ENERGY OF AN INTUITIONISTIC FUZZY GRAPH

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Abstract. In this paper, the concept of energy of fuzzy graph is extended to the energy of an intuitionistic fuzzy graph. We have defined the adjacency matrix of an intuitionistic fuzzy graph and the energy of an intuitionistic fuzzy graph is defined in terms of its adjacency matrix. These concepts are illustrated with real time example. The lower and upper bound for the energy of an intuitionistic fuzzy graph are also derived.

Keywords: fuzzy set, intuitionistic fuzzy set, fuzzy graph, energy of fuzzy graph.

1. Introduction

Fuzzy set was introduced by Zadeh [18] whose basic component is only a membership function. The generalization of Zadeh's fuzzy set, called intuitionistic fuzzy set was introduced by Atanassov [2] which is characterized by a membership function and a non-membership function. In Zadeh's fuzzy set, the sum of membership degree and a non-membership degree is equal to one. In Atanassov intuitionistic fuzzy set, the sum of membership degree and a non-membership degree does not exceed one.

The foundation for graph theory was laid in 1735 by Leonhard Euler when he solved the "Konigsberg bridges" problem. Many real life problems can be represented by graph. In computer science, graphs are used to represent networks of communications, data organization, computational devices, the flow of computation, etc. The link structure of a website could be represented by a directed graph in which the vertices are the web pages available at the website and a directed

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edge from page A to page B exists if and only if A contains a link to B [13]. A similar approach can be taken to problems in travel, biology, computer chip design and many other fields. Hence graph theory is widely used in solving real time problems. But when the system is large and complex it is difficult to extract the exact information about the system using the classical graph theory. In such cases fuzzy graph is used to analyze the system.

The first definition of fuzzy graphs was proposed by Kafmann [10] in 1973, from the Zadeh's fuzzy relations [18] [19] [20]. But Rosenfeld [14] introduced another elaborated definition including fuzzy vertex and fuzzy edges and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov [3]. I. Gutman [6] in 1978 introduced the concept of "graph energy" as the sum of the absolute values of the eigen values of the adjacency matrix of the graph. Certain bounds on energy are discussed in [4], [12] and [7]. Energy of different graphs including regular [8], non-regular [9], circulant [16] and random graphs [17] is also under study. Energy is defined for signed graphs in [5] and for weighted graphs in [15]. The energy of graph is extended to the energy of fuzzy graph in [1]. In this paper, we extend the concept of energy of fuzzy graph to the energy of an intuitionistic fuzzy graph. This paper is organized as follows. In Section 2, we give all the basic definitions related to fuzzy sets, intuitionistic fuzzy sets, energy of graph and energy of fuzzy graph. In Section 3, we define the energy of an intuitionistic fuzzy graph. The lower and upper bounds for the energy of an intuitionistic fuzzy graph are also derived. In Section 4, we illustrate these concepts by taking the website of http://www.pantechsolutions.net/ . In Section 5, we give the conclusion.

2. Preliminaries

2.1. Energy of graph

Definition 2.1. A graph G is a pair of set (V, E), denoted by $G = (V, E)$, where V is a set of vertices and E is a set of edges. Each edge in E is a pair of vertices in V. Each edge is associated with a set consisting of either one or two vertices called its endpoints.

Definition 2.2. A graph in which the edges are unordered vertex pair is called an undirected graph. A graph in which the edges are ordered vertex pair is called a directed graph. Hence if there is an edge from v_i to v_j in G then $(v_i, v_j) \in E$.

Definition 2.3. An edge whose endpoints are the same is called a loop. A graph without loops and parallel edges is called a simple graph. Two vertices that are connected by an edge are called adjacent. The adjacency matrix $A = [a_{ij}]$ for a graph $G = (V, E)$ is a matrix with n rows and n columns, $n = |V|$ and its entries defined by $a_{ij} = \begin{cases} 1, & \text{if } (v_i, v_j) \in E, \\ 0, & \text{otherwise.} \end{cases}$

Definition 2.4. The spectrum of a matrix is defined as a set of its eigenvalues. Let $G = (V, E)$ be a simple graph with n vertices and m edges. Let $A = [a_{ij}]$ be the adjacency matrix of G. As in a simple graph, there can be at most 1 edge between 2 vertices. So, the entries in A are either 0 or 1. The diagonal elements are zero since there are no loops. A is symmetric and so the spectrum of A is real. An eigenvalue of A are called eigenvalues of G[4] and the spectrum of A is called the spectrum of G. Energy of a simple graph $G = (V, E)$ with adjacency matrix A is defined as the sum of absolute values of eigenvalues of A. It is denoted by $E\,(G)$. That is,

$$E(G) = \sum_{i=1}^{n} |\lambda_i|,$$

where λ_i is an eigenvalue of A, $i = 1, 2..., n$. Since A is a symmetric matrix which diagonal elements are zero,

$$\sum_{i=1}^{n} \lambda_i = 0.$$

By comparing the coefficients of λ^{n-2} in the characteristic polynomial

$$\prod_{i=1}^{n} (\lambda - \lambda_i) = |A - \lambda I|\,,$$

we get

$$\sum_{1 \le i < j \le n} \lambda_i \lambda_j = -m.$$

Theorem 2.1. *Let G be a simple graph with $|V| = n$ vertices and m edges and A be the adjacency matrix of G then* $\sqrt{2m + n(n-1)|A|^{\frac{2}{n}}} \le E(G) \le \sqrt{2mn}$.

2.2. Fuzzy set

Definition 2.5. Let X be a nonempty set. A fuzzy set A in X is defined as $A = \{(x, \mu_A(x))\,/x \in X\}$, which is characterized by a membership function $\mu_A\,(x) : X \to [0, 1]$ and a fuzzy set satisfying the following condition $\mu_A(x) + \gamma_A(x) = 1$, where $\gamma_A(x) = 1 - \mu_A(x)$ is the nonmembership function (see [18]).

2.3. Intuitionistic fuzzy set

Definition 2.6. Let X be a nonempty set. An intuitionistic fuzzy set A in X is defined as $A = \{(x, \mu_A(x), \gamma_A(x))/x \in X\}$ which is characterized by a membership function $\mu_A(x) : X \to [0, 1]$ and the nonmembership function $\gamma_A(x) : X \to [0, 1]$ and satisfying the following condition

(i) $0 \le \mu_A(x) + \gamma_A(x) \le 1, \forall x \in X,$

(ii) $0 \le \mu_A(x), \gamma_A(x), \pi_A(x) \le 1, \forall x \in X,$

(iii) $\pi_A(x) = 1 - \mu_A(x) - \gamma_A(x),$

where $\pi_A(x)$ is called the intuitionistic fuzzy index of element x in A, the value denotes a measure of nondeterminacy. Obviously, if $\pi_A(x) = 0$, then the intuitionistic fuzzy set A is just a Zadeh's fuzzy set (see [2]).

2.4. Energy of fuzzy graph [1]

Definition 2.7. Let V be a nonempty set. A fuzzy subset of V is a function $\sigma : V \to [0,1]$. σ is called the membership function and $\sigma(v)$ is called the membership of v where $v \in V$. Let V_1 and V_2 be two nonempty sets and σ_1 and σ_2 be two fuzzy subsets of V_1 and V_2 respectively. Define a fuzzy subset μ of $V_1 \times V_2$ as $\mu(v_i, v_j) \le min\{\sigma_1(v_i), \sigma_2(v_j)\}$. Then, μ is called a fuzzy relation, from σ_1 to σ_2. Suppose $\sigma_1(x) = 1, \forall x \in V_1$ and $\sigma_2(y) = 1, \forall y \in V_2$. Then, μ is called a fuzzy relation from V_1 into V_2. $\mu(v_i, v_j)$ is interpreted as the strength of relation between v_i and v_j. Suppose $V_1 = V_2 = V$ and $\sigma_1 = \sigma_2 = \sigma$. Then, μ is called a fuzzy relation on σ. Suppose $V_1 = V_2 = V$ and $\sigma_1(x) = 1$, $\forall x \in V_1$, and $\sigma_2(y) = 1$, $\forall y \in V_2$. Then μ is called a fuzzy relation on V. From the above definitions, it follows that binary relations on crisp sets are particular cases of fuzzy relations. Let V be a nonempty set and σ be a fuzzy subset of V. Let μ be a fuzzy relation on σ. μ is said to be symmetric, if $\mu(v_i, v_j) = \mu(v_j, v_i)$ for $v_i, v_j \in V$. A fuzzy relation can also be expressed by a matrix called fuzzy relation matrix $M = [m_{ij}]$, where $m_{ij} = \mu(v_i, v_j)$.

A fuzzy graph $G = (V, \sigma, \mu)$ is a nonempty set V together with a pair of function (σ, μ) where σ is a fuzzy subset of V and μ is a fuzzy relation on σ. Let $M = [m_{ij}]$ be a fuzzy relation matrix defined on μ. This represents the strength of the relation between the vertices. Hence we have the following definitions.

Definition 2.8. [1] The adjacency matrix A of a fuzzy graph $G = (V, \sigma, \mu)$ is an $n \times n$ matrix where $n = |V|$ defined as $A = [a_{ij}]$, where $a_{ij} = \mu(v_i, v_j)$. Note that A becomes the usual adjacency matrix when all the nonzero membership values are 1, i.e., when the fuzzy graph becomes a crisp graph.

Definition 2.9. [1] Let $G = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. The eigenvalues of A are called eigenvalues of G. The spectrum of A is called the spectrum of G. It is denoted by Spec G.

Definition 2.10. [1] Let $G = (V, \sigma, \mu)$ be a fuzzy graph and A be its adjacency matrix. Energy of G is defined as the sum of the absolute values of the eigenvalues of A.

Theorem 2.2. [1] *Let $G = (V, \sigma, \mu)$ be a fuzzy graph with $|V| = n$ and $\mu^* = \{e_1, e_2, ..., e_m\}$. If $m_i = \mu(e_i)$ is the strength of the relation associated with the i^{th} edge, then*

$$\sqrt{2\sum_{i=1}^{m} m_i^2 + n(n-1)|A|^{\frac{2}{n}}} \le E(G) \le \sqrt{2\left(\sum_{i=1}^{m} m_i^2\right) n}.$$

3. Energy of an intuitionistic fuzzy graph

In this section, we define the energy of an intuitionistic fuzzy directed graph without loops. The link structure of a website could be represented by a directed intuitionistic fuzzy graph. The links are considered as vertices and the path between the links are considered as edges. The weightage of the each edge are considered as the number of visitors (membership value), the number of non visitors (non - membership value) and drop off (intuitionistic fuzzy index) among that link structure. The lower and upper bounds for the energy of an intuitionistic fuzzy graph are also obtained.

Definition 3.1. An intuitionistic fuzzy graph is defined as $G = (V, E, \mu, \gamma)$ where V is the set of vertices and E is the set of edges. μ is a fuzzy membership function defined on $V \times V$ and γ is a fuzzy non - membership function defined on $V \times V$. We denote $\mu(v_i, v_j)$ by μ_{ij} and $\gamma(v_i, v_j)$ by γ_{ij} such that (i) $0 \leq \mu_{ij} + \gamma_{ij} \leq 1$ (ii) $0 \leq \mu_{ij}, \gamma_{ij}, \pi_{ij} \leq 1$, where $\pi_{ij} = 1 - \mu_{ij} - \gamma_{ij}$. Hence $(V \times V, \mu, \gamma)$ is an intuitionistic fuzzy set.

Example 3.1.

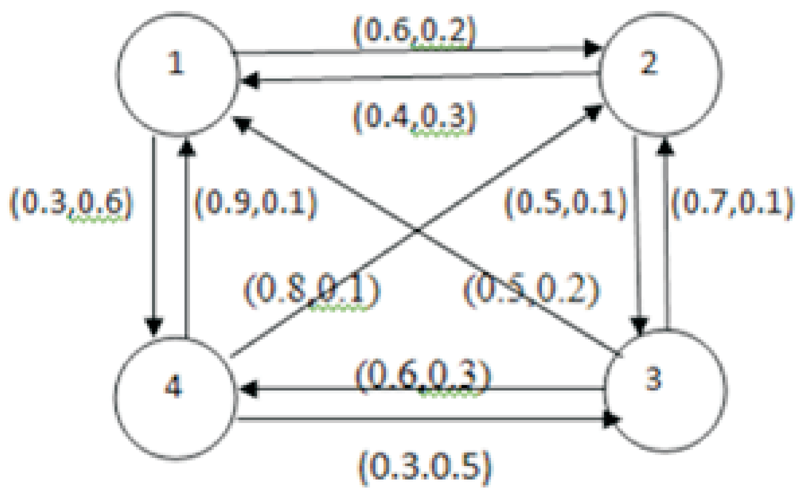

Figure 1: G1. An intuitionistic fuzzy graph

Definition 3.2. An intuitionistic fuzzy adjacency matrix of an intuitionistic fuzzy graph is defined as the adjacency matrix of the corresponding intuitionistic fuzzy graph. That is for an intuitionistic fuzzy graph $G = (V, E, \mu, \gamma)$, an intuitionistic fuzzy adjacency matrix is defined by $A(IG) = [a_{ij}]$ where $a_{ij} = (\mu_{ij}, \gamma_{ij})$. Note that μ_{ij} represents the strength of the relationship between v_i and v_j and γ_{ij} represents the strength of the non-relationship between v_i and v_j.

Example 3.2. For an intuitionistic fuzzy graph in Fig. 1, the adjacency matrix is

$$A(IG) = \begin{pmatrix} 0 & (0.6, 0.2) & 0 & (0.3, 0.6) \\ (0.4, 0.3) & 0 & (0.5, 0.1) & 0 \\ (0.5, 0.2) & (0.7, 0.1) & 0 & (0.6, 0.3) \\ (0.9, 0.1) & (0.8, 0.1) & (0.3, 0.5) & 0 \end{pmatrix}$$

Definition 3.3. The adjacency matrix of an intuitionistic fuzzy graph can be written as two matrices one containing the entries as membership values and the other containing the entries as non-membership values, i.e., $A(IG) = ((\mu_{ij}), (\gamma_{ij}))$, where

$$A(\mu_{ij}) = \begin{pmatrix} 0 & 0.6 & 0 & 0.3 \\ 0.4 & 0 & 0.5 & 0 \\ 0.5 & 0.7 & 0 & 0.6 \\ 0.9 & 0.8 & 0.3 & 0 \end{pmatrix} \text{ and } A(\gamma_{ij}) = \begin{pmatrix} 0 & 0.2 & 0 & 0.6 \\ 0.3 & 0 & 0.1 & 0 \\ 0.2 & 0.1 & 0 & 0.3 \\ 0.1 & 0.1 & 0.5 & 0 \end{pmatrix}$$

Definition 3.4. The eigenvalue of an intuitionistic fuzzy adjacency matrix $A(IG)$ is defined as (X, Y), where X is the set eigenvalues of $A(\mu_{ij})$ and Y is the set of eigenvalues of $A(\gamma_{ij})$.

Definition 3.5. The spectrum of an intuitionistic fuzzy adjacency matrix $A(IG)$ is the defined as (X, Y), where X is the set eigenvalues of $A(\mu_{ij})$ and Y is the set of eigenvalues of $A(\gamma_{ij})$.

Definition 3.6. The energy of an intuitionistic fuzzy graph $G = (V, E, \mu, \gamma)$ is defined as

$$\left(\sum_{\lambda_i \in X} |\lambda_i|, \sum_{\delta_i \in Y} |\delta_i| \right)$$

where $\sum_{\lambda_i \in X} |\lambda_i|$ is defined as an energy of the membership matrix denoted by $E(\mu_{ij}(G))$ and $\sum_{\delta_i \in Y} |\delta_i|$ is defined as an energy of the nonmembership matrix denoted by $E(\gamma_{ij}(G))$.

Example 3.3. For an intuitionistic fuzzy graph in Fig. 1,
$\text{Spec}(\mu_{ij}(G_1)) = \{1.2406, -0.7153, -0.2627 + 0.2332i, -0.2627 - 0.2332i\}$
$\text{Spec}(\gamma_{ij}(G_1)) = \{0.6441, -0.0148, -0.3146 + 0.1629i, -0.3146 - 0.1629i\}$
$E(\mu_{ij}(G_1)) = 1.2406 + 0.7153 + 0.3513 + 0.3513 = 2.6585$
$E(\gamma_{ij}(G_1)) = 0.6441 + 0.0148 + 0.3543 + 0.3543 = 1.3675$

Theorem 3.1. *Let G be an intuitionistic fuzzy directed graph (without loops) with $|V| = n$ and $|E| = m$ and $A(IG) = ((\mu_{ij}), (\gamma_{ij}))$ be an intuitionistic fuzzy adjacency matrix of G then*

(i) $\sqrt{2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}+n(n-1)|A|^{\frac{2}{n}}}\le E\left(\mu_{ij}(G)\right)\le\sqrt{2n\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}}$,

where $|A|$ is the determinant of $A(\mu_{ij})$, and

(ii) $\sqrt{2\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}+n(n-1)|B|^{\frac{2}{n}}}\le E\left(\gamma_{ij}(G)\right)\le\sqrt{2n\sum_{1\le i<j\le n}\gamma_{ij}\gamma_{ji}}$,

where $|B|$ is the determinant of $A(\gamma_{ij})$.

Proof. Upper bound. Apply Cauchy's Schwartz inequality to the n numbers $1,...,1$ and $|\lambda_1|,|\lambda_2|,...,|\lambda_n|$,

$$\sum_{i=1}^{n}|\lambda_i|\le\sqrt{n}\sqrt{\sum_{i=1}^{n}|\lambda_i|^2} \tag{1}$$

$$\left(\sum_{i=1}^{n}\lambda_i\right)^2=\sum_{i=1}^{n}|\lambda_i|^2+2\sum_{1\le i<j\le n}\lambda_i\lambda_j \tag{2}$$

By comparing the coefficients of λ^{n-2} in the characteristic polynomial

$$\prod_{i=1}^{n}(\lambda-\lambda_i)=|A-\lambda I|,$$

we get

$$\sum_{1\le i<j\le n}\lambda_i\lambda_j=-\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji} \tag{3}$$

Substituting (3) in (2), we get

$$\sum_{i=1}^{n}|\lambda_i|^2=2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji} \tag{4}$$

Substituting (4) in (1), we get

$$\sum_{i=1}^{n}|\lambda_i|\le\sqrt{n}\sqrt{2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}}=\sqrt{2n\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}}$$

$$E\left(\mu_{ij}(G)\right)\le\sqrt{2n\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}}$$

Lower bound

$$\begin{aligned}\left(E\left(\mu_{ij}\left(G\right)\right)\right)^2 &= \left(\sum_{i=1}^{n}|\lambda_i|\right)^2=\sum_{i=1}^{n}|\lambda_i|^2+2\sum_{1\le i<j\le n}|\lambda_i\lambda_j|\\ &= 2\sum_{1\le i<j\le n}\mu_{ij}\mu_{ji}+\frac{2n(n-1)}{2}AM\left\{|\lambda_i\lambda_j|\right\}\end{aligned}$$

$$
\begin{aligned}
AM\left\{\left|\lambda_i\lambda_j\right|\right\} &\geq GM\left\{\left|\lambda_i\lambda_j\right|\right\}, 1\leq i<j\leq n\\
E\left(\mu_{ij}\left(G\right)\right) &\geq \sqrt{2\sum_{1\leq i<j\leq n}\mu_{ij}\mu_{ji}+n\left(n-1\right)GM\left\{\left|\lambda_i\lambda_j\right|\right\}}\\
GM\left\{\left|\lambda_i\lambda_j\right|\right\} &= \left(\prod_{1\leq i<j\leq n}\left|\lambda_i\lambda_j\right|\right)^{\frac{2}{n(n-1)}} = \left(\prod_{i=1}^{n}\left|\lambda_i\right|^{n-1}\right)^{\frac{2}{n(n-1)}}\\
&= \left(\prod_{i=1}^{n}\left|\lambda_i\right|\right)^{\frac{2}{n}} = \left|A\right|^{\frac{2}{n}}\\
E\left(\mu_{ij}\left(G\right)\right) &\geq \sqrt{2\sum_{1\leq i<j\leq n}\mu_{ij}\mu_{ji}+n\left(n-1\right)\left|A\right|^{\frac{2}{n}}}
\end{aligned}
$$

Therefore,

$$\sqrt{2\sum_{1\leq i<j\leq n}\mu_{ij}\mu_{ji}+n\left(n-1\right)\left|A\right|^{\frac{2}{n}}}\leq E\left(\mu_{ij}\left(G\right)\right)\leq\sqrt{2n\sum_{1\leq i<j\leq n}\mu_{ij}\mu_{ji}}$$

Similarly, we can prove

$$\sqrt{2\sum_{1\leq i<j\leq n}\gamma_{ij}\gamma_{ji}+n\left(n-1\right)\left|B\right|^{\frac{2}{n}}}\leq E\left(\gamma_{ij}\left(G\right)\right)\leq\sqrt{2n\sum_{1\leq i<j\leq n}\gamma_{ij}\gamma_{ji}} \qquad \blacksquare$$

Example 3.4. (Illustration to Theorem 3.1) For an intuitionistic fuzzy graph in Fig. 1,

$E\left(\mu_{ij}\left(G_1\right)\right)=2.6585$ its Lower bound $=2.4599$ and Upper bound $=2.8844$ and $E\left(\gamma_{ij}\left(G_1\right)\right)=1.3675$ its Lower bound $=0.9878$ and Upper bound $=1.4967$.

4. Numerical examples

In this section, we explain the concept for the energy of an intuitionistic fuzzy graph and lower and upper bounds for the energy of an intuitionistic fuzzy graph through a real time example.

We have taken the website http://www.pantechsolutions.net/. This website is modeled as an intuitionistic fuzzy graph by considering the navigation of the customer. An intuitionistic fuzzy graph of this site for four different time periods is taken for each of these periods the energy of an intuitionistic fuzzy graph and its lower and upper bounds are calculated. The energy is also represented interms of bar diagram. We have taken the four links 1. microcontroller-boards, 2./log-in html, 3./ and 4./ project kits for our calculation.

Example 4.1. In the website http://www.pantechsolutions.net/ we consider four links 1. microcontroller-boards, 2./log-in html, 3./ and 4./ project kits for the period July 16, 2013 to August 15, 2013.

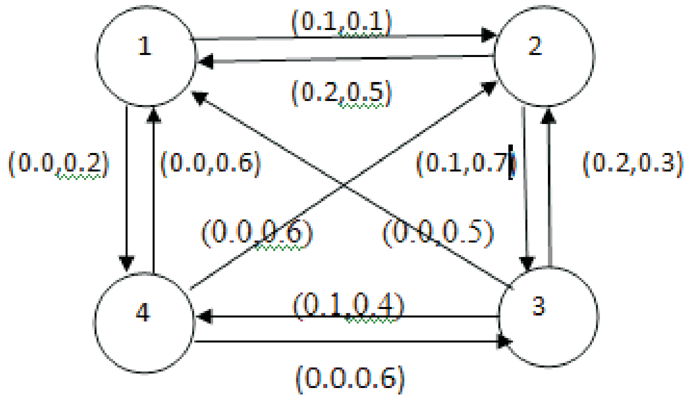

Figure 2: G2. An intuitionistic fuzzy graph

For an intuitionistic fuzzy graph in Fig. 2,

Spec($\mu_{ij}(G_2)$) $= \{-0.2000, 0.0000, 0.2000, 0.0000\}$
Spec($\gamma_{ij}(G_2)$) $= \{0.9927, -0.1841, -0.4043 + 0.2306i, -0.4043 - 0.2306i\}$
$E(\mu_{ij}(G_2)) = 0.2000 + 0.0000 + 0.2000 + 0.0000 = 0.4$ its Lower bound $= 0.2828$ and Upper bound $= 0.5657$
$E(\gamma_{ij}(G_2)) = 0.9927+0.1841+0.4655+0.4655 = 2.1078$ its Lower bound $= 1.9047$ and Upper bound $= 2.2271$.

Example 4.2. In the same website (mentioned above in example 4.1.), we consider the same four links for the period August 16, 2013 to September 15, 2013.

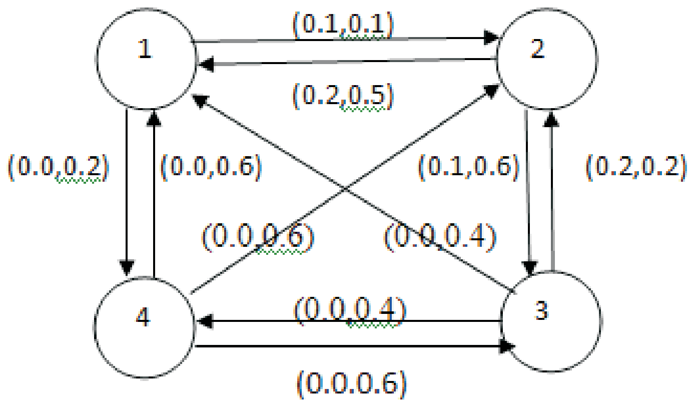

Figure 3: G3. An intuitionistic fuzzy graph

For an intuitionistic fuzzy graph in Fig.3,

$\mathrm{Spec}(\mu_{ij}(G_3)) = \{-0.2000, 0.0000, 0.2000, 0.0000\}$
$\mathrm{Spec}(\gamma_{ij}(G_3)) = \{0.8964, -0.1893, -0.3536 + 0.2114i, -0.3536 - 0.2114i\}$
$E(\mu_{ij}(G_3)) = 0.2000 + 0.0000 + 0.2000 + 0.0000 = 0.4$ its Lower bound $= 0.2828$ and Upper bound $= 0.5657$
$E(\gamma_{ij}(G_3)) = 0.8964+0.1893+0.4120+0.4120 = 1.9096$ its Lower bound $= 2.0000$ and Upper bound $= 1.7425$.

Example 4.3. In the same website (mentioned above in example 4.1.), we consider the same four links for the period September 16, 2013 to October 15, 2013.

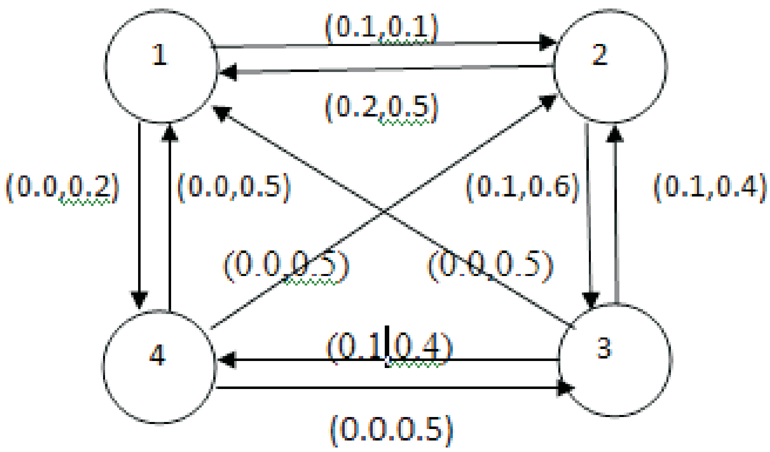

Figure 4: G4. An intuitionistic fuzzy graph

For an intuitionistic fuzzy graph in Fig.4,

$\mathrm{Spec}(\mu_{ij}(G_4)) = \{-0.1732, 0.1732, 0.0000, 0.0000\}$
$\mathrm{Spec}(\gamma_{ij}(G_4)) = \{0.9418, -0.2000, -0.3709 + 0.1053i, -0.3709 - 0.1053i\}$
$E(\mu_{ij}(G_4)) = 0.1732+0.1732+0.0000+0.0000 = 0.3464$ its Lower bound $= 0.2449$ and Upper bound $= 0.4899$
$E(\gamma_{ij}(G_4)) = 0.9418+0.2000+0.3856+0.3856 = 1.913$ its Lower bound $= 1.7855$ and Upper bound $= 2.1726$.

Example 4.4. In the same website (mentioned above in Example 4.1.), we consider the same four links for the period of October 16, 2013 to November 15, 2013.

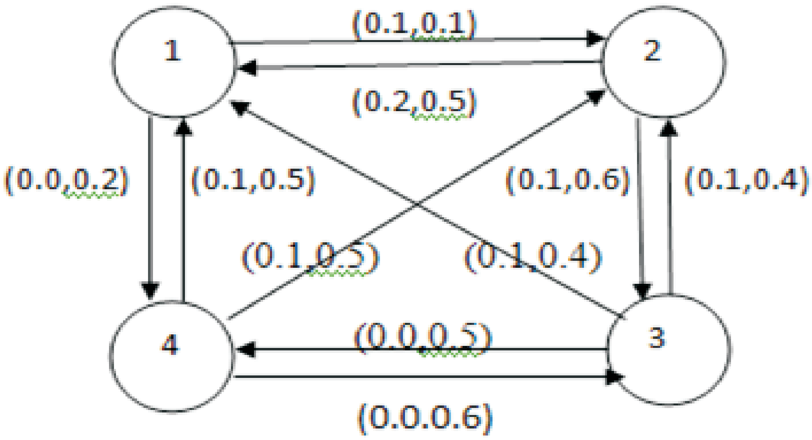

Figure 5: G5. An intuitionistic fuzzy graph

For an intuitionistic fuzzy graph in Fig. 5,

$\mathrm{Spec}(\mu_{ij}(G_5)) = \{0.0000, 0.1879, -0.1532, -0.0347\}$
$\mathrm{Spec}(\gamma_{ij}(G_5)) = \{0.9940, -0.1302, -0.4000, -0.4638\}$
$E(\mu_{ij}(G_5)) = 0.0000+0.1879+0.1532+0.0347 = 0.3758$ its Lower bound $= 0.2449$ and Upper bound $= 0.4899$
$E(\gamma_{ij}(G_5)) = 0.9940+0.1302+0.4000+0.4638 = 1.988$ its Lower bound $= 1.7997$ and Upper bound $= 2.3495$.

The following bar diagrams represents the energy of four links for the above four periods corresponding the membership and non-membership values respectively.

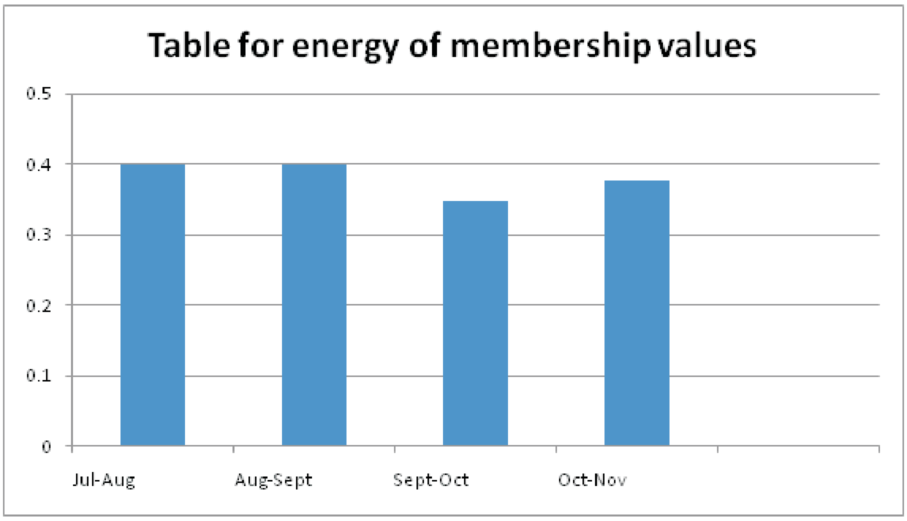

Table 1: Table for energy of membership values

From the above bar diagram the energy for the period July 16 to August 15 and August 16 to September 15 are high and same comparing the other period.

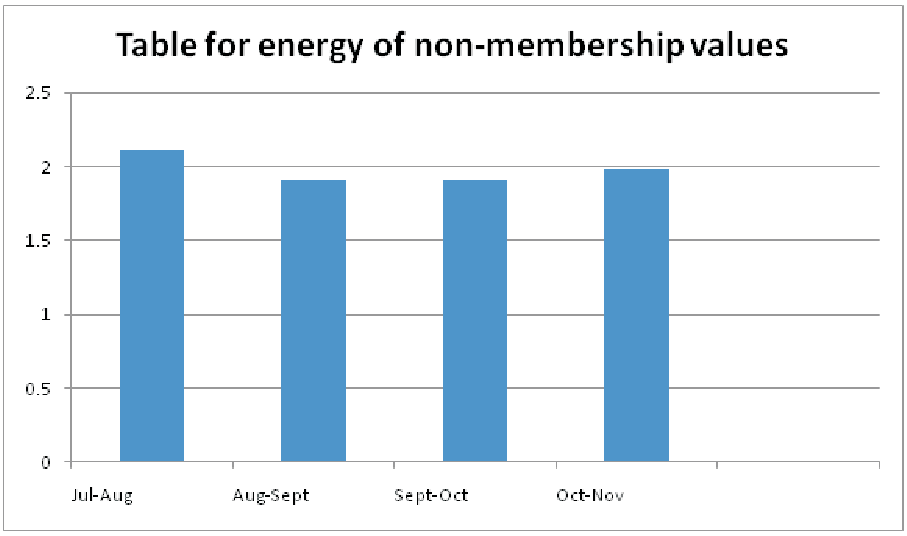

Table 2: Table for energy of non membership values

From the above bar diagram the energy for the period July 16 to August 15 is high comparing the other period.

5. Conclusion

In this paper, we defined the energy of an intuitionistic fuzzy graph interms of its adjacency matrix. The lower and upper bounds for the energy of an intuitionistic fuzzy graph are derived. These concepts are illustrated with real time example.

Acknowledgments. The authors thank the managements for their constant support towards the successful completion of this work.

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Accepted: 20.03.2014