PROPERTIES OF (γ, γ')-SEMIOPEN SETS

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Abstract. In this paper, we introduce and study the concept of (γ, γ')-θ-semiconnectedness and (γ, γ')-θ-semicomponents using (γ, γ')-θ-semiopen sets.

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1. Introduction

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real Analysis concerns the various modified forms of continuity, separation axioms etc. by utilizing generalized open sets. Kasahara [3] defined the concept of an operation on topological spaces. Umehara et. al. [9] introduced the notion of $\tau_{(\gamma,\gamma')}$ which is the collection of all (γ, γ')-open sets in a topological space (X, τ). Recently, G.S.S. Krishnan and K. Balachandran (see [4], [6], [5] and [8]) studied in this field. In [1] the authors, introduced the notion of (γ, γ')-semiopeness and investigated its fundamental properties. In this paper, we introduce and study the concept of (γ, γ')-θ-semiconnectedness and (γ, γ')-θ-semicomponents using (γ, γ')-θ-semiopen sets.

2. Preliminaries

Definition 2.1 [3] Let (X, τ) be a topological space. An operation γ on the topology τ is function from τ on to power set $\mathcal{P}(X)$ of X such that $V \subset V^{\gamma}$ for each $V \in \tau$, where V^{γ} denotes the value of γ at V. It is denoted by $\gamma : \tau \to \mathcal{P}(X)$.

Definition 2.2 [9] A subset A of a topological space (X, τ) is said to be (γ, γ')-open set if, for each $x \in A$, there exist open neighborhoods U and V of x such that $U^{\gamma} \cup V^{\gamma'} \subset A$. The complement of (γ, γ')-open set is called (γ, γ')-closed. $\tau_{(\gamma,\gamma')}$ denotes set of all (γ, γ')-open sets in (X, τ).

Remark 2.3 It is easily to see that $\tau_{(\gamma,\gamma')} = \tau_{\gamma} \cap \tau_{\gamma'}$, where τ_{γ} denotes set of all γ-open sets in (X, τ).

Definition 2.4 [2] Let A be a subset of a space X. A point $x \in A$ is said to be (γ, γ')-interior point of A, if there exist open neighborhoods U and V of x such that $U^{\gamma} \cup V^{\gamma'} \subset A$ and we denote the set of all such points by $\mathrm{Int}_{(\gamma,\gamma')}(A)$. Thus $\mathrm{Int}_{(\gamma,\gamma')}(A) = \{x \in A : x \in U \in \tau, V \in \tau$ and $U^{\gamma} \cup V^{\gamma'} \subset A$. Note that A is (γ, γ')-open if and only if $A = \mathrm{Int}_{(\gamma,\gamma')}(A)$. A set A is called (γ, γ')-closed if and only if $X \backslash A$ is (γ, γ')-open.

Definition 2.5 [9] A point $x \in X$ is called a (γ, γ')-closure point of $A \subset X$, if $(U^{\gamma} \cup V^{\gamma'}) \cap A \neq \emptyset$, for any open neighborhoods U and V of x. The set of all (γ, γ')-closure points of A is called (γ, γ')-closure of A and is denoted by $\mathrm{cl}_{(\gamma,\gamma')}(A)$. A subset A of X is called (γ, γ')-closed, if $\mathrm{cl}_{(\gamma,\gamma')}(A) \subset A$. Note that $\mathrm{cl}_{(\gamma,\gamma')}(A)$ is contained in every (γ, γ')-closed superset of A.

Definition 2.6 [1] A subset A of a space (X, τ) is said to be (γ, γ')-semiopen set, if there exists a (γ, γ')-open set O such that $O \subset A \subset \mathrm{cl}_{(\gamma,\gamma')}(O)$. The set of all (γ, γ')-semiopen sets is denoted by $SO_{(\gamma,\gamma')}(X)$. A is (γ, γ')-semiclosed if and only if $X \backslash A$ is (γ, γ')-semiopen in X. Note that A is (γ, γ')-semiclosed if and only if $\mathrm{Int}_{(\gamma,\gamma')}(\mathrm{cl}_{(\gamma,\gamma')}(A)) \subset A$.

Remark 2.7 Observe that if $\{A_i : i \in I\}$ is a collection of (γ, γ')-open set, then $\bigcup_{i \in I} A_i$ is (γ, γ')-open set. From this, it follows that $\mathrm{cl}_{(\gamma,\gamma')}(A)$ is a (γ, γ')-closed set. Moreover, if we define $\beta : \tau \to \mathcal{P}(X)$ as $\beta(U) = \mathrm{cl}_{(\gamma,\gamma')}(U)$, then β is a monotone operation on τ.

Definition 2.8 [1] Let A be a subset of a space X. The intersection of all (γ, γ')-semiclosed sets containing A is called (γ, γ')-semiclosure of A and is denoted by $s\mathrm{cl}_{(\gamma,\gamma')}(A)$. Note that A is (γ, γ')-semiclosed if and only if $s\mathrm{cl}_{(\gamma,\gamma')}(A) = A$.

Definition 2.9 [1] Let A be a subset of a space X. The union of (γ,γ')-semiopen subsets of A is called (γ,γ')-semiinterior of A and is denoted by $s\mathrm{Int}_{(\gamma,\gamma')}(A)$.

Definition 2.10 [1] A point $x \in X$ is said to be (γ,γ')-semi-θ-adherent point of a subset A of X, if $s\mathrm{cl}_{(\gamma,\gamma')}(U) \cap A \neq \emptyset$, for every $U \in SO_{(\gamma,\gamma')}(X,x)$. The set of all (γ,γ')-semi-θ-adherent points of A is called the (γ,γ')-semi-θ-closure of A and is denoted by $s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$. A subset A is called (γ,γ')-semi-θ-closed, if $s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A) = A$. A subset A is called (γ,γ')-semi-θ-open, if and only if $X\backslash A$ is (γ,γ')-semi-θ-closed.

Definition 2.11 [1] A subset A of a space X is said to be (γ,γ')-semiregular, if it is both (γ,γ')-semiopen and (γ,γ')-semiclosed. The class of all (γ,γ')-semiregular sets of X is denoted by $SR_{(\gamma,\gamma')}(A)$.

3. (γ,γ')-semi-θ-closed sets

Theorem 3.1 *For a subset A of a topological space X, $s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A) = \cap\{V : A \subset V$ and V is $SR_{(\gamma,\gamma')}(X)\}$.*

Proof. Let $x \notin s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$. Then there exists a (γ,γ')-semiopen set U containing x such that $s\mathrm{cl}_{(\gamma,\gamma')}(U) \cap A = \emptyset$. Then $A \subset X\backslash s\mathrm{cl}_{(\gamma,\gamma')}(U) = V$ (say). Thus $V \in SR_{(\gamma,\gamma')}(X)$ such that $x \notin V$. Hence $x \notin \cap\{V : A \subset V$ and V is $SR_{(\gamma,\gamma')}(X)\}$. Again, if $x \notin \cap\{V : A \subset V$ and V is $SR_{(\gamma,\gamma')}(X)\}$, then there exists $V \in SR_{(\gamma,\gamma')}(X)$ containing A such that $x \notin V$. Then $(X\backslash V)(= U$, say) is a (γ,γ')-semiopen set containing x such that $s\mathrm{cl}_{(\gamma,\gamma')}(U) \cap V = \emptyset$. This shows that $s\mathrm{cl}_{(\gamma,\gamma')}(U) \cap A = \emptyset$ so that $x \notin s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$. ∎

Corollary 3.2 *A subset A of X is (γ,γ')-semi-θ-closed if and only if $A = \cap\{V : A \subset V \in SR_{(\gamma,\gamma')}(X)\}$.*

Theorem 3.3 *For any subset A of X, $s_{(\gamma,\gamma')}\mathrm{cl}_\theta(s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A))) = s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$.*

Proof. Obviously, $s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A) \subset s_{(\gamma,\gamma')}\mathrm{cl}_\theta(s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)))$. Now, let

$$x \in s_{(\gamma,\gamma')}\mathrm{cl}_\theta(s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A))) \text{ and } U \in SO_{(\gamma,\gamma')}(X,x).$$

Then $s\mathrm{cl}_{(\gamma,\gamma')}(U) \cap s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A) \neq \emptyset$. Let $y \in s\mathrm{cl}_{(\gamma,\gamma')}(U) \cap s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$. Since $s\mathrm{cl}_{(\gamma,\gamma')}(U) \in SO_{(\gamma,\gamma')}(X,y)$, $s_{(\gamma,\gamma')}\mathrm{cl}(s_{(\gamma,\gamma')}\mathrm{cl}(U)) \cap A \neq \emptyset$, that is, $s_{(\gamma,\gamma')}\mathrm{cl}(U) \cap A \neq \emptyset$. Thus $x \in s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$. ∎

Corollary 3.4 *$s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$ is (γ,γ')-semi-θ-closed for any $A \subset X$.*

Theorem 3.5 *Intersection of arbitrary collection of (γ,γ')-semi-θ-closed sets in X is (γ,γ')-semi-θ-closed.*

Proof. Let $\{A_\alpha : \alpha \in \Lambda\}$ be any collection of (γ, γ')-semi-θ-closed sets in a topological space (X, τ) and $A = \bigcap_{\alpha \in \Lambda} A_\alpha$. Now, using Definition 2.10, $x \in s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$, in consequence, $x \in s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A_\alpha)$ for all $\alpha \in \Lambda$. Follows that $x \in A_\alpha$ for all $\alpha \in \Lambda$. Therefore, $x \in A$. Thus $A = s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$. ∎

Corollary 3.6 *For any $A \subset X$, $s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$ is the intersection of all (γ, γ')-semi-θ-closed sets each containing A.*

Remark 3.7 The following example shows that the union of (γ, γ')-semi-θ-closed sets may fail to be (γ, γ')-semi-θ-closed. Thus (γ, γ')-semi-θ-closure operator is not a Kuratowski's closure operator.

Example 3.8 Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Let $\gamma : \tau \to \mathcal{P}(X)$ and $\gamma' : \tau \to \mathcal{P}(X)$ be operators defined as follows: for every $A \in \tau$,

$$\gamma(A) = \mathrm{Int}(\mathrm{cl}\,(A))$$

$$\gamma'(A) = \begin{cases} X & \text{if A = \{a,b\},} \\ A & \text{if A} \neq \text{\{a,b\}.} \end{cases}$$

Then the subsets $\{a\}$ and $\{b\}$ are (γ, γ')-semi-θ-closed, but their union $\{a, b\}$=$A \cup B$ is not (γ, γ')-semi-θ-closed.

Example 3.9 Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Let $\gamma : \tau \to \mathcal{P}(X)$ and $\gamma' : \tau \to \mathcal{P}(X)$ be operators defined as follows: for every $A \in \tau$,

$$\gamma(A) = \begin{cases} X & \text{if A = \{a,b\},\{c\}} \\ A & \text{if A} \neq \text{\{a,b\},\{c\}.} \end{cases}$$

$$\gamma'(A) = \begin{cases} X & \text{if A = \{a,b\},\{c\},\{a,c\}} \\ A & \text{if A} \neq \text{\{a,b\},\{c\},\{a,c\}.} \end{cases}$$

$\{c\}$ is (γ, γ')-semi-θ-closed but not (γ, γ')-semiregular.

Remark 3.10 It is proved by Carpintero et. al. [1] that a (γ, γ')-semiregular set is (γ, γ')-semi-θ-closed set. In the above example, $\{c\}$ is (γ, γ')-semi-θ-closed but not (γ, γ')-semiregular. Again, for a subset A, we always have $A \subset s_{(\gamma,\gamma')}\mathrm{cl}\,(A) \subset s_{(\gamma,\gamma')}\mathrm{cl}_\theta(A)$. Therefore, every (γ, γ')-semi-θ-open set is (γ, γ')-semiopen. The following example shows that the converse is not true in general.

Example 3.11 Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ and $\gamma : \tau \to P(X)$ and $\gamma' : \tau \to P(X)$ be operators defined as follows: for every $A \in \tau$,

$$\gamma(A) = \begin{cases} \mathrm{cl}\,(A) & \text{if A = \{a\},} \\ A & \text{if A} \neq \text{\{a\},} \end{cases}$$

$$\gamma'(A) = \begin{cases} \mathrm{cl}\,(A) & \text{if A = \{b\},} \\ A & \text{if A} \neq \text{\{b\}.} \end{cases}$$

Then $\{a, b\}$ is a (γ, γ')-semiopen set but not a (γ, γ')-semi-θ-open set.

Remark 3.12 The notions (γ, γ')-openness and (γ, γ')-semi-θ-openness are independent. In Example 3.9, $\{a, b\}$ is a (γ, γ')-semi-θ-open set but not a (γ, γ')-open set, whereas in Example 3.11, $\{b\}$ is a (γ, γ')-open set but not a (γ, γ')-semi-θ-open set.

4. (γ, γ')-semi-θ-connectedness

Definition 4.1 A pair (A, B) of nonempty subsets of a topological space (X, τ) is said to be a (γ, γ')-semi-θ-separation relative to X, if $s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(A) \cap B = A \cap s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(B) = \emptyset$.

Definition 4.2 A subset N of a topological space (X, τ) is said to be (γ, γ')-semi-θ-connected relative to X (or, simply (γ, γ')-semi-θ-connected), if there exists no (γ, γ')-semi-θ-separation (A, B) relative to X such that $N = A \cup B$. If N is not (γ, γ')-semi-θ-connected, then it is called (γ, γ')-semi-θ-disconnected.

Lemma 4.3 *If (A, B) is a (γ, γ')-semi-θ-separation relative to X and P, Q are nonempty subsets of A and B respectively, then (P, Q) is also a (γ, γ')-semi-θ-separation relative to X.*

Proof. The proof is obvious. ∎

Theorem 4.4 *For a subset A of X, the following are equivalent:*

1. *A is (γ, γ')-semi-θ-connected.*
2. *If (L, M) is a (γ, γ')-semi-θ-separation relative to X and $A \subset L \cup M$, then either $A \subset L$ or $A \subset M$.*

Theorem 4.5 *Let A be a (γ, γ')-semi-θ-closed set in a topological space (X, τ). If (P, Q) is a (γ, γ')-semi-θ-separation relative to X such that $A = P \cup Q$, then P and Q are (γ, γ')-semi-θ-closed sets in X.*

Proof. We have $A = P \cup Q$, where $s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(P) \cap Q = P \cap s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(Q) = \emptyset$. Now, $A \cap s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(P) = (P \cup Q) \cap s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(P) = (P \cap s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(P)) \cup (Q \cap s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(P)) = P$. In view of Corollary 3.4 and Theorem 3.5, P is (γ, γ')-semi-θ-closed. Similarly, Q is (γ, γ')-semi-θ-closed. ∎

Theorem 4.6 *If C is a (γ, γ')-semi-θ-connected set relative to a topological space X and $C \subset T \subset s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(C)$, then T is also (γ, γ')-semi-θ-connected.*

Corollary 4.7 *The (γ, γ')-semi-θ-closure of a (γ, γ')-semi-θ-connected set is (γ, γ')-semi-θ-connected.*

Remark 4.8 Since for any subset A of X, $A \subset s_{(\gamma,\gamma')}\mathrm{cl}\,(A) \subset s_{(\gamma,\gamma')}\mathrm{cl}_{\theta}(A)$, the (γ, γ')-semiclosure of a (γ, γ')-semi-θ-connected set is (γ, γ')-semi-θ-connected set.

Theorem 4.9 *A set E in a topological space X is (γ, γ')-semi-θ-connected if and only if any two points of E are contained in some (γ, γ')-semi-θ-connected set contained in E.*

Theorem 4.10 *The union of any collection of (γ, γ')-semi-θ-connected sets, no two of which are (γ, γ')-semi-θ-separated relative to X is a (γ, γ')-semi-θ-connected set.*

Corollary 4.11 *The union of any family of (γ, γ')-semi-θ-connected sets, every pair of which has an element in common is (γ, γ')-semi-θ-connected.*

Theorem 4.12 *For a topological space (X, τ) the following statements are equivalent:*

1. *X is (γ, γ')-semi-θ-connected.*
2. *There do not exist disjoint nonempty (γ, γ')-semi-θ-clopen sets A, B in X such that $X = A \cup B$.*
3. *The only subset of X which are both (γ, γ')-semi-θ-open and (γ, γ')-semi-θ-closed are X and $\emptyset$.*

Definition 4.13 [7] A topological space (X, τ) is said to be a (γ, γ')-connected, if there does not exist a pair A, B of nonempty disjoint (γ, γ')-open subsets of X such that $X = A \cup B$, otherwise X is called (γ, γ')-disconnected.

The relation between (γ, γ')-semi-θ-connectedness and (γ, γ')-connectedness, is as follows.

Theorem 4.14 *Every (γ, γ')-semi-θ-connected space X is (γ, γ')-connected.*

Proof. If X is (γ, γ')-disconnected, there exist nonempty disjoint (γ, γ')-closed sets U and V such that $X = U \cup V$. Then U and V are (γ, γ')-semiregular sets and hence each is (γ, γ')-semi-θ-closed. Thus X is not (γ, γ')-semi-θ-connected. ∎

Remark 4.15 The converse of the above theorem is false. In fact, in Example 3.9 the topological space (X, τ) is (γ, γ')-connected but not (γ, γ')-semi-θ-connected.

5. (γ, γ')-semi-θ-components and (γ, γ')-semi-θ-quasi components

Definition 5.1 Let (X, τ) be a topological space. A maximal (γ, γ')-semi-θ-connected subset relative to X, that is, an (γ, γ')-semi-θ-connected set in X which is not properly contained in any larger (γ, γ')-semi-θ-connected set of X, is called a (γ, γ')-semi-θ-component of the space X.

Theorem 5.2 *For a topological space (X, τ), the following statements are true:*

1. *Each point of X is contained in exactly one (γ, γ')-semi-θ-component of X.*
2. *X is equal to the union of its (γ, γ')-semi-θ-components.*
3. *Each (γ, γ')-semi-θ-connected subset of X is contained in an (γ, γ')-semi-θ-component of X.*
4. *Each (γ, γ')-semi-θ-component of X is (γ, γ')-semi-θ-closed.*
5. *Two different (γ, γ')-semi-θ-components of X are disjoint.*

Proof. (1). Let $x \in X$. Consider the class $\{C_\alpha : \alpha \in \Lambda\}$ of all (γ, γ')-semi-θ-connected sets of X, each of which contains x. The class is nonempty, since $\{x\}$ belongs to the class. By Corollary 4.11, $C = \bigcup_{\alpha\in\Lambda} C_\alpha$ is a maximal (γ, γ')-semi-θ-connected set containing x that is, a (γ, γ')-semi-θ-component of X. Uniqueness of the (γ, γ')-semi-θ-component (containing x) is obvious. (2). Follows from (1). (3). By construction in (1), any (γ, γ')-semi-θ-connected set is contained in the (γ, γ')-semi-θ-component which contains any one of its points. (4). Follows from Corollary 4.7. (5). If C_1 and C_2 are two (γ, γ')-semi-θ-components and $C_1 \cap C_2 \neq \emptyset$, then by Corollary 4.11, $C_1 \cup C_2$ is a (γ, γ')-semi-θ-connected set in X. By the maximality of C_1 and C_2, $C_1 = C_1 \cup C_2 = C_2$. ■

Definition 5.3 Let A be a nonempty subset of X. Two elements x, y of A are said to be (γ, γ')-semi-θ-equivalent, written $x \backsim y$, if whenever (L, M) is a (γ, γ')-semi-θ-separation relative to X and $A = L \cup M$, we have either $x, y \in L$ or $x, y \in M$.

The relation " $\backsim$ " is an equivalence relation on A. The equivalence classes are called (γ, γ')-semi-θ-quasi components of A. We shall denote by $A(x)$ the (γ, γ')-semi-θ-quasi component of A containing x. For a nonempty subset A of X, let us call a subset P of A a (γ, γ')-semi-θ-component of A if P is (γ, γ')-semi-θ-connected and there does not exist any subset Q of A such that $P \nsubseteq Q$ and Q is (γ, γ')-semi-θ-connected.

Theorem 5.4 *Let (X, τ) be a topological space and A a nonempty subset of X. Then $A(x)$ is an (γ, γ')-semi-θ-component of A for each $x \in A$ for which $A(x)$ is (γ, γ')-semi-θ-connected.*

Proof. Let $x \in A$ for which $A(x)$ is (γ, γ')-semi-θ-connected and $B \subset A$ be a (γ, γ')-semi-θ-connected set and also assume that $A(x) \subset B$. If $Y \in B$ and $A = L \cup M$, where (L, M) is a (γ, γ')-semi-θ-separation relative to X, then either $B \subset L$ or $B \subset M$. Thus either $x, y \in L$ or $x, y \in M$. Hence $y \in A(x)$ and consequently, $A(x) = B$. Thus $A(x)$ is a (γ, γ')-semi-θ-component of A. ■

Definition 5.5 Let A be a nonempty subset of X and $x \in A$. We define $L_x(A) = \{L \subset X :$ for some $M \subset X, (L, M)$ is a (γ, γ')-semi-θ-separation relative to X satisfying $A = L \cup M$ and $x \in L\}$.

Theorem 5.6 *Let A be a nonempty subset of a topological space (X, τ) and $x \in A$. Then a (γ, γ')-semi-θ-quasi component $A(x)$ of A is the intersection of the elements of $L_x(A)$.*

Proof. Let $y \in A(x)$ and L_x be any member of $L_x(A)$. Then for some $M_x \subset X$, (L_x, M_x) is a (γ, γ')-semi-θ-separation relative to X satisfying $A = L_x \cup M_x$ and $x \in L_x$. Since $x \backsim y$, we have $x, y \in L_x$. Thus $y \in \cap L_x(A)$; hence $A(x) \subset \cap L_x(A)$. On the other hand, if $y \in \cap L_x(A)$ then obviously, $y \in A(x)$. ∎

Theorem 5.7 *Each (γ, γ')-semi-θ-quasi component Q in the topological space (X, τ) is the intersection of all (γ, γ')-semi-θ-clopen sets containing a given point belonging to Q.*

Proof. Straightforward. ∎

Corollary 5.8 *Every (γ, γ')-semi-θ-quasi component of a topological space (X, τ) is (γ, γ')-semi-θ-closed in X.*

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