GLOBAL EXPONENTIAL STABILITY OF IMPULSIVE HYBRID DYNAMICAL SYSTEMS WITH ANY TIME DELAY[1]

Xingjie Wu[2]
Yang Liu
Zongmin Qiao

School of Mathematics and Statistics
Hefei Normal University
Hefei Anhui 230601
P.R. China

Abstract. This present paper addresses global exponential stability for a class of more general linear impulsive hybrid dynamical systems with any time delay. Combined Lyapunov function methods with the Razumikhin technique, several criteria on global exponential stability are derived, which are substantially extension and generalization of the corresponding results in recent literature. Subsequently, two application examples and its numerical simulations demonstrate that the obtained stability criteria are practical and effective.

Keywords: Impulsive dynamics, Any time delay, Hybrid systems, Global exponential stability, Razumikhin technique.

1. Introduction and notation

In past few decades, the stability problems of dynamical systems with impulses and delays have attracted a great deal of attention from a variety of applications in science and engineering, see [1]-[7] and references therein. In particular, special attention has been focused on the exponential stability of such dynamical systems because it has played an important role in practical applications of dynamical systems [8]-[10].

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[2]Corresponding author. E-mail: wuxingjie06@126.com.

At the same time, the impulsive hybrid dynamical system is regarded as a special but important of impulsive differential systems with variable structure, and it has recently emerged as a new challenging area of research due to its potential applications in various fields of engineering areas, including control technology, industrial robotics, and communication engineering, etc [11]-[17]. There are several research works appeared in the literature on exponential stability of impulsive hybrid dynamical systems with time delay [18]-[21]. For example, Liu and Shen [19] presented some criteria on uniform stability and uniform asymptotic stability of invariant sets for impulsive hybrid dynamical systems with time delay. In [20], Li, Ma, and Feng proved some general criteria on global exponential stability for nonlinear impulsive and switching delayed dynamical systems. However, there is a restrictive condition that the time delay is less than the length of all the impulsive intervals, so they are generally inapplicable in some practical applications. In addition, Zhang and Sun [21] only given some results related to local uniformly stability for linear impulsive hybrid dynamical systems with time delay.

The main objective of this paper is further to investigate global exponential stability for a class of more general linear impulsive hybrid dynamical systems with any time delays. Based on the Lyapunov function methods combined with the Razumikhin techniques, several criteria on global exponential stability are derived analytically, which are nature extension and generalization of the corresponding results existing in the literature. Compared with some existing works, a distinctive feature of this work is to address global exponential stability for linear or nonlinear impulsive hybrid dynamical systems with any time delays. Furthermore, our results show that impulses do contribute to global exponential stability of dynamical systems with any time delays even if it may be chaotic or unstable itself, which can be usually used as an effective control strategy to stabilize the underlying delayed dynamical systems in some practical applications.

Let $R = (-\infty, +\infty)$ be the set of real numbers, $R^+ = [0, +\infty)$ be the set of nonnegative real numbers, and $N = \{0, 1, 2, \ldots\}$ be the set of nonnegative integer numbers. For the vector $u \in R^n$, $u^\top$ denotes its transpose. The norm of the vector u is defined as $\|u\| = \sqrt{u^\top u}$. $R^{n\times n}$ stands for $n \times n$ the set of real matrices.

Consider the following impulsive delayed linear dynamical hybrid systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-\tau(t)) + M_k x_k, & t \in [t_k, t_{k+1}), \\ x(t) = C_k x(t^-), & t = t_k,\ k \in N, \end{cases} \tag{1.1}$$

where $t \geq t_0$, $\varphi \in PC([-\tau, 0], R^n)$, $x(t) \in R^n$, A, B, C_k, $M_k \in R^{n\times n}, C_0 = I$, $M_k x_k$ is hybrid term.

The time sequence $\{t_k\}_{k=1}^{+\infty}$ satisfy $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = +\infty$, the time delay $0 \leq \tau(t) \leq \tau < +\infty$, $x(t^+) = \lim_{s\to 0^+} x(t+s)$ and $x(t^-) = \lim_{s\to 0^-} x(t+s)$.

Let $PC([-\tau, 0], R^n) = \{\phi : [-\tau, 0] \to R^n,\ \phi(t)$ is continuous everywhere except a finite number of points $\hat{t}$ at which $\phi(\hat{t}^+)$ and $\phi(\hat{t}^-)$ exist and $\phi(\hat{t}^+) = \phi(\hat{t})\}$.

For $\psi \in PC([-\tau, 0], R^n)$, the norm of ψ is defined by

$$\|\psi\|_\tau = \sup_{-\tau \le s \le 0} \|\psi(s)\|.$$

$x_t,\ x_{t^-} \in PC([-\tau, 0], R^n)$ are defined by $x_t(s) = x(t+s)$ and $x_{t^-}(s) = x(t^- + s)$ for $s \in [-\tau, 0]$, respectively.

We always assume that system (1.1) has a unique solution with respect to initial conditions. Denote by $x(t) = x(t, t_0, \varphi)$ the solution of system (1.1) such that $x_{t_0} = \varphi$. We further assume that all the solutions $x(t)$ of system (1.1) are continuous except at $t_k,\ k \in N$, at which $x(t)$ is right continuous. Let $x_k = x(t_k)$, $\Delta t_k = t_{k+1} - t_k,\ k \in N$. Obviously, $x(t) = 0$ is a solution of system (1.1), which we call the zero solution.

Definition 1.1. The zero solution of system (1.1) is said to be globally exponentially stable if there exist two constants $\lambda > 0$ and $K \ge 1$ such that for any initial value $x_{t_0} = \varphi$,

$$\|x(t, t_0, \varphi)\| \le K \|\varphi\|_\tau e^{-\lambda(t - t_0)}, \quad t \ge t_0,$$

where $(t_0, \varphi) \in R^+ \times PC([-\tau, 0], R^n)$.

Definition 1.2. *Function $V : R^+ \times R^n \to R^+$ is said to belong to the class ν_0 if*

(a) *V is continuous in each of the sets $[t_k, t_{k+1}) \times R^n$, and for each $x \in R^n$, $t \in [t_k, t_{k+1}), k \in N$, $\lim_{(t,y) \to (t_k^-, x)} V(t, y) = V(t_k^-, x)$ exists, and*

(b) *$V(t, x)$ is locally Lipschitzian in all $x \in R^n$, and for all $t \ge t_0$, $V(t, 0) \equiv 0$.*

Definition 1.3. Given a function $V : R^+ \times R^n \to R^+$, the upper right-hand derivative of V with respect to system (1.1) is defined by

$$D^+ V(t, x(t)) = \lim_{\delta \to 0^+} \sup \frac{1}{\delta} \Big(V(t + \delta, x(t + \delta)) - V(t, x(t)) \Big).$$

2. Main results

In this section, by combining the Lyapunov function and Razumikhin technique, we shall present several criteria on global exponential stability for the delayed linear hybrid dynamical system (1.1).

Theorem 2.1. *Let the $n \times n$ matrix P be symmetric and positive definite, λ_3 be the largest eigenvalue of $P^{-1}(A^T P + PA + 2PP)$, λ_4 be the largest eigenvalue of $P^{-1} B^T B$, λ_{M_k} be the largest eigenvalue of $P^{-1} C_k^T M_k^T M_k C_k$, λ_{C_k} be the largest eigenvalue of $P^{-1} C_k^T P C_k$, $\lambda_5 = \sup_{k \in N} \lambda_{M_k}$, $\lambda_6 = \sup_{k \in N} \lambda_{C_k}$ and $0 < \lambda_6 < 1$. Assume that there exist two constants $\lambda > 0$ and $\sigma > 0$, such that for all $k \in N$, the following conditions are satisfied:*

(i) $F(\lambda) = \sigma - \lambda - \Big(\lambda_3 + \dfrac{\lambda_4}{\lambda_6} e^{\lambda\tau} + \dfrac{\lambda_5}{\lambda_6}\Big) \geq 0$;

(ii) $\ln \lambda_6 < -(\sigma + \lambda)(t_{k+1} - t_k)$.

Then, for any delay $0 \leq \tau(t) \leq \tau < +\infty$, *the zero solution of the linear impulsive delayed hybrid dynamical system* (1.1) *is globally exponentially stable with convergence rate* $\dfrac{\lambda}{2}$.

Proof. Let $x(t) = x(t, t_0, \varphi)$ be any solution of the linear impulsive delayed hybrid dynamical system (1.1) with $x_{t_0} = \varphi$.

We construct a Lyapunov function as follow:

$$V(t, x(t)) = x^T(t) P x(t). \tag{2.1}$$

Let $\lambda_1 > 0$ and $\lambda_2 > 0$ are the smallest and the largest eigenvalues of P respectively, so we have

$$\lambda_1 \|x(t)\|^2 \leq V(t, x(t)) \leq \lambda_2 \|x(t)\|^2. \tag{2.2}$$

We shall prove that

$$V(t, x(t)) \leq \lambda_2 K \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}, \quad t \in [t_k, t_{k+1}),\ k \in N. \tag{2.3}$$

For $t \in [t_k, t_{k+1}),\ k \in N$, we calculate the upper right derivative of $V(t, x(t))$ along the solution of system (1.1) and have

$$\begin{aligned} D^+V(t, x(t)) &= x^T(t)(A^T P + PA)x(t) \\ &\quad + 2x^T(t - \tau(t)) B^T P x(t) + 2x_k M_k^T P x(t) \\ &\leq x^T(t)(A^T P + PA + 2PP)x(t) \\ &\quad + x^T(t - \tau(t)) B^T B x(t - \tau(t)) + x_k^T M_k^T M_k x_k \\ &\leq \lambda_3 V(t, x(t)) + \lambda_4 V(t - \tau(t), x(t - \tau(t))) \\ &\quad + x_k^T M_k^T M_k x_k. \end{aligned} \tag{2.4}$$

From condition (ii), we can get

$$-\ln \lambda_6 + \lambda\tau - (\sigma + \lambda)(t_{k+1} - t_k) > 0. \tag{2.5}$$

From (2.5), we can choose $K \geq 1$, such that

$$1 < e^{(\sigma+\lambda)(t_1 - t_0)} \leq K \leq -\ln \lambda_6 e^{\lambda\tau - (\sigma+\lambda)(t_1 - t_0)} e^{(\sigma+\lambda)(t_1 - t_0)}. \tag{2.6}$$

Then

$$\|\varphi\|_\tau^2 < \|\varphi\|_\tau^2 e^{\sigma(t_1 - t_0)} \leq K \|\varphi\|_\tau^2 e^{-\lambda(t_1 - t_0)}. \tag{2.7}$$

We first prove

$$V(t, x(t)) \leq \lambda_2 K \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}, \quad t \in [t_0, t_1), \tag{2.8}$$

To do this, we only need to prove that

$$V(t,x(t)) \le \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)}, \quad t\in[t_0,t_1). \tag{2.9}$$

If (2.9) is not true, from (2.2) and (2.8), then there exists some $\bar{t}\in(t_0,t_1)$ such that

$$\begin{aligned} V(\bar{t},x(\bar{t})) &> \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)} \ge \lambda_2\|\varphi\|_\tau^2 e^{\sigma(t_1-t_0)} \\ &> \lambda_2\|\varphi\|_\tau^2 \ge V(t_0+s,x(t_0+s)),\ s\in[-\tau,0], \end{aligned}$$

which implies that there exists some $t^*\in(t_0,\bar{t})$ such that

$$V(t^*,x(t^*)) = \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)}, \tag{2.10}$$

and

$$V(t,x(t)) \le V(t^*,x(t^*)), \quad t\in[t_0-\tau,t^*], \tag{2.11}$$

then there exists $t^{**}\in[t_0,t^*)$ such that

$$V(t^{**},x(t^{**})) = \lambda_2\|\varphi\|_\tau^2, \tag{2.12}$$

and

$$V(t^{**},x(t^{**})) \le V(t,x(t)), \quad t\in[t^{**},t^*]. \tag{2.13}$$

Hence, for any $s\in[-\tau,0]$, we have

$$\begin{aligned} V(t+s,x(t+s)) &\le \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t_1-t_0)} \\ &\le \frac{\lambda_2}{\lambda_6} e^{\lambda\tau-(\sigma+\lambda)(t_1-t_0)} e^{(\sigma+\lambda)(t_1-t_0)}\|\varphi\|_\tau^2 \\ &\le \frac{e^{\lambda\tau}}{\lambda_6} V(t^{**},x(t^{**})) \\ &\le \frac{e^{\lambda\tau}}{\lambda_6} V(t,x(t)), \quad t\in[t^{**},t^*], \end{aligned} \tag{2.14}$$

From (2.3), (2.12) and (2.13), we can get

$$\begin{aligned} x_0^T M_0^T M_0 x_0 &= x_0^T M_0^T M_0 P^{-1} P x_0 \le \lambda_5 V(t_0,x(t_0)) \\ &\le \lambda_5\lambda_2\|\varphi\|_\tau^2 \le \lambda_5 V(t,x(t)),\ t\in[t^{**},t^*], \end{aligned} \tag{2.15}$$

Thus, from condition (i), (2.4), (2.14) and (2.15), we get

$$\begin{aligned} D^+V(t,x(t)) &\le \Big(\lambda_3 + \frac{\lambda_4}{\lambda_6}e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\Big) V(t,x(t)) \\ &\le (\sigma-\lambda)V(t,x(t)), \quad t\in[t^{**},t^*]. \end{aligned} \tag{2.16}$$

From (2.7), (2.10), (2.11), (2.12) and (2.13) that

$$\begin{aligned} V(t^*, x(t^*)) &\leq V(t^{**}, x(t^{**}))e^{(\sigma-\lambda)(t^*-t^{**})} \\ &< \lambda_2\|\varphi\|_\tau^2 e^{\sigma(t_1-t_0)} \leq V(t^*, x(t^*)), \end{aligned} \tag{2.17}$$

which is a contradiction. Hence (2.8) holds and then (2.3) is true for $k = 0$.

Suppose (2.3) holds for $k = 0, 1, 2, \ldots, m$ ($m \in N, m \geq 0$), i.e.,

$$V(t, x(t)) \leq \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t-t_0)},\ t \in [t_k, t_{k+1}),\ k = 0, \ldots, m. \tag{2.18}$$

Next, we shall prove that (2.3) holds for $k = m + 1$, i.e.,

$$V(t, x(t)) \leq \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}, \quad t \in [t_{m+1}, t_{m+2}). \tag{2.19}$$

If (2.19) is not true, suppose

$$\bar{t} = \inf\{t \in [t_{m+1}, t_{m+2}) | V(t, x(t)) > \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t-t_0)}\}.$$

From condition (ii) and (2.18), we have

$$\begin{aligned} V(t_{m+1}, x(t_{m+1})) &= x^T(t_{m+1}^-)C_{m+1}^T P C_{m+1} x(t_{m+1}^-) \\ &\leq \lambda_6 V(t_{m+1}^-, x(t_{m+1}^-)) \\ &\leq \lambda_6\lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(t_{m+1}-t_0)} \\ &< \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}, \end{aligned} \tag{2.20}$$

and so $\bar{t} \neq t_{m+1}$. From the continuity of $V(t, x(t))$ in $[t_{m+1}, t_{m+2})$, we have

$$V(\bar{t}, x(\bar{t})) = \lambda_2 K\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)},\ t \in [t_{m+1}, \bar{t}] \tag{2.21}$$

By (2.20), we know that there exists some $t^* \in (t_{m+1}, \bar{t})$ such that

$$V(t^*, x(t^*)) = \lambda_6\lambda_2 e^{\lambda(t_{m+2}-t_{m+1})} K\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}, \tag{2.22}$$

and

$$V(t^*, x(t^*)) \leq V(t, x(t)) \leq V(\bar{t}, x(\bar{t})), \quad t \in [t^*, \bar{t}]. \tag{2.23}$$

From (2.19), (2.20), (2.22) and (2.23), we can obtain

$$\begin{aligned} x_{m+1}^T M_{m+1}^T M_{m+1} x_{m+1} &= x^{-T}_{m+1} C_{m+1}^T M_{m+1}^T M_{m+1} C_{m+1} x_{m+1}^- \\ &\leq \lambda_5 V(t_{m+1}^-, x(t_{m+1}^-)) \\ &= \lambda_2 e^{\lambda(t_{m+2}-t_{m+1})} K\|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} \\ &= \frac{\lambda_5}{\lambda_6} V(t^*, x(t^*)) \\ &\leq \frac{\lambda_5}{\lambda_6} V(t, x(t)), \quad t \in [t^*, \bar{t}] \end{aligned} \tag{2.24}$$

On the other hand, for any $t \in [t^*, \bar{t}]$, $s \in [-\tau, 0]$, then either $t+s \in [t_0 - \tau, t_{m+1})$ or $t+s \in [t_{m+1}, \bar{t}]$. Two cases will be discussed as follows:

If $t+s \in [t_0 - \tau, t_{m+1})$, from (2.18), we have

$$\begin{aligned} V(t+s, x(t+s)) &\leq \lambda_2 K \|\varphi\|_\tau^2 e^{-\lambda(t-t_0)} e^{-\lambda s} \\ &\leq \lambda_2 e^{\lambda\tau} e^{\lambda(t_{m+2}-t_{m+1})} K \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}. \end{aligned} \tag{2.25}$$

While, if $t+s \in [t_{m+1}, \bar{t}]$, from (2.21), then

$$\begin{aligned} V(t+s, x(t+s)) &\leq \lambda_2 K \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} \\ &\leq \lambda_2 e^{\lambda\tau} e^{\lambda(t_{m+2}-t_{m+1})} K \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)}. \end{aligned} \tag{2.26}$$

Form (2.25) and (2.26), in any case, we have for any $s \in [-\tau, 0]$,

$$\begin{aligned} V(t+s, x(t+s)) &\leq \frac{e^{\lambda\tau}}{\lambda_6} V(t^*, x(t^*)) \\ &\leq \frac{e^{\lambda\tau}}{\lambda_6} V(t, x(t)), \quad t \in [t^*, \bar{t}]. \end{aligned} \tag{2.27}$$

Finally, from (i), (2.4), (2.24) and (2.27), we have

$$D^+V(t, x(t)) \leq \Big(\lambda_3 + \frac{\lambda_4}{\lambda_6} e^{\lambda\tau} + \frac{\lambda_5}{\lambda_6}\Big) V(t, x(t)) \leq (\sigma - \lambda) V(t, x(t)).$$

Thus, in view of condition (ii), we have

$$\begin{aligned} V(\bar{t}, x(\bar{t})) &\leq V(t^*, x(t^*)) e^{(\sigma-\lambda)(\bar{t}-t^*)} \\ &< \lambda_2 e^{-(\sigma+\lambda)(t_{m+2}-t_{m+1})} e^{\lambda(t_{m+2}-t_{m+1})} K \|\varphi\|_\tau^2 e^{-\lambda(\bar{t}-t_0)} e^{(\sigma-\lambda)(\bar{t}-t^*)} \\ &< V(\bar{t}, x(\bar{t})), \end{aligned} \tag{2.28}$$

which is a contradiction. This implies the assumption is not true, and hence (2.3) holds for $k = m+1$. Therefore, we can obtain that (2.3) holds for any $k \in N$.

By (2.2) and (2.3), we have

$$\|x(t)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1} K} \|\varphi\|_\tau e^{-\frac{\lambda}{2}(t-t_0)}, \ t \geq t_0,$$

which implies that the zero solution of the linear impulsive delayed hybrid dynamical system (1.1) is globally exponentially stable with convergence rate $\frac{\lambda}{2}$ for any time delay $0 \leq \tau(t) \leq \tau < +\infty$.

Then, we complete the proof of Theorem 2.1. ∎

Corollary 2.1. *Let* λ_3, λ_4, λ_5, λ_6 *and condition* (iii) *be is precisely the same as that of Theorem* 2.1, *and assume that for all* $k \in N$,

(iii) $\Big(\lambda_3 + \dfrac{\lambda_4+\lambda_5}{\lambda_6}\Big)(t_{k+1}-t_k) < -\ln\lambda_6.$

Then, for any delay $0 \le \tau(t) \le \tau < +\infty$, *the zero solution of the linear impulsive delayed hybrid dynamical system* (1.1) *is globally exponentially stable.*

Consider the following impulsive delayed nonlinear dynamical hybrid systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + f(t, x(t), x(t-\tau(t))) + M_k x_k, & t \in [t_k, t_{k+1}), \\ x(t) = C_k x(t^-), & t = t_k,\ k \in N, \\ x_{t_0} = \varphi, \end{cases} \tag{2.29}$$

where $f : R^+ \times R^n \times R^n \to R^n$ is a continuously vector-valued function, $f(t,0,0) = 0$, which satisfies the following condition

(**H**): $\|f(t,x(t),x(t-\tau(t)))\|^2 \le l_1\|x(t)\|^2 + l_2\|x(t-\tau(t))\|^2$, where l_1 and l_2 are positive numbers.

In general, the condition (**H**) is very mild, since some general dissipative dynamical systems, such as neural networks, coupled chaos oscillators, and networks of multi-agent, etc., can be included in condition (**H**).

Theorem 2.2. *Let the* $n \times n$ *matrix* P *be symmetric and positive definite,* λ_3 *be the largest eigenvalue of* $P^{-1}(A^TP + PA + 2PP + l_1)$, λ_4 *be the largest eigenvalue of* l_2P^{-1}, λ_{M_k} *be the largest eigenvalue of* $P^{-1}C_k^TM_k^TM_kC_k$, λ_{C_k} *be the largest eigenvalue of* $P^{-1}C_k^TPC_k$, $\lambda_5 = \sup_{k\in N}\lambda_{M_k}$, $\lambda_6 = \sup_{k\in N}\lambda_{C_k}$ *and* $0 < \lambda_6 < 1$. *Assume both conditions* (i) *and* (ii) *are satisfied in Theorem* 2.1.

Then, for any time delay $0 \le \tau(t) \le \tau < +\infty$, *the zero solution of the nonlinear impulsive delayed dynamical hybrid system* (2.29) *is globally exponentially stable with convergence rate* $\dfrac{\lambda}{2}$.

Proof. Similarly to the proof of Theorem 2.1, by the Lyapunov function (2.1), for $t \in [t_k, t_{k+1})$, we calculate the upper right derivative of $V(t,x(t))$ along the solution of system (2.29) and have

$$\begin{aligned} D^+V(t,x(t)) &= x^T(t)(A^TP + PA)x(t) \\ &\quad + 2x^T(t)Pf(t,x(t),x(t-\tau(t))) + 2M_kx_kx(t) \\ &\le x^T(t)(A^TP + PA + PP + l_1P^{-1}P)x(t) \\ &\quad + l_2x^Tx(t-\tau(t))x(t-\tau(t)) + x_k^TM_k^TM_kx_k \\ &\le \lambda_3V(t,x(t)) + \lambda_4V(t-\tau(t), x(t-\tau(t))) + x_k^TM_k^TM_kx_k. \end{aligned} \tag{2.30}$$

The rest of the proof is precisely the same as Theorem 2.1, so we omit it. Theorem 2.2 is proven. ∎

Corollary 2.2. *Let λ_3, λ_4, λ_5 and λ_6 be is precisely the same as that of Theorem 2.2, and assume that for all $k \in N$,*

$$\text{(iv)} \quad \Big(\lambda_3 + \frac{\lambda_4 + \lambda_5}{\lambda_6}\Big)(t_{k+1} - t_k) < -\ln \lambda_6.$$

Then, for any delay $0 \le \tau(t) \le \tau < +\infty$, the zero solution of the nonlinear impulsive delayed hybrid dynamical system (2.29) *is globally exponentially stable.*

Remark 2.1. Obviously, Corollary 2.1 has extended the Theorem 1 of [21] concerning the local uniform stability to the global exponential stability for the linear impulsive delayed hybrid dynamical system (1.1) under the same conditions. Therefore, Theorem 2.1 is an important improvement and generalization of the main results in [21]. In addition, it is interested that if the linear impulsive delayed hybrid dynamical systems are local uniform stable, then it must be globally exponentially stable.

Remark 2.2. Theorem 2.1, 2.2 and Corollary 2.1, 2.2 are valid for any time delay $0 \le \tau(t) \le \tau < +\infty$. Therefore, our results are more practically applicable than those in the literature since the restrictive condition that the time delays are less than the length of all the impulsive intervals is actually removed here (see for example: Theorem 3.1 of [20]). Furthermore, our results show that impulse and hybrid term do contribute to the global exponential stability of linear or nonlinear impulsive delayed hybrid dynamical systems even if the corresponding systems without impulses are chaotic or unstable.

Remark 2.3. Let $h(\lambda) = (-\ln \lambda_6) / \left(2\lambda + \lambda_3 + \dfrac{\lambda_4}{\lambda_6} e^{\lambda\tau} + \dfrac{\lambda_5}{\lambda_6}\right)$, in Theorem 2.1, the satisfaction of both (i) and (ii) is equivalent to $\Delta t_k < h(\lambda)$, if we require that the exponential convergence rate of system (1.1) or (2.29) is larger than or equal to any given $\dfrac{\lambda^*}{2} > 0$, we can choose any suitable $\Delta t_k < \triangle t_k^*$, such that system (1.1) or (2.29) is globally exponentially stable with exponential convergence rate $\dfrac{\lambda}{2} \ge \dfrac{\lambda^*}{2} > 0$, where $\Delta t_k^* = h(\lambda^*)$.

3. Application examples

In this section, we shall discuss the applications of the above theoretic criteria. Two examples and their simulations are given to show that our main results are practical.

Example 3.1. Consider the following linear impulsive delayed hybrid dynamical systems:

$$
(3.1)\quad \begin{cases} \dot{x}(t) = \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{4} & 5 \end{pmatrix} x(t) + \begin{pmatrix} 2 & 2 \\ \frac{1}{3} & 6 \end{pmatrix} x(t-\tau(t)) + \begin{pmatrix} \frac{1}{k+1} & 0 \\ 0 & \frac{1}{k+1} \end{pmatrix} x_k, \\ \qquad\qquad t \in [t_k, t_{k+1}), \\ x(t) = \begin{pmatrix} \frac{3k+1}{k^2+1} & 0 \\ 0 & \frac{4k+1}{k^2+1} \end{pmatrix} x(t^-), \qquad t = t_k,\ k \in N, \end{cases}
$$

where $x(t) = (x_1(t), x_2(t))^T \in R^2$.

Let $P = \begin{pmatrix} 9 & 0 \\ 0 & 12 \end{pmatrix}$, then $\lambda_3 = 34.0371$, $\lambda_4 = 3.4449$, $\lambda_5 = 0.1302$, $\lambda_6 = 0.5208$, which implies Theorem 2.1 hold. By condition (iii) of Corollary 2.1, if $\Delta t_k < 0.0159$ the zero solution of system (3.1) is globally exponentially stable for any time delay. The numerical simulation with $\Delta t_k = 0.013$, $\tau = 0.3$ and initial functions $(\varphi_1(t), \varphi_2(t))$, where

$$
(3.2)\quad \varphi_1(t) = \begin{cases} 0, & t \in [-0.3, 0), \\ -1, & t = 0, \end{cases} \qquad \varphi_2(t) = \begin{cases} 0, & t \in [-0.3, 0), \\ 1, & t = 0, \end{cases}
$$

is given in Fig. 1, the global exponential convergence rate is 0.9861. The system (3.1) without impulses and hybrid term is unstable with the initial functions (3.2) (see Fig. 2).

It should be noted that the global exponential stability cannot be derived by applying existing criterion in [21], where only local uniform stability is proved under the same conditions. Therefore, Theorem 2.1 or Corollary 2.1 substantially extends and improves the main results of [21].

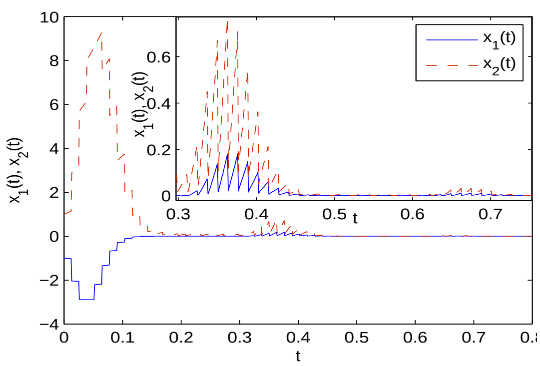

Fig. 1. Global exponential stability of system (3.1) with $\Delta t_k = 0.013$.

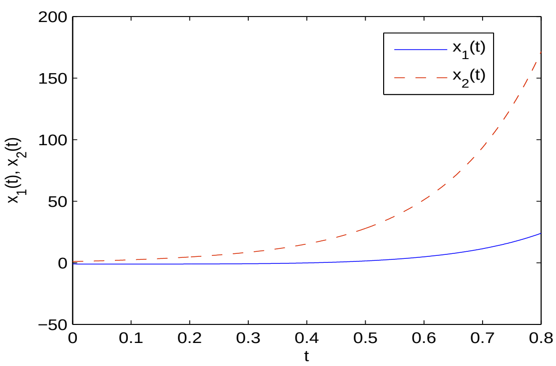

Fig. 2. Instability of system (3.1) without impulses and hybrid term.

Example 3.2. Consider the following Machey-Glass [22] nonlinear delayed impulsive hybrid model:

$$(3.3)\begin{cases} \dot{x}(t) = -0.68x(t) + \dfrac{1.7x(t-\tau(t))}{1+x^8(t-\tau(t))} + \dfrac{3}{k+2}x_k, & t \in [t_k, t_{k+1}), \\ x(t) = \dfrac{k+1}{2k^2+1}x(t^-), & t = t_k,\ k \in N, \end{cases}$$

where $0 \le \tau(t) \le \tau < +\infty$. The corresponding system (3.3) without impulse and hybrid term is a chaotic system. Fig. 3 shows its simulation results for $\tau = 4$ with the initial function:

$$(3.4) \qquad \varphi(t) = \begin{cases} 0, & t \in [-4, 0), \\ 1.2, & t = 0, \end{cases}$$

It is easy to see that condition (**H**) holds for

$$f(t, x(t), x(t-\tau(t))) = \frac{1.7x(t-\tau(t))}{1+x^8(t-\tau(t))}$$

with $l_1 = 0$ and $l_2 = 1.7$. By taking $P = 13$, $\lambda_3 = 24.64$, $\lambda_4 = 0.0452$, $\lambda_5 = 0.1731$, $\lambda_6 = 0.0769$, then all the conditions of Theorem 2.2 are satisfied. By the condition (iv) of Corollary 2.2, if $\Delta t_k < 0.0933$ the zero solution of system (3.3) is globally exponentially stable for any time delay. The numerical simulation with $\Delta t_k = 0.092$ and the initial function (3.4) is given in Fig. 4, the global exponential convergence rate is 0.0422.

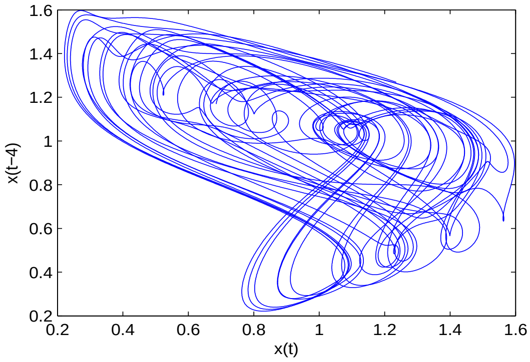

Fig. 3. Chaotic behaviors of system (3.3) without impulse and hybrid term.

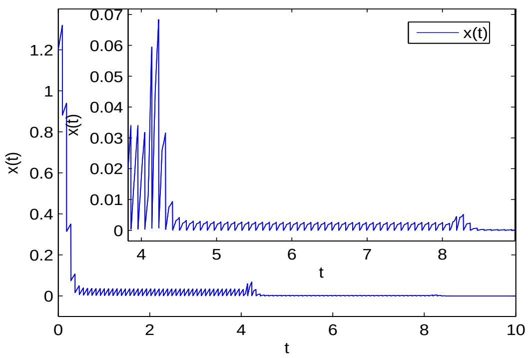

Fig. 4. Global exponential stability of system (3.3) with $\Delta t_k = 0.092$.

By Remark 2.3, let $P = 13, \tau = 4$, Fig. 5 shows the relationship between λ and Δt_k in system (3.3). By Remark 2.3, in Theorem 2.2, the satisfaction of both (i) and (ii) is equivalent to $\Delta t_k < h(\lambda)$. Hence, in Fig. 5, **D** can denote the globally exponentially stable region.

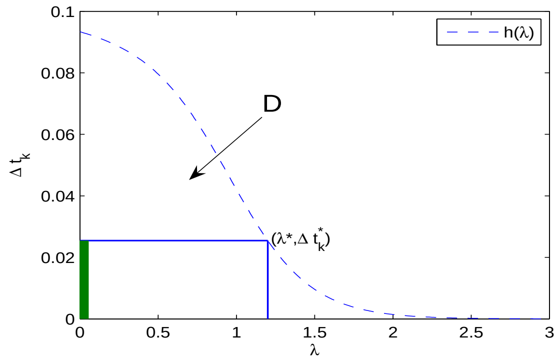

Fig. 5. Globally exponentially region of system (3.3) for $(\lambda, \Delta t_k)$.

It can been seen from Example 3.2 that the impulses hybrid terms can contribute to global exponential stability for nonlinear delayed dynamical systems even if the corresponding systems without impulses and hybrid terms may be unstable or chaotic itself, which can be usually used as an effective control strategy to stabilize the underlying delayed dynamical systems in some practical applications.

4. Conclusions

In this paper, the global exponential stability criteria of linear and nonlinear impulsive hybrid dynamical systems for any time delay are investigated. By employing the methods of Razumikhin technique and Lyapunov function, some global exponential stability criteria have been established. Our results have improved and generalized some of the known results existing in the literature. Two examples are given to illustrate the theoretical results. Furthermore, it is shown that the impulses and hybrid term can stabilize an unstable or even chaotic dynamical system, which is particularly meaningful for control and design of hybrid dynamical systems in practice.

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