easy to see that $\phi(x)$ and $\psi(x)$ are bipolar fuzzy K-subalgebras for $x \in A$ and $x \in B$, respectively. Hence (ϕ, A) and (ψ, B) are bipolar fuzzy soft K-subalgebras.

We state the following propositions without their proofs.

Proposition 3.4. *Let (ϕ, A) and (ψ, B) be bipolar fuzzy soft K-subalgebras over $\mathcal{K}$, then $(\phi, A)\widetilde{\cap}(\psi, B)$ and $(\phi, A) \wedge (\psi, B)$ are bipolar fuzzy soft K-subalgebras over $\mathcal{K}$.*

Proposition 3.5. *Let (ϕ, A) and (ψ, B) be bipolar fuzzy soft K-subalgebras over $\mathcal{K}$. If $A \cap B = \emptyset$ then $(\phi, A)\widetilde{\cup}(\psi, B)$ is a bipolar fuzzy soft K-subalgebra of $\mathcal{K}$.*

Proposition 3.6. *Let (ϕ, A) and (ψ, B) be bipolar fuzzy soft K-subalgebras over $\mathcal{K}$. If $\phi(x) \subseteq \psi(x)$ for all $x \in A$, (ϕ, A) is a bipolar fuzzy soft subalgebra of (ψ, B).*

Theorem 3.7. *Let (ϕ, A) be a bipolar fuzzy soft K-subalgebra over $\mathcal{K}$ and let $\{(\varphi_i, B_i) \mid i \in I\}$ be a nonempty family of bipolar fuzzy soft K-subalgebras of (ϕ, A). Then*

(a) $\widetilde{\bigcap_{i\in I}}(\varphi_i, B_i)$ *is a bipolar fuzzy soft K-subalgebra of (ϕ, A),*

(b) $\bigwedge_{i\in I}(\varphi_i, B_i)$ *is a bipolar fuzzy soft K-subalgebra of* $\bigwedge_{i\in I}(\phi, A)$,

(c) *If $B_i \cap B_j = \emptyset$ for all i, $j \in I$, then* $\widetilde{\bigvee_{i\in I}}(\varphi_i, B_i)$ *is a bipolar fuzzy soft K-subalgebra of* $\widetilde{\bigvee_{i\in I}}\phi, A)$.

Definition 3.8. Let (ϕ, A) be a bipolar fuzzy soft set over U. For each $s \in [0, 1]$, the set $(\phi, A)^s = (\phi^s, A)$ is called an s-level soft set of (ϕ, A), where $\phi^s_\varepsilon = \{x \in U \mid \mu^+_{\phi_\varepsilon}(x) \geq s, \mu^-_{\phi_\varepsilon}(x) \leq -s\}$ for all $\varepsilon \in A$. Clearly, $(\phi, A)^s$ is a soft set over U.

Example 3.9. Consider the K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where $G = \{e, a, a^2, a^3\}$ is the cyclic group of order 4 and $\odot$ is given by Cayley's table in Example 3.3. 0.2-level soft set of $(\phi, A) = \{e_1, e_2, e_3\}$.

Theorem 3.10. *Let (ϕ, A) be a bipolar fuzzy soft set over $\mathcal{K}$. (ϕ, A) is a bipolar fuzzy soft K-subalgebra if and only if $(\phi, A)^s$ is a soft K-subalgebra over $\mathcal{K}$ for each $s \in [0, 1]$.*

Proof. Suppose that (ϕ, A) is a bipolar fuzzy soft K-subalgebra. For each $s \in [0, 1]$, $\varepsilon \in A$ and x_1, $x_2 \in \phi^s_\varepsilon$, then $\mu^+_{\phi_\varepsilon}(x_1) \geq s$, $\mu^+_{\phi_\varepsilon(x_2)} \geq s$ and $\mu^-_{\phi_\varepsilon}(x_1) \leq -s$, $\mu^-_{\phi_\varepsilon(x_2)} \leq -s$. From definition 3.1, it follows that ϕ^s_ε is a bipolar fuzzy K-subalgebra over $\mathcal{K}$. Thus $\mu^+_{\phi_\varepsilon}(x_1 \odot x_2) \geq \min(\mu^+_{\phi_\varepsilon}(x_1), \mu^+_{\phi_\varepsilon}(x_2))$, $\mu^+_{\phi_\varepsilon}(x_1 \odot x_2) \geq s$, $\mu^-_{\phi_\varepsilon}(x_1 \odot x_2) \leq \max(\mu^-_{\phi_\varepsilon}(x_1), \mu^-_{\phi_\varepsilon}(x_2))$, $\mu^-_{\phi_\varepsilon}(x_1 \odot x_2) \leq -s$. This implies that

$x_1 \odot x_2 \in \phi_\varepsilon^s$, i.e., ϕ_ε^s is a K-subalgebra over $\mathcal{K}$. According to Definition 3.8, $(\phi, A)^s$ is a soft K-subalgebra over $\mathcal{K}$ for each $s \in [0,1]$.

Conversely, assume that $(\phi, A)^s$ is a soft K-subalgebra over $\mathcal{K}$ for each $s \in [0,1]$. For each $\varepsilon \in A$ and $x_1, x_2 \in G$, let $s = \min\{\mu^+_{\phi_\varepsilon}(x_1), \mu^+_{\phi_\varepsilon}(x_2)\}$ and let $-s = \max\{\mu^-_{\phi_\varepsilon}(x_1), \mu^-_{\phi_\varepsilon}(x_2)\}$, then $x_1,\ x_2 \in \phi_\varepsilon^s$. Since ϕ_ε^s is a K-subalgebra over $\mathcal{K}$, then $x_1 \odot x_2 \in \phi_\varepsilon^s$. This means that $\mu^+_{\phi_\varepsilon}(x_1 \odot x_2) \geq \min(\mu^+_{\phi_\varepsilon}(x_1), \mu^+_{\phi_\varepsilon}(x_2))$, $\mu^-_{\phi_\varepsilon}(x_1 \odot x_2) \leq \max(\mu^-_{\phi_\varepsilon}(x_1), \mu^-_{\phi_\varepsilon}(x_2))$, i.e., ϕ_ε^s is a bipolar fuzzy K-subalgebra over $\mathcal{K}$. According to Definition 3.1, (ϕ, A) is a bipolar fuzzy soft K-subalgebra over $\mathcal{K}$. This completes the proof. ∎

Proposition 3.11. *Let $\mathcal{K}$ be a K-algebra built on Abelian group. Let (ϕ,A), (ψ,B) and (φ,C) be bipolar fuzzy soft K-subalgebras over $\mathcal{K}$ where A, B and C are subsets of E. Then*

(1) $(\phi,A)\widetilde{o}((\psi,B)\widetilde{\cup}(\varphi,C)) = ((\phi,A)\widetilde{o}(\psi,B))\widetilde{\cup}((\phi,A)\widetilde{o}(\varphi,C))$,

(2) $(\phi,A)\widetilde{o}((\psi,B) \Cup (\varphi,C)) = ((\phi,A)\widetilde{o}(\psi,B)) \Cup ((\phi,A)\widetilde{o}(\varphi,C))$,

(3) $((\phi,A)\widetilde{\cup}(\psi,B))\widetilde{o}(\varphi,C) = ((\phi,A)\widetilde{o}(\varphi,C))\widetilde{\cup}((\psi,B)\widetilde{o}(\varphi,C))$,

(4) $((\phi,A) \Cup (\psi,B))\widetilde{o}(\varphi,C) = ((\phi,A)\widetilde{o}(\varphi,C)) \Cup ((\psi,B)\widetilde{o}(\varphi,C))$.

Proof. The proofs follow from the definitions of operations. ∎

Proposition 3.12. *Let $\mathcal{K}$ be a K-algebra built on Abelian group. Let (ϕ,A), (ψ,B) and (φ,C) be bipolar fuzzy soft K-subalgebras over $\mathcal{K}$ where A, B and C are subsets of E. Then*

(5) $(\phi,A)\widetilde{o}_R((\psi,B) \Cup (\varphi,C))\widetilde{\subset}((\phi,A)\widetilde{o}_R(\psi,B)) \Cup ((\phi,A)\widetilde{o}_R(\varphi,C))$,

(6) $(\phi,A)\widetilde{o}_R((\psi,B)\widetilde{\cup}(\varphi,C))\widetilde{\subset}((\phi,A)\widetilde{o}_R(\psi,B))\widetilde{\cup}((\phi,A)\widetilde{o}_R(\varphi,C))$,

(7) $((\phi,A)\widetilde{\cup}(\psi,B))\widetilde{o}_R(\varphi,C)\widetilde{\subset}((\phi,A)\widetilde{o}_R(\varphi,C))\widetilde{\cup}((\psi,B)\widetilde{o}_R(\varphi,C))$,

(8) $((\phi,A) \Cup (\psi,B))\widetilde{o}_R(\varphi,C)\widetilde{\subset}((\phi,A)\widetilde{o}_R(\varphi,C)) \Cup ((\psi,B)\widetilde{o}_r(\varphi,C))$.

Proof. The proofs follow from the definitions of operations. ∎

Definition 3.13. Let $f : X \to Y$ and $g : A \to B$ be two functions, A and B are parametric sets from the crisp sets X and Y, respectively. Then the pair (f, g) is called a *bipolar fuzzy soft function* from X to Y.

Definition 3.14. Let (ϕ, A) and (ψ, B) be two bipolar fuzzy soft sets over G_1 and G_2, respectively and let (f, g) be a bipolar fuzzy soft function from G_1 to G_2.

(1) The image of (ϕ, A) under the bipolar fuzzy soft function (f, g), denoted by $(f, g)(\phi, A)$, is the bipolar fuzzy soft set on $\mathcal{K}_2$ defined by $(f, g)(\phi, A) = (f(\phi), g(A))$, where for all $k \in g(A), y \in G_2$

$$\mu^+_{f(\phi)_k}(y) = \begin{cases} \bigvee\limits_{f(x)=y} \bigvee\limits_{g(a)=k} \phi_a(x) & \text{if } x \in g^{-1}(y), \\ 1 & \text{otherwise,} \end{cases}$$

$$\mu^-_{f(\phi)_k}(y) = \begin{cases} \bigwedge\limits_{f(x)=y} \bigwedge\limits_{g(a)=k} \phi_a(x) & \text{if } x \in g^{-1}(y), \\ -1 & \text{otherwise.} \end{cases}$$

(2) The preimage of (ψ, B) under the bipolar fuzzy soft function (f, g), denoted by $(f, g)^{-1}(\psi, B)$, is the bipolar fuzzy soft set over $\mathcal{K}_1$ defined by $(f, g)^{-1}(\psi, B) = (f^{-1}(\psi), g^{-1}(B))$, where for all $a \in g^{-1}(A)$, for all $x \in G_1$,

$$\mu^+_{f^{-1}(\psi)_a}(x) = \mu^+_{\psi_{g(a)}}(f(x)),$$

$$\mu^-_{f^{-1}(\psi)_a}(x) = \mu^-_{\psi_{g(a)}}(f(x)).$$

Definition 3.15. Let (f, g) be a bipolar fuzzy soft function from $\mathcal{K}_1$ to $\mathcal{K}_2$. If f is a homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ then (f, g) is said to be *bipolar fuzzy soft homomorphism*, If f is a isomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$ and g is one-to-one mapping from A onto B then (f, g) is said to be bipolar fuzzy soft isomorphism.

Theorem 3.16. *Let (ψ, B) be a bipolar fuzzy soft K-subalgebra over $\mathcal{K}_2$ and let (f, g) be a bipolar fuzzy soft homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$. Then $(f, g)^{-1}(\psi, B)$ is a bipolar fuzzy soft K-subalgebra over $\mathcal{K}_1$.*

Proof. Let $x_1, x_2 \in G_1$, then

$$\begin{aligned} f^{-1}(\mu^+_{\psi_\varepsilon})(x_1 \odot x_2) &= \mu^+_{\psi_{g(\varepsilon)}}(f(x_1 \odot x_2)) = \mu^+_{\psi_{g(\varepsilon)}}(f(x_1) \odot f(x_2)) \\ &\geqslant \min\{\mu^+_{\psi_{g(\varepsilon)}}(f(x_1)), \mu^+_{\psi_{g(\varepsilon)}}(f(x_2))\} \\ &= \min\{f^{-1}(\mu^+_{g_\varepsilon})(x_1), f^{-1}(\mu^+_{g_\varepsilon})(x_2)\}, \end{aligned}$$

$$\begin{aligned} f^{-1}(\mu^-_{\psi_\varepsilon})(x_1 \odot x_2) &= \mu^-_{\psi_{g(\varepsilon)}}(f(x_1 \odot x_2)) = \mu^-_{\psi_{g(\varepsilon)}}(f(x_1) \odot f(x_2)) \\ &\leqslant \max\{\mu^-_{\psi_{g(\varepsilon)}}(f(x_1)), \mu^-_{\psi_{g(\varepsilon)}}(f(x_2))\} \\ &= \max\{f^{-1}(\mu^-_{g_\varepsilon})(x_1), f^{-1}(\mu^-_{g_\varepsilon})(x_2)\}. \end{aligned}$$

Hence $(f, g)^{-1}(\psi, B)$ is a bipolar fuzzy soft K-subalgebra over $\mathcal{K}_1$. ∎

Remark. Let (ϕ, A) be a bipolar fuzzy soft K-subalgebra over $\mathcal{K}_1$ and let (f, g) be a bipolar fuzzy soft homomorphism from $\mathcal{K}_1$ to $\mathcal{K}_2$. Then $(f, g)(\phi, A)$ may not be a bipolar fuzzy soft K-subalgebra over $\mathcal{K}_2$.

4. Generalized bipolar fuzzy soft $K-$ algebras

Definition 4.1. Let c be a point in a non-empty set G. If $\gamma \in (0,1]$ and $\delta \in [-1,0)$ are two real numbers, then the bipolar fuzzy $c(\gamma,\delta) =< x, c_\gamma, c_\delta >$ is called a bipolar fuzzy point in G, where γ(resp, δ) is the positive degree of membership (resp, negative degree of membership) of $c(\gamma,\delta)$ and $c \in G$ is the support of $c(\gamma,\delta)$. Let $c(\gamma,\delta)$ be a bipolar fuzzy in G and let $A =< x, \mu_A^+, \mu_A^- >$ be a bipolar fuzzy in G. Then $c(\gamma,\delta)$ is said to belong to A, written $c(\gamma,\delta) \in A$ if $\mu_A^+(c) \geq \gamma$ and $\mu_A^-(c) \leq \delta$. We say that $c(\gamma,\delta)$ is quasicoincident with A, written $c(\gamma,\delta)qA$, if $\mu_A^+(c) + \gamma > 1$ and $\mu_A^-(c) + \delta < -1$. To say that $c(\gamma,\delta) \in \vee qA$ (resp, $c(\gamma,\delta) \in \wedge qA$) means that $c(\gamma,\delta) \in A$ or $c(\gamma,\delta)qA$ (resp, $c(\gamma,\delta) \in A$ and $c(\gamma,\delta)qA$) and $c(\gamma,\delta)\overline{\in \vee q}A$ means that $c(\gamma,\delta) \in \vee qA$ does not hold.

Definition 4.2. A bipolar fuzzy set $A = (\mu_A^+, \mu_A^-)$ in $\mathcal{K}$ is called an $(\in, \in \vee q)$-*bipolar fuzzy $K-$ algebra* of $\mathcal{K}$ if it satisfies the following conditions:

(a) $x(s,t) \in A \Rightarrow e(s,t) \in \vee qA$,

(b) $x(s_1,t_1) \in A, y(s_2,t_2) \in A \Rightarrow (x \odot y)(\min(s_1,s_2), \max(t_1,t_2)) \in \vee qA$,

for all x, $y \in G$, s, s_1, $s_2 \in (0,1]$, t, t_1, $t_2 \in [-1,0)$.

Example 4.3. Consider the K-algebra $\mathcal{K} = (G, \cdot, \odot, e)$, where $G = \{e, a, a^2, a^3\}$ is the cyclic group of order 4 and $\odot$ is given by Cayley's table in Example 3.3. We define a bipolar fuzzy set $A : G \to [0,1] \times [-1,0]$ by

$$\mu_A^+(x) := \begin{cases} 1 & \text{if } x = e, \\ 0.5 & \text{otherwise,} \end{cases}$$

$$\mu_A^-(x) := \begin{cases} 0 & \text{if } x = e, \\ -0.3 & \text{otherwise.} \end{cases}$$

Take $s = 0.4 \in (0,1]$ and $t = -0.5 \in [-1,0)$. By routine computations, it is easy to see that A is not an $(\in, \in \vee q$- bipolar fuzzy K-subalgebra of $\mathcal{K}$.

Theorem 4.4. *Let A be a bipolar fuzzy set in a K-algebra $\mathcal{K}$. Then A is an $(\in, \in \vee q)-$ bipolar fuzzy K-subalgebra of $\mathcal{K}$ if and only if* $\mu_A^+(x \odot y) \geqslant \min(\mu_A^+(x), \mu_A^+(y), 0.5)$, $\mu_A^-(x \odot y) \leqslant \max(\mu_A^-(x), \mu_A^-(y), -0.5)$ *hold for all* $x, y \in G$.

Proof. The proof is straightforward and thus omitted. ∎

Theorem 4.5. *Let A be a bipolar fuzzy set of K-algebra of $\mathcal{K}$. Then A is an $(\in, \in \vee q)$- bipolar fuzzy K-subalgebra of $\mathcal{K}$ if and only if each nonempty $A_{(s,t)}$, $s \in (0.5,1]$, $t \in [-1,-0.5)$ is a K-subalgebra of $\mathcal{K}$.*

Proof. Assume that A is an $(\in, \in \vee q)-$ bipolar fuzzy K-subalgebra of $\mathcal{K}$ and let $s \in (0.5,1]$, $t \in [-1,0.5)$. If $x, y \in A_{(s,t)}$, then $\mu_A^+(x) \geq s$ and $\mu_A^+(y) \geq s$, $\mu_A^-(x) \leq t$ and $\mu_A^-(y) \leq t$. Thus,

$$\mu_A^+(x \odot y) \geqslant \min(\mu_A^+(x), \mu_A^+(y), 0.5) \geqslant \min(s, 0.5) = s,$$
$$\mu_A^-(x \odot y) \leqslant \max(\mu_A^-(x), \mu_A^-(y), -0.5) \leqslant \max(t, -0.5) = t,$$

and so $x \odot y \in A_{(s,t)}$. This shows that $A_{(s,t)}$ are K- subalgebras of $\mathcal{K}$. The proof of converse part is obvious. This ends the proof. ∎

Theorem 4.6. *Let A be a bipolar fuzzy set in a K-algebra $\mathcal{K}$. Then $A_{(s,t)}$ is a K-subalgebra of $\mathcal{K}$ if and only if* $\max(\mu_A^+(x \odot y), 0.5) \geqslant \min(\mu_A^+(x), \mu_A^+(y))$, $\min(\mu_A^-(x \odot y), -0.5) \leqslant \max(\mu_A^-(x), \mu_A^-(y))$ *for all $x, y \in G$.*

Proof. The proof is straightforward and thus omitted. ∎

Definition 4.7. Let (ϕ, A) be a bipolar fuzzy soft set over a K-algebra $\mathcal{K}$. Then (ϕ, A) is called an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra if $\phi(\alpha)$ is an $(\in, \in\vee q)$-bipolar fuzzy K-subalgebra of $\mathcal{K}$ for all $\alpha \in A$.

Theorem 4.8. *Let (ϕ, A) and (ψ, B) be two $(\in, \in \vee q)$-bipolar fuzzy soft K-subalgebras over a K-algebra $\mathcal{K}$. Then $(\phi, A) \wedge (\psi, B)$ is an $(\in, \in \vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$.*

Proof. Since $(\phi, A) \wedge (\psi, B) = (\varphi, C)$, where $C = A \times B$ and $\varphi(\alpha, \beta) = \phi(\alpha) \cap \psi(\beta)$ for all $(\alpha, \beta) \in C$. Now for any $(\alpha, \beta) \in C$, since (ϕ, A) and (ψ, B) are $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebras over $\mathcal{K}$, we have both $\phi(\alpha)$ and $\psi(\beta)$ are $(\in, \in \vee q)$-bipolar fuzzy K-subalgebras of $\mathcal{K}$. Thus $\varphi(\alpha, \beta) = \phi(\alpha) \cap \psi(\beta)$ is an $(\in, \in\vee q)$-bipolar fuzzy K-subalgebra of $\mathcal{K}$. Therefore, $(\phi, A) \wedge (\psi, B)$ is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$. ∎

Theorem 4.9. *Let (ϕ, A) and (ψ, B) be two $(\in, \in \vee q)$-bipolar fuzzy soft K-subalgebras over a K-algebra $\mathcal{K}$. If $A \cap B \neq \emptyset$, then $(\phi, A)\tilde{\sqcap}(\psi, B)$ is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$.*

Proof. Since $(\phi, A)\tilde{\sqcap}(\psi, B) = (\varphi, C)$, where $C = A \cap B$ and $\varphi(\alpha) = \phi(\alpha) \cap \psi(\alpha)$ for all $\alpha \in C$. Now, for any $\alpha \in C$, since (ϕ, A) and (ψ, B) are $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebras over $\mathcal{K}$, we have both $\phi(\alpha)$ and $\psi(\alpha)$ are $(\in, \in\vee q)$-bipolar fuzzy K-subalgebras of $\mathcal{K}$. Thus $\varphi(\alpha) = \phi(\alpha) \cap \psi(\alpha)$ is an $(\in, \in\vee q)$-bipolar fuzzy K-subalgebra of $\mathcal{K}$. Therefore, $(\phi, A)\tilde{\sqcap}(\psi, B)$ is an $(\in, \in \vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$. ∎

As a generalization of Theorem 3.7, we have the following result.

Theorem 4.10. *Let (ϕ, A) be an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$ and let $\{(\varphi_i, B_i) \mid i \in I\}$ be a nonempty family of $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebras of (ϕ, A). Then*

(a) $\widetilde{\bigcap_{i\in I}}(\varphi_i, B_i)$ *is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra of (ϕ, A),*

(b) $\bigwedge_{i\in I}(\varphi_i, B_i)$ *is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra of* $\bigwedge_{i\in I}(\phi, A)$,

(c) *If $B_i \cap B_j = \emptyset$ for all i, $j \in I$, then* $\widetilde{\bigvee_{i\in I}}(\varphi_i, B_i)$ *is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra of* $\widetilde{\bigvee_{i\in I}}\phi, A)$.

Proof. The proof is straightforward and thus omitted. ∎

Theorem 4.11. *Let (ϕ, A) and (ψ, B) be two $(\in, \in \vee q)$-bipolar fuzzy soft K-subalgebras over a K-algebra $\mathcal{K}$. Then $(\phi, A)\tilde{\cap}(\psi, B)$ is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$.*

Proof. Since $(\phi, A)\tilde{\cap}(\psi, B) = (\varphi, C)$, where $C = A \cup B$ and

$$\varphi(\alpha) = \begin{cases} \phi(\alpha) & \text{if } \alpha \in A - B, \\ \psi(\alpha) & \text{if } \alpha \in B - C, \\ \phi(\alpha) \cap \psi(\alpha) & \text{if } \alpha \in A \cap B. \end{cases}$$

for all $\alpha \in C$.

Now, for any $\alpha \in C$, we consider the following cases.

Case 1. $\alpha \in A-B$. Then $\varphi(\alpha) = \phi(\alpha)$ is an $(\in, \in\vee q)$-bipolar fuzzy K-subalgebra of $\mathcal{K}$ since (ϕ, A) is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$.

Case 2. $\alpha \in B-A$. Then $\varphi(\alpha) = \psi(\alpha)$ is an $(\in, \in\vee q)$-bipolar fuzzy K-subalgebra of $\mathcal{K}$ since (ψ, B) is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$.

Case 3. $\alpha \in A \cap B$. Then $\varphi(\alpha) = \phi(\alpha) \cap \psi(\alpha)$ is an $(\in, \in \vee q)$-bipolar fuzzy K-subalgebra of $\mathcal{K}$ by the assumption. Thus, in any case, $\varphi(\alpha)$ is an $(\in, \in\vee q)$-bipolar fuzzy K-subalgebra of $\mathcal{K}$. Therefore, $(\phi, A)\tilde{\cap}(\psi, B)$ is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$. ∎

Theorem 4.12. *Let (ϕ, A) and (ψ, B) be two $(\in, \in \vee q)$-bipolar fuzzy soft$K$-subalgebras over a K-algebra $\mathcal{K}$. If A and B are disjoint, then $(\phi, A)\tilde{\cup}(\psi, B)$ is an $(\in, \in\vee q)$-bipolar fuzzy soft K-subalgebra over $\mathcal{K}$.*

Proof. The proof is straightforward and thus omitted. ∎

5. Conclusions

Molodtsov introduced the concept of soft set theory as a general mathematical tool for dealing with uncertainty and vagueness, and many researchers have created some models to solve problems in decision making and medical diagnosis. In soft computing and uncertain modeling, soft sets can be combined with other mathematical tools. Bipolar fuzzy sets and soft sets are two different mathematical tools for representing vagueness and uncertainty. In this paper we have applied these mathematical tools in combination to study vagueness and uncertainty in K- algebras. We have introduced the notion of an $(\in, \in \vee q)-$ bipolar fuzzy soft K-subalgebra. The natural extension of this research work is connected with the study of: (i) fuzzy soft rough K-algebras, and (ii) bipolar fuzzy rough K-algebras.

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