ON THOMPSON'S CONJECTURE FOR ALTERNATING GROUP A_{26}

Shitian Liu
Yanhua Huang

School of Science
Sichuan University of Science and Engineering
Zigong Sichuan, 643000
China
e-mails: liust@suse.edu.cn
maggie434100@163.com

Abstract. Let G be a group. Denote by $N(G)$ the set of nonidentity orders of conjugacy classes of elements in G. For groups A_{10}, A_{16} and A_{22} which are uniquely determined by $N(G)$, theses degrees are $p+3$ and $p+4$ is a prime with $p=7,13,19$. If $p+4$ is composite, then whether can the groups A_{p+3} be characterized by $N(G)$. In this paper, we give an example for A_{p+3} with $p+4$ composite, namely, we proved that if G is a group with trivial center and $N(G)=N(A_{26})$, then $G\cong A_{26}$.

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1. Introduction

Let G be a finite group. Denote by $N(G)$ the set of nonidentity orders of conjugacy classes of elements in G. Related to $N(G)$, J.G. Thompson gave the following conjecture.

Thompson's Conjecture. (see [11, Question 12.38]). *If L is a finite simple non-Abelian group, G is a finite group with trivial center, and $N(G)=N(L)$, then $G\cong L$.*

Let $\pi(G)$ denote the set of all prime divisors of $|G|$. Let $GK(G)$ be a graph with vertex set $\pi(G)$ such that two primes p and q in $\pi(G)$ are joined by an edge if G has an element of order $p\cdot q$. We set $s(G)$ denote the number of connected components of the prime graph $GK(G)$. A classification of all finite simple groups with disconnected prime graph was obtained in [10], [14]. Based on these results, Thompson's conjecture was proved valid for all finite simple groups with $s(G) \geq 2$ (see [2], [3]). So whether there is a group with connected prime graph for which Thompson's conjecture would be true? Recently, the groups A_{10}, A_{16} and A_{22} were proved valid for this conjecture (see [13], [6] and [15], respectively). As the development of this topics, we will prove that Thompson's conjecture is true for the alternating group A_{26} of degree 26.

We will introduce some notations used to the proof of the main theorem. Let A_n and S_n denote the alternating and symmetric groups of degree n, respectively. Let G be a group. Set $Aut(G)$ denotes the automorphism group of G. Let $\omega(G)$ denote the set of element order of G. The notations are standard (see [5], for instance).

2. Some lemmas

In this section we will give some preliminary results.

Lemma 2.1 [13, Lemma 1.2] and [1, Lemma 2.3] *Let $x, y \in G$, $(|x|, |y|) = 1$, and $xy = yx$. Then*

(1) $C_G(xy) = C_G(x) \cap C_G(y)$;

(2) $|x^G|$ *divides* $|(xy)^G|$;

(3) *If* $|x^G| = |(xy)^G|$, *then* $C_G(x) \leq C_G(y)$.

Lemma 2.2 [13, Lemma 3] *If P and H are finite groups with trivial centers, and $N(P) = N(G)$, then $\pi(P) = \pi(H)$.*

Lemma 2.3 [13, Lemma 4] *Suppose that G is a finite group with trivial center and p is a prime from $\pi(G)$ such that p^2 does not divide $|x^G|$ for all x in G. Then a Sylow p-subgroup of G is elementary abelian.*

Lemma 2.4 [13, Lemma 5] *Let K be a normal subgroup of G, and $\overline{G} = G/K$.*

(1) *If $\overline{x}$ is the image of an element x of G in $\overline{G}$. Then $|\overline{x}^{\overline{G}}|$ divides $|x^G|$.*

(2) *If $(|x|, |K|) = 1$, then $C_{\overline{G}}(\overline{x}) = C_G(x)K/K$.*

(3) *If $y \in K$, then $|y^K|$ divides $|y^G|$.*

3. Main theorem and its proof

In this section, we give the main theorem and its proof.

Theorem 3.1 *Let G be a finite group with trivial center and $N(G) = N(A_{26})$. Then G is isomorphic to A_{26}.*

Proof. We divide the proof into the following lemmas.

Lemma 3.2 *Let $L = A_{26}$. Then the following hold.*

(1) $|L| = 2^{22} \cdot 3^{10} \cdot 5^{6} \cdot 7^{3} \cdot 11^{2} \cdot 13^{2} \cdot 17 \cdot 19 \cdot 23$.

(2) *numbers* $\frac{26!}{23\cdot 3}$ *and* $\frac{26\cdot 25\cdot 24}{3}$ *are maximal and minimal with respect to divisibility in the set* $N(L)$*, respectively.*

(3) *for any* $n \in N(L)$*, numbers* 17^2*,* 19^2 *and* 23^2 *do not divide* n*.*

(4) $23'$*-numbers in* $N(L) \setminus \{1\}$ *are*

$2^{4} \cdot 5^{2} \cdot 13$,

$2^{22} \cdot 3^{9} \cdot 5^{6} \cdot 7^{3} \cdot 11^{2} \cdot 13^{2} \cdot 17 \cdot 19$.

$19'$*-numbers in* $N(L) \setminus \{1\}$ *are*

$2^{4} \cdot 5^{2} \cdot 13$,

$2^{7} \cdot 3^{2} \cdot 5^{3} \cdot 11 \cdot 13 \cdot 23$,

$2^{4} \cdot 3 \cdot 5^{3} \cdot 7 \cdot 11 \cdot 13 \cdot 23$,

$2^{2} \cdot 3^{2} \cdot 5^{2} \cdot 7 \cdot 11 \cdot 13 \cdot 23$,

$2^{4} \cdot 5^{2} \cdot 7 \cdot 11 \cdot 13 \cdot 23$,

$2^{5} \cdot 3 \cdot 5 \cdot 11 \cdot 13 \cdot 23$,

$2 \cdot 3 \cdot 5^{2} \cdot 13 \cdot 23$,

$2^{20} \cdot 3^{10} \cdot 5^{6} \cdot 7^{3} \cdot 11^{2} \cdot 13^{2} \cdot 17 \cdot 23$,

$2^{22} \cdot 3^{8} \cdot 5^{6} \cdot 7^{3} \cdot 11^{2} \cdot 13^{2} \cdot 17 \cdot 23$,

$2^{19} \cdot 3^{9} \cdot 5^{6} \cdot 7^{3} \cdot 11^{2} \cdot 13^{2} \cdot 17 \cdot 19 \cdot 23$,

$2^{20} \cdot 3^{9} \cdot 5^{6} \cdot 7^{3} \cdot 11^{2} \cdot 13^{2} \cdot 17 \cdot 23$,

$2^{22} \cdot 3^{10} \cdot 5^{6} \cdot 7^{2} \cdot 11^{2} \cdot 13^{2} \cdot 17 \cdot 23$.

$17'$*-numbers in* $N(L) \setminus \{1\}$ *are*

$2^{4} \cdot 5^{2} \cdot 13$,

$2^{7} \cdot 3^{2} \cdot 5^{3} \cdot 11 \cdot 13 \cdot 23$,

$2^{4} \cdot 3 \cdot 5^{3} \cdot 7 \cdot 11 \cdot 13 \cdot 23$,

$2^{2} \cdot 3^{2} \cdot 5^{2} \cdot 7 \cdot 11 \cdot 13 \cdot 23$,

$2^{4} \cdot 5^{2} \cdot 7 \cdot 11 \cdot 13 \cdot 23$,

$2^{5} \cdot 3 \cdot 5 \cdot 11 \cdot 13 \cdot 23$,

$2 \cdot 3 \cdot 5^{2} \cdot 13 \cdot 23$,

$2^{8} \cdot 3^{2} \cdot 5^{3} \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 23$,

$2^{7} \cdot 5^{3} \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 23$,

$2^{5} \cdot 3^{3} \cdot 5^{3} \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 23$,

$2^5 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 23$,
$2^5 \cdot 3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 23$,
$2^7 \cdot 3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 19 \cdot 23$,
$2^{22} \cdot 3^8 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{20} \cdot 3^9 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{20} \cdot 3^{10} \cdot 5^5 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{22} \cdot 3^6 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{18} \cdot 3^{10} \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{22} \cdot 3^{10} \cdot 5^6 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{19} \cdot 3^9 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{21} \cdot 3^7 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{22} \cdot 3^9 \cdot 5^5 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{21} \cdot 3^9 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{20} \cdot 3^9 \cdot 5^5 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{17} \cdot 3^9 \cdot 5^5 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{19} \cdot 3^8 \cdot 5^5 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{15} \cdot 3^6 \cdot 5^5 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$.

(5) *the numbers*
$2^{22} \cdot 3^{10} \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 17 \cdot 19 \cdot 23$,
$2^{22} \cdot 3^8 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 19 \cdot 23$,
$2^{22} \cdot 3^{10} \cdot 5^6 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 23$,
$2^{22} \cdot 3^9 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19$
are maximal elements of $N(L)$ *by divisibility.*

(6) *for any* $n \in N(L)\backslash\{1\}$, *$n$ is divisible by 11 or 13.*

Proof. The information is obtained via GAP [7] or [8]. ∎

Lemma 3.3 *Let G be a finite group with trivial center and $N(G) = N(L)$. Then* $|L| \mid |G|$ *and* $\pi(G) = \pi(L) = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$.

Proof. Since $|x^G||C_G(x)| = |G|$, then every member form $N(G)$ divides the order of G and $|L| \mid |G|$. So by Lemmas 2.2 and 3.2, we have that $\pi(G) = \pi(L)$. ∎

Lemma 3.4 *Suppose that G is a finite group with trivial center and $N(G) = N(L)$. Let $p \in \{17, 19, 23\}$. Then the Sylow p-subgroup S of G is of order p. There does not exist an element of order* $17 \cdot 19$, $17 \cdot 23$ *and* $19 \cdot 23$.

Proof. By Lemma 2.3, S is elementary abelian. If $|x| = p$, then we have that $|x^G|$ is a p'-number.

Suppose that $p = 23$ and $|S| \geq 23^2$. We can assume that there exists an element y of G with that $|y^G| = 2^{22} \cdot 3^{10} \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 17 \cdot 19 \cdot 23$ by Lemma 3.2.

Assume that $23 \mid |y|$. Let $|y| = 23t$. Since S is elementary abelian, the numbers 23 and t are coprime. Put $u = y^{23}$, $v = y^t$. Then $y = uv$, $C_G(uv) = C_G(u) \cap C_G(v)$ by Lemma 2.1. Therefore $|v^G| \mid |y^G| = 2^{22} \cdot 3^{10} \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 17 \cdot 19 \cdot 23$. On the other hand, the element v is of order 23, and therefore, $|v^G| = 2^4 \cdot 5^2 \cdot 13$, $2^{22} \cdot 3^9 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19$. It follows that $13 \mid |v^G|$, a contradiction

Assume that $23 \nmid |y|$. Let x be an element of $C_G(y)$ of order 23. Then $C_G(xy) = C_G(x) \cap C_G(y)$, and therefore $|x^G| \mid |(xy)^G|$ and $|y^G| \mid |(xy)^G|$. Since S is abelian, $S \leq C_G(x)$. It follows that $23 \nmid |x^G|$ and so, $|x^G| = 2^4 \cdot 5^2 \cdot 13$, $2^{22} \cdot 3^9 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19$, a contradiction.

For cases "$p = 19$ and $p = 17$", we can get the same result as "$p = 23$".

There is no element of order $17 \cdot 19$, $17 \cdot 23$, or $19 \cdot 23$ in G. ∎

Lemma 3.5 *Suppose that G is a finite group with trivial center and $N(G) = N(L)$. Assume that $p \in \{2, 5, 7, 11, 13\}$, P is a Sylow p-subgroup of G, and $Z(P)$ is its center. Then for every element $x \in Z(P)\backslash\{1\}$, the order of the centralizer $C_G(x)$ is coprime to 23 if $p = 2$, coprime to 23 if $p = 5$, coprime to* 23 *if $p = 7$, coprime to* $17 \cdot 19 \cdot 23$ *if $p = 11$, and coprime to* $17 \cdot 19 \cdot 23$ *if $p = 13$.*

Proof. By Lemma 3.4, $|G|_p = |L|_p$ for $p \in \{17, 19, 23\}$. For any $1 \neq x \in Z(P)$, we get the desired results since $N(G) = N(L)$. ∎

Lemma 3.6 *Let G be a finite group and $p \in \pi(G)$. If $p^2 \nmid |G|$, then G has a normal series $1 \leq K \leq L \leq G$, such that L/K is a simple group and $p \in \pi(L/K)$.*

Proof. Since G is a finite group, G has a chief series. So let $G_0 \leq G_1 \leq G_2 \cdots \leq G_r = G$ be a chief series of G. There exists some t, such that $1 \leq t \leq r$ and $p \in \pi(G_t) \setminus \pi(G_{t-1})$. Let $L = G_t$ and $K = G_{t-1}$, then $1 \leq K \leq L \leq G$ is a normal series of G and L/K is a chief factor of G. Therefore L/K is a minimal normal subgroup of G/K. We know that the chief factors are characteristically simple. Also every characteristically simple group is a simple group or a product of isomorphic simple groups. So L/K is a simple group or a product of isomorphic simple groups. Since $p \in \pi(L/K)$ and $p^2 \nmid |L/K|$, it follows that L/K is a simple group. ∎

Lemma 3.7 *There is a normal series $1 \leq K \leq L \leq G$ such that $L/K \cong A_{26}$.*

Proof. By Lemma 3.6, we have that there is a normal series $1 \leq K \leq L \leq G$ such that $M := L/K$ is isomorphic to a direct product of nonabelian simple groups $S_1, S_2, \cdots, S_n$ which are listed in [16]. Since G has not a Hall $\{17, 19, 23\}$-subgroup, then the numbers 17, 19 and 23 divide the order of exactly one of these groups. Without generality, we assume that the numbers $17, 19, 23$ divide S_1. Obviously, $S_1 \lhd \overline{G}$. Let $G^* = \overline{G}/S_1$ and $M^* = M/S_1$. We prove that $n = 1$.

Assume that $n \geq 2$. Then a Sylow 2-subgroup P_2 of G^* is nontrivial and its center $Z(P_2)$ has a nontrivial intersection with M^*. Let x be a nontrivial element of $S_2 \times \cdots \times S_n$ with that its image in $\overline{G}$ lies in $Z(P_2)$. Since $y \in Z(S_1)$, then there is an element of order $2 \cdot 23$. It follows that there exists an element y with $|y^G| = 2^{19} \cdot 3^{10} \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19$, which contradicts Lemma 3.2(4). So $n = 1$.

Therefore

$$L/K \leq \overline{G} \leq Aut(L/K)$$

with L/K a nonabelian simple group.

For $p \in \{17, 19, 23\}$, we have that $|G|_p = p$ by Lemma 3.4 and by Lemma 3.6, $p \in \pi(L/K)$. It follows from [16], that $L/K \cong A_n$ where $n = 23, 24, 25, 26, 27, 28$.

If $L/K \cong A_{23}$, then there exist an element $\overline{x}$ of $\overline{G}$ with that

$$\overline{x}^{\overline{G}} = 2^{18} \cdot 3^9 \cdot 5^4 \cdot 7^3 \cdot 11^2 \cdot 13 \cdot 17 \cdot 19.$$

Let x be the preimage of $\overline{x}$ of $\overline{G}$ in G. Then since $|\overline{x}^{\overline{G}}|$ is maximal in $N(G)$, this forces $|x^G| = |\overline{x}^{\overline{G}}|$. It follows that $13 \mid |C_G(x)|$. Then there is an element of order $13 \cdot 23$, and by the proof of Lemma 3.4, we get a contradiction.

Similarly, we can rule out these cases "$L/K \cong A_{24}$, and $L/K \cong A_{25}$".

If $L/K \cong A_{27}$, then there exist an element $\overline{x}$ of $\overline{G}$ with that

$$\overline{x}^{\overline{G}} = 2^{20} \cdot 3^{12} \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19$$

which contradicts Lemma 3.2(4).

If $L/K \cong A_{28}$, then there exist an element $\overline{x}$ of $\overline{G}$ with that

$$\overline{x}^{\overline{G}} = 2^{22} \cdot 3^{13} \cdot 5^5 \cdot 7^5 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19$$

which contradicts Lemma 3.2(4).

Therefore, $L/K \cong A_{26}$. ∎

Lemma 3.8 $G \cong A_{26}$.

Proof. By Lemma 3.7,

$$A_{26} \leq \overline{G} \leq Aut(A_{26}) \cong S_{26}.$$

If $\overline{G} \cong S_{26}$, then there exist an element $\overline{x}$ of $\overline{G}$ with that

$$\overline{x}^{\overline{G}} = 2^{19} \cdot 3^8 \cdot 5^6 \cdot 7^3 \cdot 11^2 \cdot 13^2 \cdot 17 \cdot 19$$

which contradicts Lemma 3.2(4).

So $\overline{G} \cong A_{26}$. Then we define the normal series $1 \leq K \leq G$ into the chief ones. We prove that $K = 1$. If $K \neq 1$, then order consideration, we have that $G/K \cong A_{26}$ and $|K| = 2$. It follows that $Z(G) = K$ and there is an element of order $2 \cdot 23$, a contradiction.

Therefore, $K = 1$ and $G \cong A_{26}$.

This completes the proof of the Lemma and also of the main theorem. ∎

4. Some applications and problem

Chen et al in [4] proved that the group A_{10} can be characterized by its order and two special conjugacy classes sizes. Then obviously, we also have the following result.

Corollary 4.1 *Let G be a finite group with trivial center. Assume that $N(G) = N(A_{26})$ and $|G| = |A_{26}|$. Then $G \cong A_{26}$.*

We know that the alternating group A_n with $n = 10, 16, 22, 26$, are characterized by $N(G)$. Then, by [2], [3], we have the following.

Corollary 4.2 *Let G be a finite group with trivial center. Assume that $N(G) = N(A_n)$ with $n \leq 26$. Then $G \cong A_n$.*

Related to **Thompson's conjecture**, we give the following problem.

Problem. *Are the alternating groups A_{p+3} of degree $p + 3$ where p is a prime, characterized by $N(G)$?*

Shi gave the following conjecture.

Conjecture [12] *Let G be a group and H a finite simple group. Then $G \cong H$ if and only if* (a) $\omega(G) = \omega(H)$ *and* (b) $|G| = |H|$.

Then, we have the following corollary.

Corollary 4.3 *Let G is a group and n an integer with $n \leq 26$. Then $G \cong A_n$ if and only if $\omega(G) = \omega(A_n)$ and $|G| = |A_n|$.*

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