# A COMMUTATIVE REGULAR MONOID ON ROUGH SETS

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**Abstract.** In this paper, we intend to study an algebraic approach on rough sets. We introduce the concept of rough semigroup, rough monoid and rough ideals on the set of all rough sets for the given information system together with the operation Praba $\Delta$. We illustrate these concepts through examples.

**Keyword:** roughset, semigroup, monoid, regular semigroup, ideals.

## 1. Introduction

The concept of rough set theory was introduced by Z. Pawalak [12] in 1982. This formal tool was implemented to process incomplete information in the information systems. Rough set theory is an extention of set theory and it is defined by a pair of sets called lower and upper approximations. In the content of data analysis, this concept will be used to discover fundamental patterns in data, remove redundancies and generate decision rules. Also rough set theory

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will be applied in several field like computational intelligence such as machine learning, intelligent systems, pattern recognition, knowledge discovery, expert systems and others [14], [10], [6], [2], [4]. Zadeh[15] introduced the concept of fuzzy sets in his paper. B. Praba and R. Mohan [13] discussed the concept of rough lattice. In this paper the authors considered an information system $I = (U, A)$. A partial ordering relation was defined on $T = \{RS(X) \mid X \subseteq U\}$. The least upper bound and greatest lower bound were established. They have also defined the operation Praba $\Delta$. N. Kuroki and P.P. Wang [9] discussed some properties of lower and upper approximations with respect to the normal subgroup. R. Biswas and S. Nanda [1] introduced the notion of rough groups and rough subgroups. Also the authors T.B. Iwinski [8] Z. Bonikowaski [3] have studied algebraic properties of rough sets. Then the concept of rough fuzzysets and fuzzy rough sets was introduced by D. Dubois, H. Parade [5] and Nick C. Fiala [11] discussed about semigroup, monoid and group models of groupoid identities in his paper. In the recent and past, rough set theory has triggered many researchers all around the world. The concept of rough set theory is the approximation space such as lower and upper approximations of a set determined by attributes. The pair of lower and upper approximation is called rough set also in rough set theory data can be represented in the form of an information system. An information system is a pair $I = (U, A)$ where $U$ is a non empty finite set of objects, called universal set and A is a nonempty set of fuzzy attributes defined by $\mu_a : U \to [0, 1]$, $a \in A$, is a fuzzy set.

*Indiscernibility* is a core concept of rough set theory and it is defined as an equivalence between objects. Objects in the information system about which we have the same knowledge forms an equivalence relation.

Formally, any set $P \subseteq A$, there is an associated equivalence relation called $P$-*Indiscernibility* relation defined as follows,

$$IND(P) = \{(x, y) \in U^2 | \forall a \in P, \mu_a(x) = \mu_a(y)\}.$$

The partition induced by $IND(P)$ consists of equivalence classes defined by

$$[x]_p = \{y \in U | (x, y) \in IND(P)\}.$$

For any $X \subseteq U$, define the lower approximation space

$$\underline{P}(x) = \{x \in U | [x]_p \subseteq X\}.$$

Also, define the upper approximation space $\overline{P}(x) = \{x \in U | [x]_p \cap X \neq \phi\}$.

For every subset $X \in U$, there is an associated rough set

$$RS(X) = (\underline{P}(x), \overline{P}(x)).$$

In this paper, we consider an information system $I = (U, A)$ where $U$ is a non empty finite set of objects, called universal set and A is a nonempty set of fuzzy attributes and let $T = \{RS(X) \mid X \subseteq U\}$ be the set of all rough sets on $U$. A binary relation Praba $\Delta$ [13] is defined on$T$ then we prove that $(T, \Delta)$ is a commutative regular monoid of idempotents. Also we characterize the principal ideals generated by these idempotents. The paper is organized as follows.

In Section 2, we give the necessary definitions pertaining to rough set theory and semigroup theory.

In Section 3, we define the binary operation Praba $\Delta$ [13] on $T$ and prove that $(T, \Delta)$ is a commutative regular monoid of idempotents called as the rough monoid.

In Section 4, a characterization for the lower and upper approximation of the principal ideal generated by the elements of $T$ is discussed in detail and section 5 illustrates these concepts with example.

Section 6 gives the conclusion.

## 2. Preliminaries

In this section, we present some preliminaries on rough sets and algebraic structures.

### 2.1. Rough sets

Let $I = (U, A)$ be an information system and for any subset $X$ of $U$ and $(\underline{P}(x), \overline{P}(x))$ are the lower and upper approximations respectively as defined in previous section.

**Definition 2.1.** [rough set] A rough set corresponding to $X$, where $X$ is an arbitrary subset of $U$ in the approximation space $P$, we mean the ordered pair $RS(X) = (\underline{P}(x), \overline{P}(x))$.

**Remarks 2.1.** [13] If $X \subseteq U$, then $X \subseteq \bigcup_{i=1}^{r} X_{\alpha_i}$ where none, one or more of the equivalence classes are contained in $X$. Here $X_{\alpha_i}$, $i = 1, 2, ..., r$ are the equivalence classes induced by $Ind(P)$.

**Definition 2.2.** [13] If $X \subseteq U$, then the number of equivalence classes (Induced by Ind(P)) contained in $X$ is called as the Ind. weight of $X$. It is denoted by $IW(X)$.

**Example 2.1.** [13] Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ and $A = \{a_1, a_2, a_3, a_4\}$ where each $a_i$ ($i = 1$ to 4) is a fuzzy set whose membership values are shown in Table 1.

Table 1:

| $A/U$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ |
|---|---|---|---|---|
| $x_1$ | 0 | 0.1 | 0.3 | 0.2 |
| $x_2$ | 1 | 0.6 | 0.7 | 0.3 |
| $x_3$ | 0 | 0.1 | 0.3 | 0.2 |
| $x_4$ | 1 | 0.6 | 0.7 | 0.3 |
| $x_5$ | 0.8 | 0.5 | 0.2 | 0.4 |
| $x_6$ | 1 | 0.6 | 0.7 | 0.3 |

Let $X = \{x_1, x_3, x_5, x_6\}$ and $P = A$. Then the equivalence classes induced by $IND(P)$ are given below.

$$[x_1]_p = \{x_1, x_3\} \quad (1)$$
$$[x_2]_p = \{x_2, x_4, x_6\} \quad (2)$$
$$[x_5]_p = \{x_5\} \quad (3)$$

Hence,

$$\underline{P}(x) = \{x_1, x_3, x_5\} \quad \text{and}$$
$$\overline{P}(x) = \{x_1, x_2, x_3, x_4, x_5, x_6\}.$$

Therefore

$$RS(X) = (\{x_1, x_3, x_5\}, \{x_1, x_2, x_3, x_4, x_5, x_6\}).$$

Note that the upper approximation space consists of those objects that are possibly members of the target set $X$. The set $U - \overline{P}(x)$ represents the negative region containing the set of objects that can be definitely ruled out to be the members of the target set $X$. The boundary region given by the set difference $\underline{P}(x) - \overline{P}(x)$ consists of those objects that can neither be ruled in nor ruled out as the members of the target set $X$. In the previous example the negative region is an empty set and the boundary is $\underline{P}(x) - \overline{P}(x) = \{x_2, x_4, x_6\}$. Throughout this paper, we use this same Example 2.1 to illustrate our concepts.

**Example 2.2.** Let $U = \{x_1, x_2, \ldots, x_6\}$ as in Table 1. The equivalence classes induced by $Ind(P)$ are

$$[x_1]_p = \{x_1, x_3\}$$
$$[x_2]_p = \{x_2, x_4, x_6\}$$
$$[x_5]_p = \{x_5\}$$

Let $X = \{x_1, x_4, x_5\} \subseteq U$ then by definition, Ind. weight of $X = IW(X) = 1$ (since there is only one equivalence class $[x_5]_p = \{x_5\}$ present in X).

**Definition 2.3.** [13] Let $X, Y \subseteq U$. The Praba $\Delta$ is defined as $X \Delta Y = X \cup Y$, if $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$.
If $IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y)$,
then identify the equivalence class obtained by the union of $X$ and $Y$. Then delete the elements of that class belonging to $Y$. Call the new set as $Y$. Now, obtain $X \Delta Y$. Repeat this process until

$$IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y).$$

**Example 2.3.** [13] Let $U = \{x_1, x_2, \ldots, x_6\}$ as in Table 1.
Let $X = \{x_2, x_4, x_5\}, Y = \{x_1, x_6\} \subseteq U$ then by definition,

$$IW(X) = 1; \; IW(Y) = 0; \; IW(X \cup Y) = 2; \; IW(X \cap Y) = 0.$$

Here,

$$IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y).$$

The new equivalence class formed in $X \cup Y$ is $[x_2]_p$. As $x_6 \in Y$ and $x_6$ is an element of $[x_2]_p$, delete $x_6$ from $Y$. Now the new $Y$ is $\{x_1\}$. Now for $X = \{x_2, x_5, x_6\}$ and $Y = \{x_1\}$. Finding $IW(X \cup Y)$,

$$IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y).$$

Therefore, $X \Delta Y = X \cup Y = \{x_1, x_2, x_4, x_5\}$.

### 2.2. Algebraic structures

**Definition 2.4.** [Groupoid] [11], [7] A groupoid consists of a non-empty set equipped with a binary operation $*$, and it is denoted by $(S, *)$.

**Definition 2.5.** [Semigroup] [11], [7] A *semigroup* $(S, *)$ is a groupoid that is *associative* $(x * y) * z = x * (y * z)$ for all $x, y, z \in S)$.

**Definition 2.6.** [Monoid] [11], [7] A *semigroup* $(S, *)$ is said to be a monoid if it contains an identity element $e \in S$ such that $e * x = x * e = x$ for all $x \in S$.

**Definition 2.7.** [Commutative monoid] [11], [7] A *monoid* $(S, *)$ which satisfies commutative axiom namely $x*y = y*x$ for all $x, y \in S$, is known as Commutative monoid

**Definition 2.8.** [Idempotent] [11], [7] An element $x$ in a groupoid $(S, *)$ is said to be idempotent, if $x * x = x$.

**Definition 2.9.** [Regular semigroup] [11], [7] A semigroup $(S, *)$ is said to be regular, if there exists an element $y \in S$ such that $x = x * y * x$ for all $x \in S$.

**Definition 2.10.** [Right (Left) ideal] [7] A nonempty subset $I$ of a semigroup $(S, *)$ is a right (left) ideal, if it satisfies $I * S \subseteq I$ $(S * I \subseteq I)$.

**Definition 2.11.** [Ideal] A nonempty subset $I$ of a semigroup $(S, *)$ is said to be an ideal, if it is both right and left ideal.

**Definition 2.12.** [Principal ideal] [7] Let $(S, *)$ be a semigroup and for any element $a \in S$ such that $a * S$ $(S * a)$ is called as principal right (left) ideal generated by a and $S * a * S$ is called as principal ideal of $S$

**Example 2.4.** Ideals $nZ$ of the semigroup $Z$ are all principal and in fact all ideals of $Z$ are principal.

In the following section, a binary operation Praba $\Delta$ [13] is defined on the set of all rough sets and its algebraic structure is studied.

## 3. Monoids on Rough sets

Throughout this section, we consider an information system $I = (U, A)$. Now, for any $X \subseteq U$, $RS(X) = (\underline{P}(X), \overline{P}(X))$, and let $T = \{RS(X)|X \subseteq U\}$ be the set of all rough sets on $I$

**Definition 3.1.** [Binary operation] Let $T$ be the collection of rough sets and let $\Delta : T \times T \to T$ such that $\Delta(RS(X), RS(Y)) = RS(X\Delta Y)$

**Theorem 3.1.** *Let $I = (U, A)$ be an information system where $U$ be the universal (finite) set and $A$ be the set of attributes and $T$ be the set of all rough sets then $(T, \Delta)$ is a Monoid.*

**Proof.**

**(a) Closure axiom:** For $X, Y \subseteq U$, and letting $X\Delta Y = Z \subseteq U$,

$$\implies RS(X\Delta Y) = RS(Z) \in T$$

**(b) Associative axiom:** For all $X, Y, Z \subseteq U$ and $RS(X), RS(Y), RS(Z) \in T$ such that

$$RS(X\Delta(Y\Delta Z)) = RS((X\Delta Y)\Delta Z)),$$

i.e., to prove $RS(X)\Delta(RS(Y)\Delta RS(Z)) = (RS(X)\Delta RS(Y))\Delta RS(Z)$,

$$\implies RS(X\Delta(Y\Delta Z)) = RS((X\Delta Y)\Delta Z). \tag{4}$$

**Claim:**

(i) $\underline{P}(X\Delta(Y\Delta Z)) = \underline{P}((X\Delta Y)\Delta Z))$

(ii) $\overline{P}(X\Delta(Y\Delta Z)) = \overline{P}((X\Delta Y)\Delta Z))$

Proof for Claim (i): For $x \in \underline{P}(X\Delta(Y\Delta Z))$, we have

$$\begin{aligned} & \{x \in U|[x]_p \subseteq X\Delta(Y\Delta Z)\} \\ \implies & i.e.,\ [x]_p \subseteq X \ or\ [x]_p \subseteq Y\Delta Z \\ \implies & [x]_p \subseteq X \ or\ [x]_p \subseteq Y \ or\ [x]_p \subseteq Z \\ \implies & [x]_p \subseteq X\Delta Y \ or\ [x]_p \subseteq Z \\ \implies & [x]_p \subseteq (X\Delta Y)\Delta Z \\ \implies & x \in \underline{P}((X\Delta Y)\Delta Z) \\ \implies & \underline{P}(X\Delta(Y\Delta Z)) \subseteq \underline{P}((X\Delta Y)\Delta Z). \end{aligned}$$

Similarly, we prove that

$$\begin{aligned} & \underline{P}((X\Delta Y)\Delta Z) \subseteq \underline{P}(X\Delta(Y\Delta Z)) \\ \therefore\ & \underline{P}((X\Delta Y)\Delta Z) = \underline{P}(X\Delta(Y\Delta Z)). \end{aligned}$$

Proof for claim (ii): For $x \in \overline{P}(X\Delta(Y\Delta Z))$, we have

$$
\begin{aligned}
& \{x \in U | [x]_p \cap (X\Delta(Y\Delta Z)) \neq \phi\} \\
\Longrightarrow\quad & i.e., \ \ [x]_p \cap X \neq \phi \ or \ [x]_p \cap (Y\Delta Z) \neq \phi \\
\Longrightarrow\quad & [x]_p \cap X \neq \phi \ or \ [x]_p \cap Y \neq \phi \ or \ [x]_p \cap Z \neq \phi \\
\Longrightarrow\quad & [x]_p \cap (X\Delta Y) \neq \phi \ or \ [x]_p \cap Z \neq \phi \\
\Longrightarrow\quad & [x]_p \cap ((X\Delta Y)\Delta Z) \neq \phi \\
\Longrightarrow\quad & x \in \overline{P}((X\Delta Y)\Delta Z) \\
\Longrightarrow\quad & \overline{P}(X\Delta(Y\Delta Z)) \subseteq \overline{P}((X\Delta Y)\Delta Z).
\end{aligned}
$$

Similarly, we prove that

$$
\begin{aligned}
& \overline{P}((X\Delta Y)\Delta Z) \subseteq \overline{P}(X\Delta(Y\Delta Z)) \\
\therefore\quad & \overline{P}((X\Delta Y)\Delta Z) = \overline{P}(X\Delta(Y\Delta Z)).
\end{aligned}
$$

Thus $RS(X\Delta(Y\Delta Z)) = RS((X\Delta Y)\Delta Z))$. Hence $(T, \Delta)$ is a semigroup. This Semigroup is called as rough semigroup.

**(c) Identity axiom:** For $RS(X) \in T$ there exists $RS(\phi) \in T$ such that

$$
\begin{aligned}
& RS(X)\Delta RS(\phi) = RS(X\Delta\phi) = RS(X) \\
\therefore\quad & RS(X\Delta\phi) = RS(\phi\Delta X) = RS(X).
\end{aligned}
$$

Thus $(T, \Delta)$ is a monoid. This monoid is called as the rough monoid. ■

**Theorem 3.2.** $(T, \Delta)$ *is a commutative rough monoid.*

**Proof.** From Theorem 3.1, we have $(T, \Delta)$ is a rough monoid. Now, it is enough to prove $\Delta$ is commutative, i.e., for any $X, Y \subseteq U$ and $RS(X), RS(Y) \in T$, such that $RS(X\Delta Y) = RS(Y\Delta X)$, i.e.,

**Claim:**

(i) $\underline{P}(X\Delta Y) = \underline{P}(Y\Delta X)$,

(ii) $\overline{P}(X\Delta Y) = \overline{P}(Y\Delta X)$.

Proof of Claim (i): For $x \in \underline{P}(X\Delta Y)$, i.e., $[x]_P \subseteq (X\Delta Y)$

$$
\begin{aligned}
\Longrightarrow\quad & [x]_P \subseteq X \ or \ [x]_P \subseteq Y \\
\Longrightarrow\quad & [x]_P \subseteq Y \ or \ [x]_P \subseteq X \\
\Longrightarrow\quad & [x]_P \subseteq (Y\Delta X) \\
\Longrightarrow\quad & \underline{P}(X\Delta Y) \subseteq \underline{P}(Y\Delta X).
\end{aligned}
$$

Similarly, we prove that

$$
\begin{aligned}
& \underline{P}(Y\Delta X) \subseteq \underline{P}(X\Delta Y) \\
\therefore\quad & \underline{P}(X\Delta Y) = \underline{P}(Y\Delta X)
\end{aligned}
$$

Proof of claim (ii): For $x \in \overline{P}(X\Delta Y)$, i.e., $[x]_P \cap (X\Delta Y) \neq \phi$

$$\begin{aligned} &\Longrightarrow && [x]_P \cap X \neq \phi \textit{ or } [x]_P \cap Y \neq \phi \\ &\Longrightarrow && [x]_P \cap Y \neq \phi \textit{ or } [x]_P \cap X \neq \phi \\ &\Longrightarrow && [x]_P \cap (Y\Delta X) \neq \phi \\ &\Longrightarrow && \overline{P}(X\Delta Y) \subseteq \overline{P}(Y\Delta X). \end{aligned}$$

Similarly, we prove that

$$\begin{aligned} & \overline{P}(Y\Delta X) \subseteq \overline{P}(X\Delta Y) \\ \therefore \quad & \overline{P}(X\Delta Y) = \overline{P}(Y\Delta X). \end{aligned}$$

Hence, $RS(X\Delta Y) = RS(Y\Delta X)$. Thus $(T, \Delta)$ is a commutative monoid. This commutative monoid is called as commutative rough monoid. ∎

**Theorem 3.3.** *$(T, \Delta)$ is a regular rough monoid of idempotents.*

**Proof.** To prove: $(T, \Delta)$ is a regular monoid. We need to prove that, for any $RS(X) \in T$, there exist $RS(Y) \in T$ such that

$$RS(X)\Delta RS(Y)\Delta RS(X) = RS(X).$$

For $RS(X) \in T$, take $Y = E_X = \{x \in U \mid [x]_P \subseteq X\}$.
Then $Y\Delta X = X$ and hence $X\Delta Y\Delta X = X$.

$$\therefore RS(X)\Delta RS(Y)\Delta RS(X) = RS(X\Delta Y\Delta X) = RS(X).$$

Hence

$$(T, \Delta) \text{ is a regular monoid .}$$

Now, for $RS(X) \in T$,

$$RS(X)\Delta RS(X) = RS(X\Delta X) = RS(X).$$

This implies that $RS(X)$ is an idempotent in $T$, i.e., all elements of $T$ are idempotent. Hence $(T, \Delta)$ is a regular monoid of idempotents called as regular rough monoid of idempotents on $T$. ∎

## 4. Rough Ideals

In this section, we discuss about the principal ideals of commutative regular monoid of idempotents $(T, \Delta)$.

**Definition 4.2.** [Rough ideal] Consider the commutative regular monoid of idempotents $(T, \Delta)$. For any $RS(X) \in T$, $RS(X)T$ is the principal ideal generated by $RS(X)$. This ideal is called as the principal rough ideal on $T$.

**Remarks 4.1.** The Rough ideal is $RS(X)T = \{RS(X)\Delta RS(Y) \mid RS(Y) \in T\}$.

Next, we prove a representation theorem for these principal ideals and for any information system $I = (U, A)$, $T = \{RS(X) : X \subseteq U\}$ and $X = E_X \cup P_X$ where $E_X$ be the union of equivalence classes which is completely contained in $X$, i.e., $E_X = \{x \in U \mid [x]_P \subseteq X\}$ and $P_X = \bigcup_{i=1}^{r} X_{\alpha_i}$ where $X_{\alpha_i}$ are the proper subset of all indiscernable classes also $\overline{P_X}$ be the union of equivalence classes containing the elements of $P_X$.

For example, let $X = \{x_1, x_2, x_3\}$ as in Table 1. Then, $X = E_X \cup P_X$, where $E_X = \{x_1, x_3\}$, $P_X = \{x_2\}$ and $\overline{P_X} = \{x_2, x_4, x_6\}$.

Using this characterization for any subset $X$ of $U$, we have the following theorem representing the lower and upper approximation of the elements of the rough ideals.

**Theorem 4.1.** *If $I$=$(U, A)$ be an information system and $T$=$\{RS(X) : X \subseteq U\}$, for any $RS(Y) \in T$, then $RS(X\Delta Y) = (E_X \cup E_Y, E_X \cup \overline{P_X} \cup E_Y \cup \overline{P_Y})$*

**Proof.** For $X \subseteq U$, $X = E_X \cup P_X$ and for $Y \subseteq U$, $Y = E_Y \cup P_Y$

$$RS(X\Delta Y) = (\underline{P}(X\Delta Y), \overline{P}(X\Delta Y)) \tag{5}$$

$$\underline{P}(X\Delta Y) = [x]_P \subseteq X\Delta Y$$

$$= E_X \cup E_Y \tag{6}$$

$$\overline{P}(X\Delta Y) = [x]_P \cap X\Delta Y \neq \phi$$

$$= E_X \cup \overline{P_X} \cup E_Y \cup \overline{P_Y} \tag{7}$$

From (5), (6) and (7) we have,

$$RS(X\Delta Y) = (E_X \cup E_Y, E_X \cup \overline{P_X} \cup E_Y \cup \overline{P_Y}) \tag{8}$$

This completes the proof. ∎

In the following section the described concepts are illustrated through examples.

## 5. Examples

**Example 5.1.** Let us consider The following information system $I = (U, A)$, where $U = \{x_1, x_2, \ldots, x_6\}$ as in table 1, then $(T, \Delta)$ is a Commutative Monoid.

As we have, from table 1, $|T| = 18$ [13], also the Cayley's table of $(T, \Delta)$ will be very large. So, we have illustrated through one simple example.

(Closure axiom) Let $X = \{x_1, x_2, x_3, x_4\} \subseteq U$, $Y = \{x_5, x_6\} \subseteq U$ from equations (1), (2) and (3). We have, $X\Delta Y = X \cup Y$ if $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$ but from $X \cup Y$ we have $IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y)$. so by definition of $Ind.weight$ deleting the elements from the new class, we get $X\Delta Y = X \cup Y = \{x_1, x_2, x_3, x_4, x_5\}$.

Therefore, $RS(X\Delta Y) = \{[x_1]_p \cup [x_5]_p, [x_1]_p \cup [x_2]_p \cup [x_5]_p\}$.
Hence, $RS(X\Delta Y) \in T$.

(Associative axiom) Let us consider from closure $X\Delta Y = \{x_1, x_2, x_3, x_4, x_5\}$ and let $Z = \{x_2, x_3\}$ then from the definition of *Ind.weight* we have $(X\Delta Y)\Delta Z = \{x_1, x_2, x_3, x_4, x_5\}$

(9) Therefore, $RS((X\Delta Y)\Delta Z) = \{[x_1]_p \cup [x_5]_p, [x_1]_p \cup [x_2]_p \cup [x_5]_p\}$.

Also, $Y\Delta Z = Y \cup Z$ if $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$ so, $Y\Delta Z = \{x_2, x_3, x_5, x_6\}$ and $X\Delta(Y\Delta Z) = \{x_1, x_2, x_3, x_4, x_5\}$

(10) Therefore, $RS(X\Delta(Y\Delta Z)) = \{[x_1]_p \cup [x_5]_p, [x_1]_p \cup [x_2]_p \cup [x_5]_p\}$.

From (9) and (10), we have $RS((X\Delta Y)\Delta Z) = RS(X\Delta(Y\Delta Z))$.

(Identity axiom) For $U=\{x_1, x_2, ..., x_6\}$ as in Table 1 and $X=\{x_1, x_2, x_3, x_4\} \subseteq U$, there exist an empty set $\phi$ such that

(11) $$RS(X\Delta\phi) = RS(\phi\Delta X) = RS(X) = \{[x_1]_p, [x_1]_p \cup [x_2]_p\}.$$

Thus, we have $(T, \Delta)$ is a Rough monoid.

(Commutative axiom) Let us consider, $U = \{x_1, x_2, ..., x_6\}$ as in Table 1 and let $X = \{x_1, x_2, x_3, x_4\} \subseteq U$, $Y = \{x_5, x_6\} \subseteq U$ also from equations (1), (2) and (3). We have, $X\Delta Y = X \cup Y$ if $IW(X \cup Y) = IW(X) + IW(Y) - IW(X \cap Y)$ but from $X \cup Y$ we have $IW(X \cup Y) > IW(X) + IW(Y) - IW(X \cap Y)$. so by definition of *Ind.weight* deleting the elements from the new class, we get $X\Delta Y = X \cup Y = \{x_1, x_2, x_3, x_4, x_5\} = Y \cup X = Y\Delta X$.
Therefore, $RS(X\Delta Y) = \{[x_1]_p \cup [x_5]_p, [x_1]_p \cup [x_2]_p \cup [x_5]_p\} = RS(Y\Delta X)$.

$(T, \Delta)$ is a Commutative rough monoid.

**Example 5.2.** (Regular Monoid) Let us consider, $U = \{x_1, x_2, \ldots, x_6\}$ as in Table 1 and let $X = \{x_1, x_2, x_3\} \subseteq U$ , $Y = \{x_4, x_6\} \subseteq U$ also from equations (1), (2) and (3). We have, $X\Delta Y = X \cup Y = \{x_1, x_2, x_3\}$ then

$$RS(X\Delta Y) = RS(X) = \{[x_1]_p, [x_1]_p \cup [x_2]_p\} = RS(X\Delta Y\Delta X).$$

### 5.1. Examples for Ideals

- The ideals for $RS(X_1)\Delta T$ are $RS(U)$, $RS(X_1)$, $RS(X_1 \cup X_2)$, $RS(X_1 \cup X_3)$, $RS(X_1 \cup \{x_2\})$, $RS(X_1 \cup \{x_2\} \cup X_3)$.
- The ideals for $RS(X_3)\Delta T$ are $RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(X_3), RS(U)$, $RS(\{x_1\} \cup X_3), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup X_2 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3)$, $RS(\{x_1\} \cup \{x_2\} \cup X_3)$.

- The ideals for $RS(\{x_1\})\Delta T$ are $RS(U), RS(X_1), RS(X_1 \cup X_2), RS(X_1 \cup X_3)$, $RS(X_1 \cup \{x_2\})$, $RS(\{x_1\} \cup X_2)$, $RS(\{x_1\} \cup X_3)$, $RS(\{x_1\} \cup X_2 \cup X_3)$, $RS(X_1 \cup \{x_2\} \cup X_3)$, $RS(\{x_1\} \cup X_2 \cup X_3)$, $RS(\{x_1\})$, $RS(\{x_1\} \cup \{x_2\})$, $RS(\{x_1\} \cup \{x_2\} \cup X_3)$.

## 6. Conclusion

In this paper, we have introduced a new operation Praba $\Delta$ on the set of all rough sets $T$ for a given information system $I = (U, A)$. Also we have proved $(T, \Delta)$ is a commutative regular monoid of idempotents. We also gave a characterization for the principal rough ideal in $T$. The future study is to investigate and explore this regular monoid.

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