INFLUENCE OF MICROPOLAR PARAMETERS ON THE STABILITY DOMAIN IN A RAYLEIGH-BÉNARD CONVECTION PROBLEM – A RELIABLE NUMERICAL STUDY

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Abstract. The main objective of this paper is the determination in the parameter space of the neutral hypersurface which separates the domain of stability from the instability domain in a problem of stationary convection in a micropolar fluid [1]. The influence of each of the micropolar parameters on the eigenparameter represented by the Rayleigh number is investigated using a spectral-Galerkin method based on expansion functions proved to assure an exponential convergence and small computational time expenses.

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1. Introduction

Micropolar fluids were first presented as a particular case of microfluids in which the coupling between the spin of each particle and the macroscopic velocity field was taken into account [6]. The constitutive equations for micropolar fluids are considered in fact non-Newtonian fluid models [6] since apart from the classical velocity field, a microrotation vector and a gyration parameter are introduced in order to perform an analysis of the kinematics of microrotation. All materials, whether natural or synthetic, posses microstructure. Micropolar fluid models are very useful in explaining the flow of colloidal suspensions, liquid crystal, fluids with additives and animal bloods. The theory of micropolar fluids takes into account the microscopic effects arising from their local structure and microrotation of the fluids elements. Extensive reviews on this theory and its application can be found in recent books of Eringen [7] and Lukasziewicz [9].

Thermal effects in micropolar fluid flow problems have been extensively investigated due also to a large number of applications in engineering problems. It is now well known ([12]-[20]) that stationary convection is the preferred mode for onset of Rayleigh-Bénard convection in Eringen's micropolar fluids. Thus, the determination of a criterion for the onset of convection in a horizontal micropolar fluid layer heated from below follows a classical problem procedure. Ramma Rao [12] studied the onset of convection for a heat conducting micropolar fluid layer between two rigid boundaries. Sastry and Rao [13] and also Qin and Kaloni [10] report on the instability of a rotating micropolar fluid. Siddheshwar and Pranesh [19] investigated the individual effects of a nonuniform temperature gradient and a magnetic field on convection in micropolar fluids.

In most of the aforementioned papers, the linear stability of the micropolar fluid layer has been investigated in the rigid boundaries case or for free boundaries case using severe truncations in the series (in the Galerkin method used). With this in mind, the purpose of the present paper is to numerically examine the influence of the micropolar parameters on the stability of a micropolar fluid layer heated from below in the case of free, isothermal, spin-vanishing boundaries. The results provide important characteristics of the stability domain for various values of the micropolar parameters and show a good agreement with existing results in the classical case.

The outline of the paper is as follows: in Section 2 the eigenvalue problem governing the linear stability of the flow is presented along with the proposed numerical method (the spectral-Galerkin method based on a certain class of polynomial approximation). Section 3 is devoted to numerical results that can be used to better understand the specific features of the stability domain at a qualitative level. The paper ends with a conclusions section summarizing the main results of the paper.

2. The mathematical model: the analytical and numerical approaches

The onset of convection in the horizontal layer of micropolar fluid heated from below is governed by the conservation of mass, balance of linear momentum, balance of moment of momentum, respectively [1]

$$
\begin{aligned}
&-\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + (1+K)\nabla^2\mathbf{u} + K\nabla\times\phi + R\theta\mathbf{k} = 0,\\
&\mathbf{j}\Big(\frac{\partial \phi}{\partial t} + \mathbf{u}\cdot\nabla\phi\Big) = K(\nabla\times\mathbf{u} - 2\phi) - C_0\nabla\times\nabla\times\phi + C_1\nabla\nabla\cdot\phi,\\
&Pr\Big(\frac{\partial \theta}{\partial t} + \mathbf{u}\cdot\nabla\theta\Big) = \nabla^2\theta + Rw,
\end{aligned}
\tag{1}
$$

with $\mathbf{u} = (u, v, w)$, p, θ, ϕ the perturbations of the governing fields of velocity, pressure, temperature and spin, $C_0 = \dfrac{\gamma}{\mu l^2}$ the couple stress parameter accounts for the spin diffusion, $C_1 = \dfrac{\alpha+\beta+\gamma}{\mu l^2}$, K the parameter coupling vorticity and spin effects, $Ra = R^2$ is the Rayleigh number, $\mathbf{k}$ the unit vector in the z-direction, μ, α, β,γ coefficients of viscosity. The curl operator is then applied on $(1)_2$, and the z-component is taken. In [1] it is proven that the principle of exchange of stability holds, so the marginal stability is characterized by non-oscillatory motion, i.e., the following form of solution for the perturbations can be assumed

$$(w, \theta, \xi) = (W, \Theta, Z)(z) e^{i(a_x x + a_y y)}, \tag{2}$$

where $\xi = (\nabla \times \phi)_z = \dfrac{\partial \phi_y}{\partial x} - \dfrac{\partial \phi_z}{\partial y}$ and $a^2 = a_x^2 + a_y^2$. In these conditions, the neutral stability of the thermal conduction against normal mode perturbations (2) is governed by an eigenvalue problem consisting of a system of linear ordinary differential equations with constant coefficients and a set of boundary conditions taken on the two boundaries. Following [1], this problem is reduced to the eigenvalue problem

$$\begin{cases} (1+K)(D^2-a^2)^2 W + K(D^2-a^2)Z - a^2 R\Theta = 0, \\ K[(D^2-a^2)W + 2Z] - C_0(D^2-a^2)Z = 0, \\ (D^2-a^2)\Theta + RW = 0, \end{cases} \tag{3}$$

$$W = D^2 W = Z = \Theta = 0 \text{ at } z = 0, 1, \tag{4}$$

where $D = \dfrac{d}{dz}$ represents the derivative with respect to the spatial independent variable, a is the wavenumber, W, Θ, Z are the perturbations amplitudes for the vertical component of the velocity, temperature and spin, respectively.

The main goal in the numerical investigation of the two-point boundary value problem (3)–(4) is to determine the critical eigenvalues Ra_{cr} represented by the first eigenvalues and separating the domain of stability from the instability domain.

Spectral methods arise from the fundamental problem of approximation of a function on an interval and are very much successful in obtaining the numerical solution of ordinary differential equations. They can be used successfully to solve classes of equations by expanding the solution function as a finite series of very smooth basis functions. In practice, summations in the expansions of the unknown functions are truncated to some finite number of terms in the approximation, N, for which the higher order terms become essentially negligible. During the last few decades optimization became a very important objective which has expanded in many directions. For this purpose new algorithmic and theoretical techniques have been developed.

Polynomials based spectral methods are widely used compared to trigonometric functions based ones since they have the characteristic of a very rapid convergence and also reduce the computational evaluations especially in the orthogonal polynomials case. The eigenvalues are computed here by a Galerkin type spectral method involving combinations of Chebyshev polynomials which are defined on $[-1,1]$. The first step of the numerical investigation consisted in shifting the interval $[a,b]=[0,1]$ into $[-1,1]$ using the transformation $x=\dfrac{2z-(a+b)}{b-a}$, i.e. $x=2z-1$. This method would transform the problem into the following generalized algebraic eigenvalue problem

$$\mathbf{A}\mathbf{u}_\mathbf{N} = R\mathbf{B}\mathbf{u}_\mathbf{N}, \tag{5}$$

where $\mathbf{A}=\begin{pmatrix}\mathbf{A_1}\\ \mathbf{1}_{bc}^{+}\\ \mathbf{1}_{bc}^{-}\end{pmatrix}$ and $\mathbf{B}=\begin{pmatrix}\mathbf{B_1}\\ \mathbf{0}_{bc}\\ \mathbf{0}_{bc}\end{pmatrix}$ corresponding to

$$\mathbf{A_1} = \begin{pmatrix}(1+K)(4D^2-a^2\mathcal{I})^2 & K(4D^2-a^2\mathcal{I}) & \mathcal{O}\\ K(4D^2-a^2\mathcal{I}) & -C_0(4D^2-a^2\mathcal{I})+2K\mathcal{I} & \mathcal{O}\\ \mathcal{O} & \mathcal{O} & 4D^2-a^2\mathcal{I}\end{pmatrix},$$

$$\mathbf{B_1} = \begin{pmatrix}\mathcal{O} & \mathcal{O} & a^2\mathcal{I}\\ \mathcal{O} & \mathcal{O} & \mathcal{O}\\ -\mathcal{I} & \mathcal{O} & \mathcal{O}\end{pmatrix}$$

completed with the boundary conditions for ± 1. The D^2, $\mathcal{I}$, $\mathcal{O}$ represent the second order differentiation matrix, the unitary matrix and the zero entries matrix of order $N+1$ and the vector $\mathbf{u_N}$ contains the values of the approximations coefficients of W, Z, Θ with respect to the approximation functions in the discretization process, i.e., $\mathbf{u_N}=(\widehat{W}_0,\ldots,\widehat{W}_N,\widehat{Z}_0,\ldots,\widehat{Z}_N,\widehat{\Theta}_0,\ldots,\widehat{\Theta}_N)^T$. In the following, this $5(N+1)\times 3(N+1)$ system is modified in order to have only Dirichlet type boundary conditions.

By taking into account the order of differentiation in (3) - (4) we introduce the new variable $\Psi := (D^2-a^2)W$, a "D^2" strategy that we successfully applied in previous papers [4], [8]. Multiplying the first equation by $(1+K)$ and the last one by $-a^2$, the two-point boundary value problem (3) - (4) can be rewritten as the second order system described by a differential operator defined on the Sobolev space $[H^2(-1,1)]^4$, $L:\mathcal{A}\to[H^2(-1,1)]^4$ with

$$\mathcal{A}=\{\mathbf{U}=(\Psi,W,Z,\Theta),\mathbf{U}\in[H^2(-1,1)]^4,\mathbf{U}=\mathbf{0}\text{ at } x=\pm1\}$$

and

$$L\mathbf{U}=\mathbf{0}, \tag{6}$$

$$L=\begin{pmatrix} -(1+K)\mathcal{I} & (1+K)(4D^2-a^2\mathcal{I}) & \mathcal{O} & \mathcal{O} \\ (1+K)(4D^2-a^2\mathcal{I}) & \mathcal{O} & K(4D^2-a^2\mathcal{I}) & -a^2R\mathcal{I} \\ \mathcal{O} & K(4D^2-a^2\mathcal{I}) & 2K\mathcal{I}-C_0(4D^2-a^2\mathcal{I}) & \mathcal{O} \\ \mathcal{O} & -a^2R\mathcal{I} & \mathcal{O} & -(4D^2-a^2\mathcal{I}) \end{pmatrix}.$$

The operator L is selfadjoint. Let us consider the inner product $(L\mathbf{U},\mathbf{V})$ obtained by multiplying the equation from (6) by $\mathbf{V}=(\Psi^*,W^*,Z^*,\Theta^*)\in\mathcal{A}$ respectively, by adding the results and then integrating the obtained sum over $(-1,1)$. Integrating by parts in this expression and taking into account the boundary conditions, we are lead to $(L\mathbf{U},\mathbf{V})=(\mathbf{U},L^*\mathbf{V})$ with L^* defined as L, i.e., $L=L^*$.

We define the functional $J:\mathcal{A}\to\mathbb{R}$, $J(\mathbf{U})=(L\mathbf{U},\mathbf{U})$ with

$$\begin{aligned} J(\mathbf{U}) \;=\;& -(1+K)\int_{-1}^{1}\Psi^2dx+2K\int_{-1}^{1}Z^2dx+C_0\int_{-1}^{1}(4(DZ)^2+a^2Z^2)dx \\ &+\int_{-1}^{1}4(D\Theta)^2+a^2\Theta^2dx--2(1+K)\int_{-1}^{1}(4D\Psi DW+a^2W\Psi)dx \\ &-2K\int_{-1}^{1}(4DZDW+a^2ZW)dx-2a^2R\int_{-1}^{1}\Theta W\,dx. \end{aligned}$$

Then the following variational principle

$$L\mathbf{U}=0$$

holds iff $\delta J(U)=0$, in other words $\mathbf{U}$ makes the functional stationary, where δ defines the Lagrange variation of J.

In this case, the finite dimensional space X_N on which we choose to construct the eigensolution (Ψ,W,Z,Θ) would be defined by functions $(\phi_k)_{k=\overline{0,N}}$ satisfying Dirichlet type boundary conditions in ±1 such that this eigensolution belongs to $\mathcal{A}$, i.e.,

$$X_N=span\{\phi_0,\phi_1,\ldots,\phi_N\}.$$

An approximation $\mathbf{U}_N\in X_N\setminus\{\mathbf{0}\}$ of the solution has the form $\mathbf{U}_N=(\Psi_N W_N Z_N \Theta_N)^T$, with the N index related to the discretization process. The approximations Ψ_N, W_N, Z_N, Θ_N of the unknown functions are then defined as truncated series of trial functions

$$\begin{aligned} \Psi_N&=\sum_{k=0}^{N}\widehat{\Psi}_k\phi_k(x), \quad W_N=\sum_{k=0}^{N}\widehat{W}_k\phi_k(x), \\ Z_N&=\sum_{k=0}^{N}\widehat{Z}_k\phi_k(x), \quad \Theta_N=\sum_{k=0}^{N}\widehat{\Theta}_k\phi_k(x). \end{aligned} \tag{7}$$

with the unknown Fourier coefficients $\widehat{\Psi}_k$, $\widehat{W}_k$, $\widehat{Z}_k$, $\widehat{\Theta}_k$, $(k = 0, 1, 2, ..., N)$. The spectral approximation $\mathbf{U}_N$ is obtained by imposing the vanishing of the projection of the residual on the same finite dimensional space X_N

$$(L\mathbf{U}_N, \mathbf{U}_N) = 0. \tag{8}$$

Exponential convergence can be achieved with an appropriate expansion functions basis in the discretization process. The Shen basis functions [14] defined by

$$S_{1k}(x) = T_k(x) - T_{k+2}(x),\ \ k = 0, 1, \ldots, N,$$

are considered to define the space X_N in our case, which means that

$$\phi_k(x) = S_{1k}(x),\ \ k = \overline{0, N}.$$

Then we have

$$X_N = span\{\phi_0(x) = 2 - 2x^2, \phi_1(x) = 4x - 4x^3, \phi_2(x) = -8x^4 + 10x^2 - 2, \ldots\}.$$

The confidence in this choice is based on the accuracy offered by these functions in our previous investigations [5]. These functions form an orthogonal set of polynomials on $L^2(-1, 1)$ leading to convenient sparse matrices of the algebraic system obtained by minimizing the residual.

For the spectral-Galerkin method the algebraic equation allowing the recovery of the critical value of the Rayleigh number, i.e., the secular equation, has the form

$$\begin{vmatrix} -(1+K)g_{ik}^{00} & (1+K)(4g_{ik}^{20} - a^2 g_{ik}^{00}) & \mathcal{O} & \mathcal{O} \\ (1+K)(4g_{ik}^{20} - a^2 g_{ik}^{00}) & \mathcal{O} & K(4g_{ik}^{20} - a^2 g_{ik}^{00}) & -a^2 R g_{ik}^{00} \\ \mathcal{O} & K(4g_{ik}^{20} - a^2 g_{ik}^{00}) & 2K g_{ik}^{00} - C_0(4g_{ik}^{20} - a^2 g_{ik}^{00}) & \mathcal{O} \\ \mathcal{O} & -a^2 R g_{ik}^{00} & \mathcal{O} & -(4g_{ik}^{20} - a^2 g_{ik}^{00}) \end{vmatrix} = 0, \tag{9}$$

$i, k = 0, \ldots, N$, where

$$g_{ik}^{lm} = \int_{-1}^{1} D^l S_{1i}(x) D^m S_{1k}(x) \omega(x) dx,$$

the weight ω defined as usual for the Chebyshev polynomials case, i.e.,

$$\omega(x) = \frac{1}{\sqrt{1 - x^2}}.$$

This equation is obtained by imposing the condition that nonzero eigenvectors exists. Clearly, the matrices simplifies since, based on the orthogonality property of the Shen basis polynomials, g_{ik}^{00} defines the identity matrix.

Before discussing the results we note below the physical significance of the parameters:

Parameter	Nature of effect	Physical reason
K $0 \leq K \leq 1$	Stabilizing	Increase in K indicates the increase in the concentration of microelements. This elements consume the greater part of the energy of the system in developing the gyrational velocities of the fluid and as a result the onset of convection is delayed.
C_0 $0 \leq C_0 \leq m$ $m:$ finite, real	Destabilizing (as C_0 increase)	Increase in C_0 decrease the couple stress of the fluid which causes a decrease in microrotation and hence makes the system more unstable.

Table 1. Why and how of the stabilizing/destabilizing effects of the micropolar parameters ([13]).

3. Numerical analysis results

There are several physical and numerical aspects of the problem we are focused on. First, we are interesting in obtaining a good approximate value of the eigenparameter represented by the Rayleigh number for all values of the physical parameters. This was made possible by solving the secular equation, which is an algebraic equation, defining the hypersurface involving Ra, a, K, C_0. Then, the critical value of the Rayleigh number at which the instability sets in is to be obtained in each parameters space.

As used by many, severely truncated (simple polynomials based) series offer approximate values of the Rayleigh number. We employed the spectral-Galerkin method to examine the nature of the eigenparameter Ra using a family of simple polynomials (denoted by $p1$) which are constructed so as to satisfy the boundary conditions of the problem on $0, 1$, without satisfying an orthogonality relation. We write

$$\Psi = a_1 x(1-x), \qquad W = a_2 x(1-x),$$
$$Z = a_3 x(1-x), \qquad \Theta = a_4 x(1-x).$$

Replacing these expressions in (6) and imposing the condition that the left-hand side of the obtained equations be orthogonal to the same expansion polynomials, we get an algebraic system of equation in a_i, $i = 1, 2, 3, 4$, of which we seek a nontrivial solution. This condition leads us to the secular equation of the form $F(Ra, a, K, C_0) = 0$. Due to the severe truncation in the expansion terms used by many, the numerical results are not the best ones.

a	K	$Ra(p1)$	$Ra(SB1)$	realative error in $Ra(p1)$
2	0.00001	686.0034307	667.0170434	2.846462
2.22	0.1	708.7990715	690.0629974	2.715125
2.22	0.01	678.4219683	660.4888697	2.715125
2.22	0	675.0467348	657.5153226	2.666312
2.22	0.00001	675.0501098	657.5186120	2.666312
2.22	0.000001	675.0470726	657.5156523	2.666312
2.233	0.00001	675.0050713	657.5418665	2.655831
2.3	0.00001	675.7233956	658.5858859	2.602167
2	0.00003	686.0102898	667.0237150	2.846461
2.233	0.00003	675.0118211	657.5484428	2.655831
2.3	0.00003	675.7301527	658.5924715	2.602167
2	0.00005	686.0171505	667.0303855	2.846461
2.233	0.00005	675.0185710	657.5550171	2.655831
2.3	0.00005	675.7369098	658.5990561	2.602167
2	0.00007	686.0240097	667.0370551	2.846461
2.233	0.00007	675.0253209	657.5615924	2.655831
2.3	0.00007	675.7436674	658.6056429	2.602167
2	0.0001	686.0343006	667.0470600	2.846461
2.233	0.0001	675.0354464	657.5714558	2.655831
2.3	0.0001	675.7538029	658.6155207	2.602167

Table 2. Numerical values of the Rayleigh number for very small values of the micropolar parameter K and for $C_0 = 0$.

Table 1 documents comparison of results obtained using the Shen basis with those of the conventional polynomials-based numerical solution. The relative error shown in Table 1 clearly points out the fact that we cannot obtain the best numerical results by severely truncated representations of ordinary polynomials.

Spectral methods are well known for their exponential convergence which imply that only a small number of terms are necessary in the approximation in order to get good numerical results. In spectral methods, the main target is to minimize the residual term (8) as much as possible. In order to validate the accuracy of the method for the proposed expansion Shen basis with respect to the spectral parameter N, we considered the classical case of Rayleigh–Bénard convection (here $K = 0$, $C_0 = 0$). The linear stability of the motionless state of an infinite layer of homogeneous fluid heated from below has been studied by Chandrasekhar [2] by means of classical normal modes and found that the critical Rayleigh number for the case of stress-free boundaries is $Ra_c = 657.511$ defined for a rounded value of a, i.e., $a = 2.22$. The investigation of the classical case gives us the following results: at $N = 3$ the relative error is around $4.66E-4$, at $N = 4$ this error is less than $6.08E-6$. Most important, these computations require small computational time. A larger value of the spectral parameter increases the computational time without offering a major decrease of the relative error. Clearly, the error is rapidly decreasing (Fig. 1).

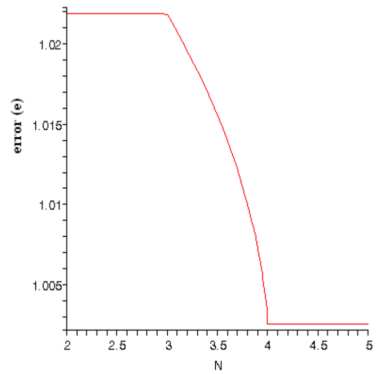

Fig. 1. The relative error (e) in the case $K = 0$, $C_0 = 0$, $a = 2.22$.

More than that, with this choice of the spectral parameter N, no convergence problems were encountered. As a result, all the numerical results presented in the paper are for $N = 4$.

Graphs depicting the stability characteristics for the case of stationary convection are presented for all the physical parameters. The sensitiveness of the critical Rayleigh number Ra_{cr} to changes in the micropolar parameters K, C_0 are presented in Figs. 2, 3, 4. The graphs in Figs. 2, 3 show a linear dependence of the Rayleigh number on K.

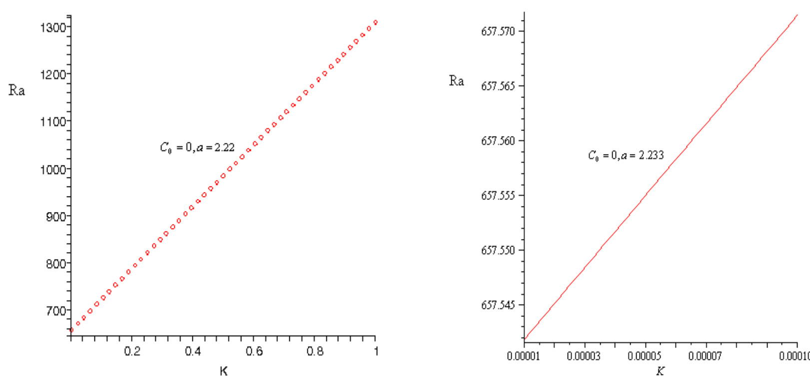

Fig. 2. Linear dependence of the critical Rayleigh number versus K for various values of C_0 and a.

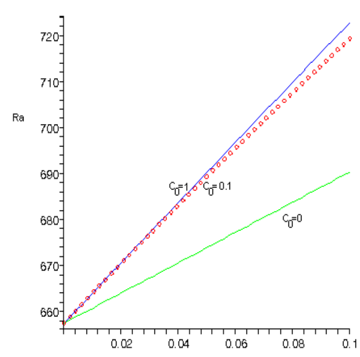

Fig. 3. The small difference in neutral curves in the (K, Ra) domain for various values of C_0 and $a = 2.233$.

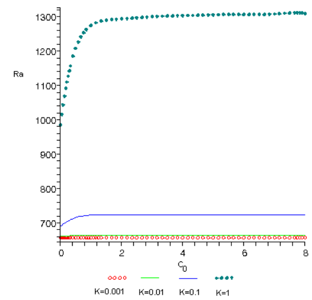

Fig. 4. Variations of the Rayleigh numbers with K in the (C, Ra) domain for a fixed $a = 2.233$.

The large influence of the coupling parameter K on Ra_c is also emphasized in Fig. 4. The neutral curves defining the dependence of the Rayleigh number on the values of the wavenumber a have a similar shape as the ones of classical Rayleigh–Bénard convection (Fig. 5). The curves are almost identical for small values of K.

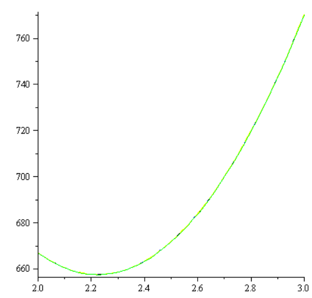

Fig. 5. Comparative neutral curves in the (a, Ra) domain for a fixed $C_0 = 0$ and $K = 0$ and $K = 0.1$.

Computations reveal that an increase in the values of C_0, say $C_0 > 100$, do not greatly influence the values of the critical Rayleigh number. For instance, for values between $C_0 = 100$ and $C_0 = 600$ we have an increase in the value of the Rayleigh number by less than 10^{-1}. This is documented in Table 3.

C_0	$Ra(K = 0)$	$Ra(K = 1)$
100	657.5153231	1314.586918
200	657.5153236	1314.808633
300	657.5153236	1314.882604
400	657.5153231	1314.919603
500	657.5153246	1314.941806
600	657.5153231	1314.956608

Table 3. Numerical values of the critical Rayleigh number showing the small influence of the C_0 parameter, for $a = 2.22$, $K = 0$ and $K = 1$.

The critical values of the Rayleigh number for various values of the micropolar parameters are tabulated in Table 4. It can be emphasized that all values of C_0, $K \in [0, 1]$ the critical wavenumber is $a = 2.22$.

K	C_0	Ra	a_{critic}	C_0	Ra	a_{critic}	C_0	Ra	a_{critic}
0.001	0	660.8029006	2.22	1	658.1727934	2.22	2	658.1724999	2.22
0.003	0	667.3780520	2.22	1	659.4874691	2.22	2	659.4873509	2.22
0.005	0	673.9532056	2.22	1	660.8017901	2.22	2	660.8020266	2.22
0.007	0	680.5283613	2.22	1	662.1157553	2.22	2	662.1165241	2.22
0.009	0	687.1035123	2.22	1	663.4293672	2.22	2	663.4308431	2.22
0.011	0	693.6876653	2.22	1	664.746242	2.22	2	664.7449895	2.22
0.2	0	723.2668569	2.22	1	787.2878606	2.22	2	788.1415850	2.22
0.4	0	789.0183884	2.22	1	913.7768538	2.22	2	917.0603873	2.22
0.6	0	854.7699250	2.22	1	1037.228602	2.22	2	1044.338290	2.22
0.8	0	920.5214526	2.22	1	1157.865368	2.22	2	1170.038385	2.22
1	0	986.2729905	2.22	1	1275.888240	2.22	2	1294.220609	2.22
0.001	3	658.1728242	2.22	10	658.1727934	2.22	100	658.1728386	2.22
0.003	3	659.4877356	2.22	10	659.4874691	2.22	100	659.4878656	2.22
0.005	3	660.8025294	2.22	10	660.8017901	2.22	100	660.8028903	2.22
0.007	3	662.1172045	2.22	10	662.1157553	2.22	100	662.1179090	2.22
0.009	3	663.4317616	2.22	10	663.4293672	2.22	100	663.4329258	2.22
0.011	3	664.7462007	2.22	10	664.746242	2.22	100	664.7479385	2.22
0.2	3	788.4312423	2.22	10	788.84113636	2.22	100	789.0006196	2.22
0.4	3	918.1936604	2.22	10	919.8143517	2.22	100	920.4503999	2.22
0.6	3	1046.832936	2.22	10	1050.437810	2.22	100	1051.864689	2.22
0.8	3	1174.378368	2.22	10	1180.714314	2.22	100	1183.243519	2.22
1	3	1300.858256	2.22	10	1310.646636	2.22	100	1314.586919	2.22

Table 4. Critical values of a in various parameters spaces.

For Dirichlet boundary conditions case, methods based on trigonometric Fourier series in which the unknown functions are expanded upon complete sets of functions which satisfy all boundary conditions can also be considered for numerical evaluations. The problem can be investigated in this case directly on the physical interval $[0, 1]$. Taking into account the order of differentiation of the equations and in the boundary conditions, the solutions for the unknown functions separate into completely even or odd solution functions. The lowest eigenvalue is usually obtained for the even solution, so we study the problem only for this case. The most simple solution function, $\sin(\pi z)$, satisfying the boundary conditions that can be used to represent such an approximative solution is

$$(\Psi, W, Z, \Theta) = (\widehat{\Psi}, \widehat{W}, \widehat{Z}, \widehat{\Theta}) \sin(\pi z), \tag{1}$$

where $\widehat{\Psi}, \widehat{W}, \widehat{Z}, \widehat{\Theta}$ are the unknown Fourier coefficients. The secular equation leads to:

$$Ra = R^2 = \frac{\{(\pi^2 + a^2)[C_0(1 + K)(\pi^2 + a^2) + K^2 + 2K\}(\pi^2 + a^2)}{a(2K + C_0(\pi^2 + a^2))}$$

The above relation enables us to evaluate the value of $Ra = R^2$ at which the instability sets for various values of a and for various values of the micropolar parameters. Numerical results for Rayleigh-Bénard convection ($K = 0$, $C_0 = 0$) cannot be obtained from the equation. Yet, numerical evaluations performed for instance for $C_0 = 0$, $K = 10^{-19}$ leads to a numerical value of $Ra_c = 657.5117337$ for $a = 2.22$ which is in good agreement with the one obtained by us, $Ra = 657.5153231$. The work on modifying the Shen-basis-reliant numerical approach is being presently pursued by the authors in a study of oscillatory convection and in a study of finite amplitude convection in different continua.

4. Conclusions

The conditions for the onset of convection in a linear stability problem of a micropolar fluid have been analyzed in this paper. Using the Boussinesq approximation, the linearized stability theory and the normal mode analysis, the eigenvalue problem for the case of two free boundaries is obtained.

We have described and applied a general spectral-Galerkin method for the numerical investigation of this eigen-boundary-value problem. The influences of all physical parameters have been systematically examined. Comparison with the corresponding results in the classical case were also performed and a excellent agreement is noted for existing results. The computations were performed with the dimensionless perturbation wavenumber ranging from 2 to 3 since the critical Rayleigh numbers are obtainable for a in this interval. The following conclusions can be drawn from the numerical results obtained by the Shen-basis-reliant approach:

(a) The Shen-basis-reliant (SBR) approach that has exponential convergence and that which uses small computational time, compared to severely truncated Galerkin expansion methods or trigonometric Fourier series method, yields a better approximation of the eigenvalue of the problem. Table 1 lends confidence on SBR approach. Truncation beyond the 4^{th} term in the SBR approach gives excellent results.

(b) The relative error in the computation involving Newtonian liquids is very small for a four term spectral expansion. Our computations reveal that this is true in the micropolar fluid problem as well. The very nature of the method forces the first few terms in the SBR expansion to hold key information on the problem. This is the reason for having exponential convergence in the method.

(c) The SBR approach very neatly captures the small changes in the eigenvalues for small variations in the parameters.

(d) All the micropolar effects increase the stability domains of the problem.

(e) The influence of C_0 on Ra_c only for small range of values and the linear variation of Ra_c with $K \in [0, 1]$ are also captured by SBR approach.

(f) The micropolar effects have a marginal influence on the cell size at the onset of convection.

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References

[1] Ahmadi, G., *Stability of a micropolar fluid layer heated from below*, Int. J. Engng. Sci., 14 (1976), 81-89.

[2] Chandrasekhar, S., *Hydrodynamic and hydromagnetic stability*, Oxford, England: Clarendon Press, 1961.

[3] Cheng, C.I., *Natural convection heat and mass transfer from a sphere in micropolar fluids with constant wall temperature and concentration*, International Communications in Heat and Mass Transfer, 35 (6) (2008), 750-755.

[4] Dragomirescu, F.I., Gheorghiu, C.I., *Analytical and numerical solutions to an electrohydrodynamic stability problem*, Appl. Math. Comp., 216 (2010), 3718-3727.

[5] Dragomirescu, F.I., Siddheshwar, P.G., *Spectral analysis in a micropolar fluid flow problem*, Aplimat-Journal of Applied Mathematics, 4 (1) (2011), 67-76.

[6] Eringen, A.C., *Theory of micropolar fluids*, J. Math. Mech., 16 (1966), 1-18.

[7] Eringen, A.C., *Microcontinuum field theories*, II. *Fluent media*, Springer, New York, 2001.

[8] Gheorghiu C.I., Dragomirescu, F.I., *Spectral methods in linear stability. Applications to thermal convection with variable gravity field*, Appl. Num. Math., 59 (6) (2009), 1290-1302.

[9] Lukasziewicz, G., *Micropolar fluids: theory and application*, Birkhḧauser, Basel, 1999.

[10] Qin, Y., Kaloni, P.N., *A thermal instability problem in a rotating micropolar fluid*, 30 (9) (1992), 1117-1126.

[11] Rani, N., Tomar, S.K., *Thermal convection problem of micropolar fluid subjected to hall current*, Appl. Math. Modelling, 34 (2) (2010), 508-519.

[12] Rama Rao, K.V., *Numerical solution of the thermal instability of a micropolar fluid layer between rigid boundaries*, Acta Mechanica, 32 (1979), 79-88.

[13] Sastry, V.U.K, Ramamohan Rao, V., *Numerical solution of thermal instability of a rotating micropolar fluid layer*, Int. J. Engng. Sci., 21 (5) (1983), 449-461.

[14] Shen, J., *Efficient spectral-Galerkin method*. II. *Direct solvers of second and fourth order eqautions by using Chebyshev polynomials*, SIAM J. Sci. Comput., 16 (1995), 74-87.

[15] Siddheshwar, P.G., Srikrishna, C.V., *Linear and nonlinear analyses of convection in a micropolar fluid occupying a porous medium*, Int. J. Nonlinear Mechanics, 38 (10) (2003), 1561-1579.

[16] Siddheshwar, P.G., Pranesh, S., *Magnetoconvection in fluids with suspended particles under* $1g$ *and* μg, J. Aerospace Sci. Tech., France, 6 (2002), 105-114.

[17] Siddheshwar, P.G., Desseaux, A., *Effects of homogeneous chemical reaction on solutal analog of Rayleigh–Bénard convection in a micropolar fluid*, Int. J. Appl. Mech. Engng., Poland, 7 (2) (2002), 513-525.

[18] Siddheshwar, P.G., Rudraiah, N., *Effect of non-uniform basic temperature gradient on the onset of Marangoni Convection in a fluid with suspended particles*, J. Aerospace Sci. Tech., France, 4 (8) (2000), 517-523.

[19] Siddheshwar, P.G., Pranesh, S., *Effects of non-uniform temperature gradient and magnetic field on the onset of convection in fluids with suspended particles under microgravity conditions*, Ind. J. Engng. and Matls. Scis., 8 (2) (2001), 77-83.

[20] Siddheshwar, P.G., Pranesh, S., *Suction-injection effects on the onset of Rayleigh-Benard-Marangoni convection in a fluid with suspended particles*, Acta Mechanica, 152 (2001), 241-252.

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