ON γ-s-URYSOHN CLOSED AND γ-s-REGULAR CLOSED SPACES

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Abstract. In this paper, we introduced and studied γ-s-Urysohn spaces and γ-s-regular closed spaces. Several characterizations and properties of these classes of spaces have been obtained.

Keywords. γ-closed (open), γ-interior (closure), γ-semi-open(closed),γ-s-Urysohn,γ-s-regular, γ-s-adherent, γ-irresolute.

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1. Introduction

S. Kasahara [14] introduced and discussed an operation γ of a topology τ into the power set $P(X)$ of a space X. H. Ogata [16] introduced the concept of γ-open sets and investigated the related topological properties of the associated topology τ_γ and τ by using operation γ.

S. Hussain and B. Ahmad [1]-[6] and [10]-[13] continued studying the properties of γ-operations on topological spaces and investigated many interesting results. Recently, B. Ahmad and S. Hussain [2], [13] defined and discussed γ-semi-open sets in topological spaces. They explored many interesting properties of γ-semi-open sets. It is interesting to mention that γ-semi-open sets generalized γ-open sets introduced by H. Ogata [17].

In 1982, S.P. Arya and M.P. Bhamini introduced the concept of s-Urysohn. A space X is said to be s-Urysohn, if for any two distinct points x and y of X, there exist semi-open sets U and V containing x and y respectively such that $cl(U) \cap cl(V) = \emptyset$. The concept of s-regular space was introduced by Mahaeshwari and Prasad in 1975. In 1984, Arya and Bhamini introduced and discussed s-regular-closed sets [8].

In 2009, S. Hussain and B. Ahmad defined a new axiom called γ-s-regularity. It is interesting to mention that this axiom is a generalization of the axiom of s-regularity [15] as well as semi-regularity [9].

In this paper, we introduced and studied γ-s-Urysohn spaces and γ-s-regular closed spaces. A γ-s-Urysohn (resp., γ-s-regular [12]) space X is said to be γ-s-Urysohn-closed (resp., γ-s-regular-closed), if it is γ-closed in every γ-s-Urysohn (respt. γ-s-regular [12]) space in which it can be embedded. Several characterizations and properties of these classes of spaces have been obtained.

First, we recall some definitions and results used in this paper. Here after, we shall write a space in place of a topological space.

Preliminaries

Definition 2.1. [14] Let X be a space. An operation $\gamma : \tau \to P(X)$ is a function from τ to the power set of X such that $V \subseteq V^\gamma$, for each $V \in \tau$, where V^γ denotes the value of γ at V. The operations defined by $\gamma(G) = G$, $\gamma(G) = \text{cl}(G)$ and $\gamma(G) = \text{intcl}(G)$ are examples of operation γ.

Definition 2.2. [14] Let $A \subseteq X$. A point $x \in A$ is said to be γ-interior point of A if there exists an open nbd N of x such that $N^\gamma \subseteq A$ and we denote the set of all such points by $int_\gamma(A)$. Thus

$$int_\gamma(A) = \{x \in A : x \in N \in \tau \ \text{ and } \ N^\gamma \subseteq A\} \subseteq A.$$

Note that A is γ-open [14] iff $A = int_\gamma(A)$. A set A is called γ-closed [1] iff $X - A$ is γ-open.

Definition 2.3. [16] A point $x \in X$ is called a γ-closure point of $A \subseteq X$, if $U^\gamma \cap A \neq \phi$, for each open nbd U of x. The set of all γ-closure points of A is called γ-closure of A and is denoted by $\text{cl}_\gamma(A)$. A subset A of X is called γ-closed, if $\text{cl}_\gamma(A) \subseteq A$. Note that $\text{cl}_\gamma(A)$ is contained in every γ-closed superset of A.

Definition 2.4. [16] An operation γ on τ is said to be regular, if for any open nbds U, V of $x \in X$, there exists an open nbd W of x such that $U^\gamma \cap V^\gamma \supseteq W^\gamma$.

Definition 2.5. [16] An operation γ on τ is said to be open, if for any open nbd U of each $x \in X$, there exists γ-open set B such that $x \in B$ and $U^\gamma \supseteq B$.

Definition 2.6. [13] A subset A of a space X is said to be a γ-semi-open set, if there exists a γ-open set O such that $O \subseteq A \subseteq \text{cl}_\gamma(O)$. The set of all γ-semi-open sets is denoted by $SO_\gamma(X)$. A is γ-semi-closed if and only if $X - A$ is γ-semi-open in X. Note that A is γ-semi-closed if and only if $\mathrm{int}_\gamma cl_\gamma(A) \subseteq A$ [2].

Definition 2.7. [2] Let A be a subset of a space X. The intersection of all γ-semi-closed sets containing A is called γ-semi-closure of A and is denoted by $\mathrm{scl}_\gamma(A)$. Note that A is γ-semi-closed if and only if $\mathrm{scl}_\gamma(A) = A$. The set of all γ-semi-closed subsets of A is denoted by $SC_\gamma(A)$.

Definition 2.8. [2] Let A be a subset of a space X. The union of γ-semi-open subsets contained in A is called γ-semi-interior of A and is denoted by $sint_\gamma(A)$.

Definition 2.9. [13] An operation γ on τ is said be semi-regular, if for any semi-open sets U, V of $x \in X$, there exists a semi-open W of x such that $U^\gamma \cap V^\gamma \supseteq W^\gamma$.

Note that, if γ is semi-regular operation, then intersection of two γ-semi-open sets is γ-semi-open [13].

Definition 2.10. [2] A subset A of a space X is said to be γ-semi-regular, if it is both γ-semi-open and γ-semi-closed. The class of all γ-semi-regular sets of X is denoted by $SR_\gamma(A)$. Note that, if γ is a regular operation, then the union of γ-semi-regular sets is γ-semi-regular.

Definition 2.11. [16] A space X is said to be γ-T_2 space, if for each distinct points $x, y \in X$ there exists open sets U, V such that $x \in U$, $y \in V$ and $U^\gamma \cap V^\gamma = \phi$.

Definition 2.12. [3] Let X be a space and $A \subseteq X$. A point $x \in X$ is said to be a γ-adherent point of A, if $U^\gamma \cap A \neq \phi$, for every γ-open set U such that $x \in U$. A point x is a γ-adherent point for A if and only if $x \in \mathrm{cl}_\gamma(A)$.

3. γ-S-Urysohn closed spaces

Definition 3.1. A space X is said to be γ-s-Urysohn, if for any two distinct points x and y there exist γ-semi-open sets U and V such that $x \in U$, $y \in V$ and $\mathrm{cl}_\gamma(U) \cap \mathrm{cl}_\gamma(V) = \phi$.

Definition 3.2. A γ-s-Urysohn space X is said to be γ-s-Urysohn-closed, if it is γ-closed in every γ-s-Urysohn space in which it can be embedded.

Definition 3.3. A filter base ξ is said to be a γ-s-Urysohn filter base, if whenever x is not γ-adherent point of ξ, there exists a γ-semi-open set U containing x such that $\mathrm{cl}_\gamma(U) \cap \mathrm{cl}_\gamma(F) = \phi$, for some $F \in \xi$. A γ-open cover δ of X is said to be a γ-s-Urysohn cover, if there exists a γ-semi-open cover ϱ of X such that for each $V \in \varrho$, there is a $U \in \delta$ such that $cl_\gamma(V) \subseteq U$.

We are interesting in characterize the γ-s-Urysohn-closed space when it is a γ-T_2 and γ-s-Urysohn space.

Theorem 3.4. *Let X be a γ-s-Urysohn and γ-T_2 space and γ be a regular and open operation. Then the following are equivalent:*

(1) *X is γ-s-Urysohn-closed.*

(2) *Every γ-s-Urysohn cover δ of X has a finite subfamily δ^* such that the γ-closures of members of δ^* cover X.*

(3) *Every γ-open γ-s-Urysohn filter base has nonempty γ-adherence.*

Proof. (1) $\Rightarrow$ (2). Let X be γ-s-Urysohn-closed and let δ be a γ-s-Urysohn cover of X. Suppose on the contrary that for no finite subfamily of δ, the γ-closures of the members of the subfamily cover X. Let $p \notin X$ and let $Y = \{p\} \cup X$. Let $\Im = \tau \cup \{\{p\} \cup \{X - \cup_{i=1}^{n} cl_\gamma(U_i), U_i \in \delta\}\}$. Then $\Im$ is a topology on Y. Define an operation $\gamma\Im$ on $\Im$ as follows:

$$\gamma\Im(U) = \gamma(U) \quad \text{if } U \in \tau \text{ and}$$
$$\gamma\Im(U) = U \quad \text{otherwise.}$$

We shall prove that $(Y, \Im)$ is $\gamma\Im$-s-Urysohn. Let x and y be two distinct points in Y. If $x \neq p \neq y$, there exist disjoint γ-semi-open sets H and K containing x and y respectively such that $\mathrm{cl}_{\gamma\Im}(H) \cap \mathrm{cl}_{\gamma\Im}(K) = \emptyset$. Suppose now, that one of x and y, say is p. Since δ is a γ-s-Urysohn cover of X, there exists a γ-semi-open cover ϱ of X such that for each $V \in \varrho$, there exists a $U_V \in \delta$ where $cl_\gamma(V) \subseteq U_V$. Let $x \in V \in \varrho$. Now, $\{p\} \cup (X - cl_\gamma(U_V))$ is $\gamma\Im$-open(and hence $\gamma\Im$-semi-open) set containing $y = p$. Since $\mathrm{cl}_\gamma(V) \subseteq U_V$. $\mathrm{cl}_{\gamma\Im}(V) \cap (\{p\} \cup (X - U_V)) = \emptyset$. That is, $\mathrm{cl}_{\gamma\Im}(V) \cap \mathrm{cl}_{\gamma\Im}(\{p\} \cup (X - \mathrm{cl}_\gamma(U_V))) = \emptyset$, since $\gamma\Im$ is regular. Hence $(Y, \Im)$ is $\gamma\Im$-s-Urysohn. But X is not a γ-closed subset of $(Y, \Im)$ since $p \in cl_{\gamma\Im}(X)$. This is a contradiction to the fact that X is γ-s-Urysohn-closed. Hence (2) is true.

(2) $\Rightarrow$ (3). We suppose contrarily that ξ be a γ-s-Urysohn filter base without any γ-adherent point. Then $\delta = \{X - \mathrm{cl}_\gamma(F) : F \in \xi\}$ is a γ-open cover of X. We shall prove that δ is a γ-s-Urysohn cover. Let $x \in X$. Since x is not a γ-adherent point of ξ, there exists a γ-semi-open set V_x such that $\mathrm{cl}_\gamma(V_x) \cap \mathrm{cl}_\gamma(F_x) = \emptyset$ for some $F_x \in \xi$. Therefore, there exist $X - \mathrm{cl}_\gamma(F_x) \in \delta$ such that $\mathrm{cl}_\gamma(V_x) \subseteq X - \mathrm{cl}_\gamma(F_x)$. Hence, $\varrho = \{V_x : x \in X\}$ and $\mathrm{cl}_\gamma(V_x) \subseteq X - \mathrm{cl}_\gamma(F_x)$ is a γ-semi-open over of X. Thus δ is a γ-s-Urysohn cover. Therefore, $X = \bigcup_{i=1}^{n} \mathrm{cl}_\gamma(X - \mathrm{cl}_\gamma(F_{x_i}) \subseteq \bigcup_{i=1}^{n} \mathrm{cl}_\gamma(X - F_{x_i}) = \bigcup_{i=1}^{n} (X - (F_{x_i})$, since each F_{x_i} is γ-open. Thus $X = X - \bigcap_{i=1}^{n} (F_{x_i})$ which means that $\bigcap_{i=1}^{n} (F_{x_i}) = \phi$, a contradiction. Thus (3) is proved.

(3) $\Rightarrow$ (1). Contrarily, suppose that X be a γ-s-Urysohn space which is not γ-s-Urysohn-closed. Let Y be a γ-s-Urysohn space in which X is embedded. If possible, suppose that X is not a γ-closed subset of Y. Let $p \in \mathrm{cl}_\gamma(X) - X$ and $\delta = \{U \cap X$ where U is a γ-open subset of Y containing $p\}$. Then δ is a γ-s-Urysohn filter base in X. Since X is a γ-s-Urysohn and $\gamma - T_2$, it can be easily verified that δ has no γ-adherent point in X. This is a contradiction. This completes the proof.

Theorem 3.5. *Every γ-clopen subset of a γ-s-Urysohn-closed space is γ-s-Urysohn-closed.*

Proof. Let X be a γ-s-Urysohn-closed and let $Y \subseteq X$ be γ-open. Let ξ be a γ-s-Urysohn filter base in Y with empty γ-adherence. Y being γ-open is γ-s-Urysohn. Also ξ is a γ-open filter base in X. We shall claim that ξ is a γ-s-Urysohn filter base in X. If every point of x is a γ-adherent point of ξ in X, then ξ is of course γ-s-Urysohn in X. Let x be a point in X which is not a γ-adherent point of ξ in X. Since every γ-semi-open subset V of X containing x have nonempty intersection with every $F \in \xi$. Then $V \cap Y$ is a γ-semi-open subset of X [2] and hence of Y. Since Y is a γ-semi-closed subset of X, $x \in Y$. Thus $V \cap Y$ is a γ-semi-open subset of Y containing x having nonempty intersection with every member of ξ. Also, Y being γ-open, every γ-semi-open subset of Y is of the form $V \cap Y$, where V is a γ-semi-open subset of X. Hence every γ-semi-open subset of Y containing x intersects every member of ξ, which is a contradiction to the fact that ξ is a γ-s-Urysohn filter base in Y. Y being γ-closed. ξ can not have a γ-adherent point in X, since it has empty γ-adherent in Y. This is a contradiction. Hence the proof.

Definition 3.6. A point x is said to be a γ-s-adherent point of a filter base ξ, if $x \in scl_\gamma(F)$, for every $F \in \xi$.

Definition 3.7. [17] Let (X, τ) and (Y, δ) be spaces. Let $\gamma :\rightarrow P(X)$ and $\beta : \delta \rightarrow P(Y)$ be operations. Let $(X \times Y, \tau \times \delta)$ be the product space and let $\rho : \tau \times \delta \rightarrow P(X \times Y)$ be an operations on $\tau \times \delta$. Then ρ is called associative with (γ, β), if $(U \times V)^\rho = U^\gamma \times V^\beta$ holds for each nonvoid $U \in \tau$ and nonvoid $V \in \delta$.

It is known in [17] that, if $A \subseteq X$ and $B \subseteq Y$. Then $\mathrm{cl}_\rho(A \times B) = \mathrm{cl}_\gamma(A) \times cl_\beta(B)$.

Lemma 3.8. *Let X be a space and γ be a regular operation such that every γ-open filter base in X has nonempty γ-s-adherent and let Y be an arbitrary space. If ξ is a ρ-open ρ-s-Urysohn filter base in $X \times Y$, then $P_y(\xi)$ is a ρ-open ρ-s-Urysohn filter base in Y, where P_y is the projection function from $X \times Y$ onto Y. Where γ, β and ρ are operations on X, Y and $X \times Y$ respectively.*

Proof. ξ is a ρ-open filter base in $X \times Y$. Then $P_x(\xi)$ is a γ-open filter base in X and hence has a γ-s-adherent point, say $x \in X$. We shall prove that $P_x(\xi)$ is a β-s-Urysohn filter base in Y. $P_y(\xi)$ is a β-open filter base in Y. Suppose that y is not a β-adherent point of $P_y(\xi)$. Then (x, y) is not a ρ-adherent point of ξ in $X \times Y$. It is given that ξ is a ρ-s-Urysohn filter base in $X \times Y$. Hence there exists a ρ-semi-open subset $U \times V$ of $X \times Y$ containing (x, y) such that $\mathrm{cl}_\rho(U \times V) \cap \mathrm{cl}_\rho(F) = \emptyset$, for some $F \in \xi$. Since $U \times V$ is a ρ-semi-open subset of $X \times Y$ containing (x, y), U is a γ-semi-open subset of X containing x and V is a β-semi-open subset of Y containing y [2]. Also, we have $\mathrm{cl}_\rho(U \times V) \cap \mathrm{cl}_\rho(F) = \emptyset$. That is, $(\mathrm{cl}_\gamma(U) \times \mathrm{cl}_\beta(V)) \cap cl_\rho(F) = \emptyset$ for some $F \in \xi$. Hence $\mathrm{cl}_\beta(V) \cap \mathrm{cl}_\beta(P_y(F)) = \emptyset$ for some $F \in \xi$, since x is a γ-s-adherent point of $P_x(\xi)$.

4. γ-s-Regular spaces

Definition 4.1. A γ-s-regular space is said to be γ-s-regular-closed, if it is γ-closed in every γ-s-regular space in which it can be embedded.

Definition 4.2. A cover δ is said to be a γ-s-regular cover, if there exists a γ-semi-open cover ϱ such that the γ-semi-closures of members of ϱ refine δ.

Definition 4.3. A filter base is said to be a γ-s-regular filter base, if it is equivalent to a γ-semi-closed filter base.

Theorem 4.4. *Let X be a γ-T_2 space and γ be a regular operation. Then the following are equivalent for a γ-s-regular space X:*

(1) *X is γ-s-regular-closed.*
(2) *Every γ-open γ-s-regular cover has a finite subcover.*
(3) *Every γ-s-regular filter base has nonempty γ-adherence.*

Proof. (1) $\Rightarrow$ (2). Let δ be a γ-open γ-s-regular cover of X and suppose that δ does not have a finite subcover and $p \notin X$. Let $Y = \{p\} \cup X$. Let $\Im = \tau \cup \{\{p\} \cup \{X - \bigcup_{i=1}^{n} U_i : U_i \in \delta\}\}$. Then $\Im$ is a topology on Y. Define an operation $\gamma\Im$ on $\Im$ as follows: $\gamma\Im(U) = \gamma(U)$ if $U \in \tau$ and $\gamma\Im(U) = U$ otherwise. We shall prove that $(Y, \Im)$ is $\gamma\Im$-s-regular.

Case (1). Let $x \in Y$, where $x \neq p$ and B be a $\Im$-closed set containing neither x nor p. Then $Y - B$ is a $\Im$-open set containing both x and p. Since δ is a γ-s-regular cover, there exists a γ-semi-open cover ϱ of X such that $\{scl_\gamma(V) : V \in \varrho\}$ is a refinement of δ. Since $x \in X$, $x \in V$ for some $V \in \varrho$. Now, $x \in Y - B$ where $(Y - B)$ is a $\gamma\Im$-open set containing p. Hence $Y - B = \{p\} \cup \{X - \bigcup_{i=1}^{n}(U_i) : U_i \in \delta\}$. Therefore, $x \in V \cap (\{p\} \cup (X - \bigcup_{i=1}^{n}(U_i)))$. The set $V \cap (\{p\} \cup (X - \bigcup_{i=1}^{n}(U_i)))$ is a $\Im$-open set, being an intersection of $\Im$-open(and hence $\Im$-semi-open) set. Thus $V \cap (\{p\} \cup (X - \bigcup_{i=1}^{n}(U_i)))$ is a $\Im$-semi-open set containing x. Also $Y - B = \{p\} \cup (X - \bigcup_{i=1}^{n}(U_i)) = Y - \bigcup_{i=1}^{n}(U_i)$. Hence $B = \bigcup_{i=1}^{n}(U_i)$. Thus $V \cap (\{p\} \cup (X - \bigcup_{i=1}^{n}(U_i)))$ and $\bigcup_{i=1}^{n}(U_i)$ are disjoint $\Im$-semi-open sets containing x and B respectively.

Case 2. Now, let us suppose that $x = p$ and B is $\Im$-closed set not containing p. Then $Y - B = \{p\} \cup \{X - \bigcup_{i=1}^{n}(U_i) : U_i \in \delta\}$. Therefore, $\{p\} \cup \left(X - \bigcup_{i=1}^{n}(U_i)\right)$ is $\Im$-open set and hence a $\gamma\Im$-semi-open set containing p and $\bigcup_{i=1}^{n} U_i$ is a $\Im$-open set containing B.

Case 3. Suppose that $x \neq p$ and let B be a $\gamma\Im$-closed set containing p. Then $Y - B$ is a $\Im$-open set containing x and not containing p. Hence $Y - B$ is a τ-open set containing x. Therefore there exists a γ-semi-open subset V of X such that $x \in V \subseteq scl_\gamma(V) \subseteq Y - B$. Thus $(Y, \Im)$ is $\gamma\Im$-s-regular. But X is not a γ-closed subset of Y since every $\gamma\Im$-open set containing p intersect X. This is a contradiction. Hence δ has a finite subcover.

(2) $\Rightarrow$ (3). Let ξ be a γ-s-regular filter base without a γ-adherent point in X. Then $\{X - cl_\gamma(F) : F \in \xi\}$ is a γ-open cover of X. We shall prove that this is a γ-s-regular cover. Let $x \in X$. Then x is not a γ-adherent point of ξ, so that $x \notin cl_\gamma(F_x)$, for some $F_x \in \xi$. Since X is γ-s-regular, there exists a γ-semi-open set V_x containing x such that $x \in V_x \subseteq scl_\gamma(V_x) \subseteq X - \mathrm{cl}_\gamma(F_x)$. Thus $\{V_x : x \in X$ and $scl_\gamma(V_x) \subseteq X - cl_\gamma(F_x)\} = \delta$ is a γ-semi-open cover of X such that $\{scl_\gamma(V_x) : V_x \in \delta\}$ is a refinement of $\{X - \mathrm{cl}_\gamma(F_x) : F_x \in \delta\}$. Hence $\{X - cl_\gamma(F) : F \in \delta\}$ is a γ-s-regular cover of X. Therefore there exist finite many members $F_1, F_2, ..., F_n$ of δ such that $X = \bigcup_{i=1}^n (X - \mathrm{cl}_\gamma(F_i)) \subseteq \bigcup_{i=1}^n (X - F_i) = X - \bigcap_{i=1}^{n} F_i$. That is $\bigcap_{i=1}^{n} F_i = \phi$, a contradiction to the fact that δ is a filter base.

(3) $\Rightarrow$ (1). If possible, suppose that X is not γ-s-regular-closed. Let X be embedded in a γ-s-regular space Y and let $cl_{\gamma_y}(X) - X \neq \phi$. Let $p \in cl_{\gamma_y}(X) - X$. Let $\delta = \{X \cap U$: U is a nbd of p in Y$\}$ and let $\varrho = \{X \cap V : V$ is the γ-semi-closure of a γ-semi-open subset of Y containing p $\}$. Since Y is γ-s-regular, δ and ϱ are equivalent and they are filter bases in X. It can be verified that each $V \in \varrho$ is a γ-semi-closed subset of X. Hence δ is a γ-s-regular filter base in X. δ does not have a γ-adherent point in X, since X is γ-T_2. This is a contradiction. Hence the proof.

Definition 4.5. [6] A function $f : X \to Y$ is said to be γ-irresolute, if the inverse image of every γ-semi-open set is γ-semi-open.

Theorem 4.6. *If $X = \Pi_{i\in I} X_i$ is γ-s-regular-closed, then each X_i is γ-s-regular-closed provided each X_i is γ-s-regular.*

Proof. Let ξ_i be a γ-s-regular filter base in X_i. Let δ_i be the γ-semi-closed filter base in X_i which is equivalent to ξ_i. Then $\{P_i^{-1}(F_i) : F_i \in \xi_i\}$ is a filter base in X and $\{P_i^{-1}(U_i) : U_i \in \xi_i\}$ is a γ-semi-closed filter base in X, since the projection mapping is γ-irresolute[6]. Also $\{P_i^{-1}(U_i) : U_i \in \xi_i\}$ is equivalent to $\{P_i^{-1}(F_i) : F_i \in \xi_i\}$. Thus $\{P_i^{-1}(F_i) : F_i \in \xi_i\}$ is a γ-s-regular filter base in X. If $x = (x_i)$ is a γ-adherent of $\{P_i^{-1}(F_i) : F_i \in \xi_i\}$ in X. Then x_i is a γ-adherent point of ξ_i. Thus X_i is γ-s-regular-closed.

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