

ERROR LOCATING CODES DEALING WITH REPEATED BURST ERRORS

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Abstract. This paper obtains bound for linear codes which are capable to detect and locate errors which occur during the process of transmission. The kind of errors considered are known as repeated burst errors of length b (fixed) introduced by Dass and Garg [10] which has its seeds in the work carried out by Srinivas et al. [15] in connection with models of stroke-induced epilepsy which is an area of mathematical biology. An illustration for such kind of codes has also been provided.

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1. Introduction

The search for practical coding techniques on error control in digital data transmission has concentrated in two areas: error detection and error correction. A coding technique lying midway between error detection and error correction was introduced by Wolf and Elspas [16]. In this technique the block of received digits is to be regarded as subdivided into mutually exclusive sub-blocks and while decoding it is possible to detect the error and in addition the receiver is able to specify which particular sub-block contains error. Such codes are referred to as Error-Locating codes (EL-codes). They permit the location of digit errors within a sub-block of the received message block without permitting the precise location

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of the erroneous digit positions. The amount of redundancy required for such codes is not excessive and EL-codes provide an attractive alternative to conventional error detection in decision feed back communication systems. If errors are detected, the receiver requests the transmission of the corrupted block of digits and this process is repeated for each incoming block. The use of EL-codes can soften this compromise between short and long block lengths by providing an additional design parameter. Wolf and Elspas [16] studied binary codes which are capable of detecting and locating a single sub-block containing random errors. Such codes for burst errors were initiated by Dass [6].

Codes developed at the early stages were meant to detect and correct random errors, however it was noticed later that in many kinds of channels the likelihood of the occurrence of errors is more in adjacent positions rather than their occurrence in a random manner. It was in this spirit that the codes correcting single errors and double adjacent errors were developed by Abramson [1]. This idea was generalized and such errors were put in the category of errors called ‘burst errors’. A burst of length b is defined as follows:

Definition 1. A burst of length b is a vector whose only non-zero components are among some b consecutive components, the first and last of which is non-zero.

This definition was given by Fire [11] in a research report wherein he called such errors as open-loop burst errors. There is yet another kind of burst errors due to Chien and Tang [4]. It was noted by them that in several channels errors occur in the form of a burst but the end digits of the burst do not get corrupted. Channels due to Alexander *et al.* [2] fall in this category. This prompted Chien and Tang to propose a modification in the definition of a burst and they defined a burst of length b , which shall be called as CT burst of length b , as follows:

Definition 2. A CT burst of length b is a vector whose only non-zero components are confined to some b consecutive positions, the first of which is non-zero.

The nature of burst errors differs from channel to channel depending upon the behaviour of channels or the kind of errors which occur during the process of transmission. This prompted Dass [5] to further modify the definition of CT burst as follows:

Definition 3. A burst of length b (fixed) is an n -tuple whose only non-zero components are confined to b consecutive positions, the first of which is non-zero and the number of its starting positions is among the first $n - b + 1$ components.

Also, in very busy communication channels, errors repeat themselves. So is a situation when errors occur in the form of bursts. So we need to consider repeated bursts. The models studied by Srinivas *et al.* [15] fall into this category and codes developed pertaining to these may play an important role in subjects like mathematical biology. They studied the changes in the neuronal network properties during epileptiform activity *in vitro* in planar two-dimensional networks cultured on a multielectrode array, using the *in vitro* model of stroke-induced epilepsy.

A 2-repeated burst (open-loop) of length b (refer Berardi et al. [3]) is defined as follows:

Definition 4. A 2-repeated burst of length b is a vector of length n whose only non-zero components are confined to two distinct sets of b consecutive components, the first and the last component of each set being non-zero.

As an illustration, (00104200210500) is a 2-repeated burst of length 4 over GF(5).

Yet another kind of 2-repeated burst error of length b (fixed) has been studied by Dass and Garg [10] in which they defined such an error as follows:

Definition 5. A 2-repeated burst of length b (fixed) is an n -tuple whose only non-zero components are confined to 2 distinct sets of b consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - 2b + 1$ components.

As an illustration (010010000) is a 2-repeated burst of length up to 3 (fixed) whereas (0010000100000) is a 2-repeated burst of length at most 5 (fixed) over GF(2).

On the similar lines, an m -repeated burst of length b (fixed) has been defined by Dass *et al.* [8] as follows:

Definition 6. An m -repeated burst of length b (fixed) is an n -tuple whose only non-zero components are confined to m distinct sets of b consecutive digits, the first component of each set is non-zero and the number of its starting positions is among the first $n - mb + 1$ components.

The development of codes locating repeated burst errors will improve the efficiency of the communication channel as it will reduce the number of parity-check digits required in comparison with the codes dealing with the location of the usual burst error locating codes while considering such repeated bursts as single bursts.

This paper mainly presents a study of error-locating codes, in which errors occur in the form of 2-repeated bursts of length b (fixed)

This paper has been organized as follows. In section 2, we derive the necessary condition for the detection and location of 2-repeated bursts of length b (fixed) followed by a sufficient condition for the existence of such a code.

An illustration of a code locating 2-repeated bursts of length 3 (fixed) over GF(2) has also been given.

In section 3, necessary condition for the detection and location of m -repeated bursts of length b (fixed) has been given. After that a sufficient condition for the existence of such a code has been given.

In what follows, we shall consider a linear code to be a subspace of n -tuples over GF(q). The block of n digits, consisting of r check digits and $k = n - r$ information digits, is considered to be divided into s mutually exclusive sub-blocks. Each sub-block contains $t = n/s$ digits. The distance between vectors will be considered in the Hamming's sense [12].

2. 2-repeated burst of length $b(\text{fixed})$ error locating codes

In this section, we consider (n, k) linear codes over $\text{GF}(q)$ that are capable of detecting and locating all 2-repeated bursts of length $b(\text{fixed})$ within a single sub-block. An EL-code capable of detecting and locating a single sub-block containing an error which is in the form of a 2-repeated burst of length $b(\text{fixed})$ must satisfy the following conditions:

- (a) The syndrome resulting from the occurrence of a 2-repeated burst of length $b(\text{fixed})$ within any one sub-block must be distinct from the all zero syndrome.
- (b) The syndrome resulting from the occurrence of any 2-repeated burst of length $b(\text{fixed})$ within a single sub-block must be distinct from the syndrome resulting likewise from any 2-repeated burst of length $b(\text{fixed})$ within any other sub-block.

In this section we shall derive two results. The first result gives a lower bound on the number of check digits required for the existence of a linear code over $\text{GF}(q)$ capable of detecting and locating a single sub-block containing errors that are 2-repeated bursts of length $b(\text{fixed})$. In the second result, we derive an upper bound on the number of check digits which ensures the existence of such a code.

Since the code is divided into several blocks of length t each and we wish to detect a 2-repeated burst of length $b(\text{fixed})$, we may come across with a situation when the difference between $2b$ and t becomes small. We note that if $t - b + 1 < 2b$ and we consider any two 2-repeated bursts x_1 and x_2 of length $b(\text{fixed})$ such that their non-zero components are confined to first $t - b + 1$ positions then their difference $x_1 - x_2$ is again a 2-repeated burst of length $b(\text{fixed})$. However, if we do not restrict ourselves to first $t - b + 1$ positions then we may not get a 2-repeated burst of length $b(\text{fixed})$ as explained with the help of the following examples:

Example 1. Suppose $t = 7, b = 3$. Let $x_1 = (1111110)$ and $x_2 = (1111100)$. Then x_1 and x_2 are 2-repeated bursts of length 3(fixed) whereas $x_1 - x_2 = (0000010)$ is not a 2-repeated burst of length 3(fixed).

Example 2. Suppose $t = 10, b = 4$. Let $x_1 = (1111001101)$ and $x_2 = (1001001001)$. Then x_1 and x_2 are 2-repeated bursts of length 4(fixed) whereas $x_1 - x_2 = (0110000100)$ is not a 2-repeated burst of length 4(fixed). So, when $t - b + 1 < 2b$ we consider the collection of those vectors in which the non-zero components are confined to first $t - b + 1$ positions whereas when $t - b + 1 \geq 2b$ we consider the collection of those vectors in which the non-zero components are confined to some two (fixed) b consecutive positions so that patterns to be detected are not code words.

Theorem 1. *The number of parity check digits r in an (n, k) linear code subdivided into s sub-blocks of length t each, that locates a single corrupted sub-block containing errors that are 2-repeated bursts of length $b(\text{fixed})$ is at least*

$$(1) \quad \begin{cases} \log_q \{1 + s(q^{t-b+1} - 1)\} & \text{when } t - b + 1 < 2b \\ \log_q \{1 + s(q^{2b} - 1)\} & \text{when } t - b + 1 \geq 2b. \end{cases}$$

Proof. Let there be an (n, k) linear code over $GF(q)$ that locates a 2-repeated burst of length b (fixed) within a single corrupted sub-block. Maximum number of distinct syndromes available using r check bits is q^r . The proof proceeds by first counting the number of syndromes that are required to be distinct by condition (a) and (b) and then setting this number less than or equal to q^r . First we consider a sub-block, say i^{th} sub-block of length t .

In view of the observations made before Theorem 1, we discuss the following two cases:

Case 1: when $t - b + 1 < 2b$.

Let X consist of all those vectors in which all the non-zero components are confined to the first $t - b + 1$ positions of the i^{th} sub-block. We observe that the syndromes of all the elements of X should be different; else for any x_1, x_2 belonging to X having the same syndrome would imply that the syndrome of $x_1 - x_2$ which is also an element of X and hence a 2-repeated burst of length b (fixed) within the same sub-block becomes zero; in violation of condition (a). Also since the code locates a single sub-block containing errors that are 2-repeated bursts of length b (fixed), the syndromes produced by similar vectors in different sub-blocks must be distinct by condition (b). Thus, the syndromes of vectors which are 2-repeated bursts of length b (fixed) in fixed positions, whether in the same sub-block or in different sub-blocks, must be distinct. (It may be noted that different fixed components may be chosen in different sub-blocks.) As there are $(q^{t-b+1} - 1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block and there are s sub-blocks in all, so we must have at least $(1 + s(q^{t-b+1} - 1))$ distinct syndromes counting the all zero syndrome. Therefore, we must have

$$q^r \geq \{1 + s(q^{t-b+1} - 1)\} \text{ when } t - b + 1 < 2b$$

or

$$(2) \quad r \geq \log_q \{1 + s(q^{t-b+1} - 1)\} \text{ when } t - b + 1 < 2b.$$

Case 2: when $t - b + 1 \geq 2b$.

Let X consist of all those vectors in which all the non-zero components are confined to some two fixed b consecutive positions of the i^{th} sub-block. As discussed in case 1, the syndromes of all the elements of X are different. As, in this case, there are $(q^{2b} - 1)$ distinct non-zero syndromes corresponding to the vectors in any one sub-block and there are s sub-blocks in all, so we must have at least $(1 + s(q^{2b} - 1))$ distinct syndromes counting the all zero syndrome. Therefore, we must have

$$q^r \geq (1 + s(q^{2b} - 1)) \text{ when } t - b + 1 \geq 2b$$

or

$$(3) \quad r \geq \log_q \{1 + s(q^{2b} - 1)\} \text{ when } t - b + 1 \geq 2b.$$

From (2) and (3) we get the required result.

In the following result, we derive another bound on the number of check digits required for the existence of such a code. The proof is based on the technique used to establish Varsharmov-Gilbert Sacks bound by constructing a parity check matrix for such a code (refer Sacks [14], Theorem 4.7 Peterson and Weldon [13]). This technique not only ensures the existence of such a code but also gives a method for the construction of the code.

Theorem 2. *An (n, k) linear EL code over $GF(q)$ capable of detecting a 2-repeated burst of length b (fixed) within a single sub-block and of locating that sub-block can always be constructed provided that*

$$(4) \quad r > (b-1) + \log_q \left[\left\{ 1 + (q-1)q^{b-1}(t-2b+1) \right\} \cdot \left\{ 1 + (s-1) \sum_{i=1}^2 \binom{t-ib+i}{i} (q-1)^i q^{i(b-1)} \right\} \right]$$

where r is the number of check digits.

Proof. In order to prove the existence of such a code, we construct an $(n-k) \times n$ parity check matrix H for such a code by a synthesis procedure. For that we first construct a matrix H_1 from which the requisite parity check matrix H shall be obtained by reversing the order of the columns of each sub-block.

After adding $(s-1)t$ columns appropriately corresponding to the first $(s-1)$ sub-blocks, suppose that we have added the first $j-1$ columns $h_1, h_2, h_3 \dots h_{j-1}$ of the s -th sub-block to the matrix H_1 , out of which the first $b-1$ columns $h_1, h_2, h_3 \dots h_{b-1}$ may be chosen arbitrarily (non-zero). We now lay down the condition to add the j -th column h_j to H_1 as follows:

According to condition (a), for the detection of 2-repeated burst of length b (fixed) in the s^{th} sub-block h_j should not be a linear combination of immediately preceding $b-1$ columns $h_{j-b+1}, h_{j-b+2} \dots h_{j-1}$ together with any linear combination of b consecutive columns out of the first $j-b$ columns of the s^{th} sub-block. i.e.,

$$(5) \quad h_j \neq (\alpha_1 h_{j-b+1} + \alpha_2 h_{j-b+2} + \dots + \alpha_{b-1} h_{j-1}) + (\beta_1 h_{i+1} + \beta_2 h_{i+2} + \dots + \beta_b h_{i+b})$$

where $\alpha_i, \beta_i \in GF(q)$ and either all the coefficients β_i 's are zero or if the p -th coefficient β_p is the last non-zero coefficient then $b \leq p \leq j-b$.

The number of ways in which the coefficients α_i 's can be selected is q^{b-1} and to enumerate the coefficients β_i 's is equivalent to enumerate the number of bursts of length b (fixed) in a vector of length $j-b$.

This number (refer Dass [5]), including the vector of all zeros is

$$1 + (j-2b+1)(q-1)q^{b-1}.$$

So, the number of linear combinations on the right hand side of (5) is

$$(6) \quad q^{b-1}[1 + (j-2b+1)(q-1)q^{b-1}].$$

Now, according to condition (b), for the location of 2-repeated burst of length b (fixed), h_j should not be a linear combination of the immediately preceding $b - 1$ columns together with any b consecutive columns out of the remaining $j - b$ columns of the s -th sub-block together with linear combination of any two sets of b consecutive columns out of any one of the previously chosen $s - 1$ sub-blocks, the coefficient of the last column of either both or one of the sets being non-zero.

The number of 2-repeated bursts of length b (fixed) in a sub-block of length t (refer Dass *et al.* [9]) is

$$\sum_{i=1}^2 \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)}.$$

Since there are $(s - 1)$ previous sub-blocks, therefore number of such linear combinations becomes

$$(7) \quad (s - 1) \left\{ \sum_{i=1}^2 \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\}.$$

So, for the location of 2-repeated burst of length b (fixed) the number of linear combinations to which h_j can not be equal to is the product computed in expr.(6) and expr.(7). i.e.,

$$(8) \quad \text{expr.(6)} \times \text{expr.(7)}$$

Thus, the total number of linear combinations that h_j can not be equal to is the sum of linear combinations in (6) and (8).

At worst, all these combinations might yield distinct sum. Therefore, h_j can be added to the s -th sub-block of H_1 provided that

$$(9) \quad q^r > q^{b-1} \{1 + (q - 1)q^{b-1}(j - 2b + 1)\} \left[1 + (s - 1) \cdot \left\{ \sum_{i=1}^2 \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right\} \right].$$

To obtain the length of the block as t replacing j by t in the above expression we get

$$r > (b - 1) + \log_q \left[\{1 + (q - 1)q^{b-1}(t - 2b + 1)\} \cdot \left\{ 1 + (s - 1) \left(\sum_{i=1}^2 \binom{t - ib + i}{i} (q - 1)^i q^{i(b-1)} \right) \right\} \right].$$

The required matrix H can be obtained from H_1 by reversing the order of the columns in each sub-block.

Remark 1. It may be noted that it hardly matters whether we reverse the order of columns within the subblock or we reverse the order of the columns of the entire matrix H_1 .

Alternate Form 1. Let B be the largest value of b satisfying the inequality (4). Then for $b = B + 1$, the inequality (4) gets reversed and we get

$$(10) \quad r \leq B + \log_q \left[\left\{ 1 + (q-1)q^B(t-2B-1) \right\} \left\{ 1 + (s-1) \sum_{i=1}^2 \binom{t-iB}{i} (q-1)^i q^{iB} \right\} \right].$$

Example 3. For an $(27, 14)$ linear code over $\text{GF}(2)$ we construct the following parity check matrix $H(13 \times 27)$, according to the synthesis procedure given in the proof of Theorem 2 by taking $s = 3$, $t = 9$, $b = 3$.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The null space of this matrix can be used as a code to locate a sub-block of length $t = 9$ containing 2-repeated bursts of length 3(fixed). It may be easily verified that:

1. Syndromes of 2-repeated bursts of length 3(fixed) within any sub-block are all non-zero showing thereby that the code detects all 2-repeated bursts of length 3(fixed) occurring within a sub block.
2. The syndrome of the 2-repeated burst of length 3(fixed) within any sub-block is different from the syndrome of a 2-repeated burst of length 3(fixed) within any other sub-block thereby ensuring that the code locates any 2-repeated burst of length 3(fixed) occurring within single sub-block. (This has been verified through MS-Excel program).

Observation. Syndromes of some of the 2-repeated bursts of length 3(fixed) occurring within the second sub-block turn out to be the same. For coding efficiency it is desired that the syndromes of the error patterns within any sub-block are identical whenever possible.

3. Location of m -repeated burst of length b (fixed)

In this section, the results of the previous section have been extended to the case of m -repeated burst of length b (fixed).

It may be noted that an EL-code capable of detecting and locating a single sub-block containing an error which is in the form of an m -repeated burst of length b (fixed) must satisfy the following conditions:

- (c) The syndrome resulting from the occurrence of an m -repeated burst of length b (fixed) within any one sub-block must be distinct from the all zero syndrome.

- (d) The syndrome resulting from the occurrence of any m -repeated burst of length b (fixed) within a single sub-block must be distinct from the syndrome resulting likewise from any m -repeated burst of length b (fixed) within any other sub-block.

In this section, we shall derive two results. The first result gives a lower bound on the number of check digits required for the existence of a linear code over $GF(q)$ capable of detecting and locating a single sub-block containing errors that are m -repeated bursts of length b (fixed). In the second result, we derive an upper bound on the number of check digits which ensures the existence of such a code.

Theorem 3. *The number of parity check digits r in an (n, k) linear code subdivided into s sub-blocks of length t each, that locates a single corrupted sub-block containing errors that are m -repeated bursts of length b (fixed) is at least*

$$(11) \quad \begin{cases} \log_q\{1 + s(q^{t-b+1} - 1)\} & \text{when } t - b + 1 < mb \\ \log_q\{1 + s(q^{mb} - 1)\} & \text{when } t - b + 1 \geq mb. \end{cases}$$

Proof. The proof of this result is on the similar lines as that of proof of Theorem 1 so we omit the proof. ■

Remark 2. For $m = 2$, this result coincides with the Theorem 1 when 2-repeated bursts of length b (fixed) are considered.

For $m = 1$, this result is similar to the result obtained in the Theorem 1 due to Dass and Chand [7] when bursts of length b (fixed) are considered.

In the following result we derive another bound on the number of check digits required for the existence of the code considered in the Theorem 3.

Theorem 4. *A code capable of detecting an m -repeated burst of length b (fixed) within a single sub-block and of locating that sub-block can always be constructed provided that*

$$(12) \quad r > (b - 1) + \log_q \left[\left\{ \sum_{i=0}^{m-1} \binom{t - (i+1)b + i}{i} \cdot (q-1)^i q^{i(b-1)} \right\} \cdot \left\{ 1 + (s-1) \sum_{i=1}^m \binom{t - ib + i}{i} (q-1)^i q^{i(b-1)} \right\} \right]$$

where r is the number of check digits.

Proof. As in Theorem 3, we omit the proof because proof of this result is on the similar lines as that of proof of Theorem 2. ■

Remark 3. For $m = 2$, this result coincides with Theorem 2 when 2-repeated bursts of length b (fixed) are considered.

For $m = 1$, this result coincides with the result obtained in Theorem 2 due to Dass and Chand [7] when bursts of length b (fixed) are considered.

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