A NOTE ON QUASI k-IDEALS AND BI k-IDEALS IN TERNARY SEMIRINGS

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Abstract. Dutta and Kar [3] have introduced the concept of ternary semiring. In this paper, the notion of quasi $k-$ideals and bi $k-$ideals of a ternary semiring is introduced and characterizations $k-$regular ternary semirings has been given.

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1. Introduction and preliminaries

Ternary semiring is a generalization of a ternary ring which is introduced by Lister [6]. T.K. Dutta and S. Kar have initiated the notion of ternary semiring and studied their properties. Recall ([3], [4]) the followings. A non-empty set S together with binary operation, called addition and ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if S is an additive commutative semigroup satisfying the following conditions:

(i) $abc \in S$,

(ii) $(abc)de = a(bcd)e = ab(cde)$,

(iii) $(a + b)cd = acd + bcd$,

(iv) $a(b + c)d = abd + acd$,

(v) $ab(c + d) = abc + abd$

for all $a, b, c, d, e \in S$.

Let S be a ternary semiring. An element $0 \in S$ such that $0 + x = x$ and $0xy = x0y = xy0 = 0$ for all $x, y \in S$, then $'0'$is called the zero element of the ternary semiring S. In this case, S is called ternary semiring S with zero. Through out this paper, S will always denote a ternary semiring with zero. An additive subsemigroup I of S is called left (right, lateral) ideal of S if s_1s_2a (respectively as_1s_2, s_1as_2) $\in I$ for all $s_1, s_2 \in S$ and $a \in I$. If I is a left, a right and a lateral ideal of S then I is called an ideal of S. An ideal I of S is called a $k-$ideal if $x + y \in I$, $x \in S$, $y \in I$ imply that $x \in I$. If A is an ideal of a ternary semiring S then $\overline{A}=\{a \in S : a + x \in A$ for some $x \in A\}$ is called $k-$closure of A. It can easily verified that an ideal A of S is $k-$ideal if and only if $A=\overline{A}$ and also $A \subseteq \overline{A}, A \subseteq B$ implies $\overline{A} \subseteq \overline{B}$. Let A, B, C be three subsets of S. Then ABC denotes the set of all finite sums of the forms $\sum a_ib_ic_i$, where $a_i \in A, b_i \in B, c_i \in C$. An element $a \in S$ is called regular if there exists $x \in S$ such that $a = axa$. If all the elements of S are regular then S is called regular ternary semiring. An additive subsemigroup Q of a ternary semiring S is called a quasi-ideal of Sif $QSS \cap (SQS + SSQSS) \cap SSQ \subseteq Q$. Clearly, every quasi ideal of ternary semiring S is a ternary subsemiring of S.

2. Main results

Proposition 2.1. *Let S be a ternary semiring and X be a nonempty subset of S. Then*

(i) $\langle X\rangle_l = Z_0^+X + SSX$ *is the smallest left ideal generated by X,*

(ii) $\langle X\rangle_r = Z_0^+X + XSS$ *is the smallest right ideal generated by X,*

(iii) $\langle X\rangle_t = Z_0^+X + SSX + XSS + SSXSS$ *is the smallest two sided ideal generated by X,*

(iv) $\langle X\rangle_m = Z_0^+X + SXS + SSXSS$ *is the smallest lateral ideal generated by X,*

(v) $\langle X\rangle_i = Z_0^+X+SSX+XSS+SXS+SSXSS$ *is the smallest ideal generated by X,*

where $SSX, XSS, SXS, SSXSS, Z_0^+X$ the set of all finite sum of the form $\sum r_is_ix_i$, $\sum x_ip_iq_i, \sum u_ix_iv_i, \sum p_i'q_i'x_ir_i's_i', \sum n_ix_i$, where $r_i, s_i, p_i, q_i, u_i, v_i, p_i', q_i', r_i', s_i' \in S$, $x_i \in X$, $n_i \in Z_0^+$, and Z_0^+ is the set of all positive integer with zero.

The following corollary can be easily proved by the above proposition.

Corollary 2.2. *If X is a subsemigroup of $(S, +)$, then*

$$\begin{aligned}
\langle X\rangle_l &= X + SSX,\\
\langle X\rangle_r &= X + XSS,\\
\langle X\rangle_t &= X + SSX + XSS + SSXSS,\\
\langle X\rangle_m &= X + SXS + SSXSS,\\
\langle X\rangle_i &= X + SSX + XSS + SXS + SSXSS.
\end{aligned}$$

Proposition 2.3. [4] *Let S be a ternary semiring and $a \in S$. Then the principal*

(i) *left ideal generated by a is given by*

$$\langle a\rangle_l = \left\{\sum r_i s_i a + na : r_i, s_i \in S : n \in Z_0^+\right\}$$

(ii) *right ideal generated by a is given by*

$$\langle a\rangle_r = \left\{\sum a r_i s_i + na : r_i, s_i \in S : n \in Z_0^+\right\}$$

(iii) *two sided ideal generated by a is given by*

$$\langle a\rangle_t = \Big\{\sum r_i s_i a + \sum a p_i q_i + \sum p_k' q_k' a r_k' s_k' + na \\ : r_i, s_i, p_i, q_i, p_k', q_k', r_k', s_k' \in S : n \in Z_0^+\Big\}$$

(iv) *lateral ideal generated by a is given by*

$$\langle a\rangle_m = \left\{\sum r_i a s_i + \sum p_j q_j a r_j s_j + na : r_i, s_i, p_j, q_j, r_j, s_j \in S : n \in Z_0^+\right\}$$

(v) *ideal generated by a is given by*

$$\langle a\rangle_i = \Big\{\sum r_i s_i a + \sum a p_i q_i + \sum u_k a v_k + \sum p_k' q_k' a r_k' s_k' + na \\ : r_i, s_i, p_i, q_i, u_k, v_k, p_k', q_k', r_k', s_k' \in S : n \in Z_0^+\Big\}$$

where $\sum$ denote the finite sum and Z_0^+ is the set of all positive integer with zero.

Proposition 2.4. *If Q is a quasi-ideal of a ternary semiring S, then*

$$Q = Q + (SSQ \cap (SQS + SSQSS) \cap QSS).$$

Proof. The proof is obvious. ∎

Let X be a non empty subset of a ternary semiring S. The smallest quasi-ideal containing X and generated by X is denoted by $(X)_q$, that is, the intersection of all quasi-ideal of S containing X.

Proposition 2.5. *Let S be a ternary semiring and X be nonempty subset of S. Then $(X)_q = Z_0^+ X + (SSX \cap (SXS + SSXSS) \cap XSS)$.*

Proof. Let $Q = Z_0^+ X + (SSX \cap (SXS + SSXSS) \cap XSS)$. Clearly, Q is a non empty additive subsemigroup of S. Now,

$$\begin{aligned}(SSQ \cap (SQS + SSQSS) \cap QSS) &\subseteq SSQ \\ &= SS(Z_0^+ X + (SSX \cap (SXS + SSXSS) \cap XSS)) \\ &\subseteq Z_0^+ SSX + SS(SSX) \subseteq SSX.\end{aligned}$$

Similarly, $(SSQ \cap (SQS + SSQSS) \cap QSS) \subseteq (SXS + SSXSS)$ and $(SSQ \cap (SQS + SSQSS) \cap QSS) \subseteq XSS$. Therefore

$$(SSQ \cap (SQS + SSQSS) \cap QSS) \subseteq (SSX \cap (SXS + SSXSS) \cap XSS) \subseteq Q.$$

Hence Q is a quasi-ideal of S containing X. Also, it is easy to show that Q is smallest quasi-ideal of S containing X. Hence

$$Q = (X)_q = Z_0^+ X + (SSX \cap (SXS + SSXSS) \cap XSS).$$ ■

Theorem 2.6. *If Q is a quasi-ideal of a ternary semiring S and if $Q \subseteq QSS$ and $Q \subseteq SSQ$ and QSS, SSQ are $k-$ideals of S, then Q is the intersection of the left ideal $Q + SSQ$, lateral ideal $Q + SQS + SSQSS$, and right ideal $Q + QSS$.*

Proof. Let $D=(Q+QSS)\cap(Q + SQS + SSQSS)\cap(Q + SSQ)$. Clearly, $Q \subseteq D$. To show $D \subseteq Q$. Now, $Q \subseteq QSS$ and $Q \subseteq SSQ$. Therefore

$$D = QSS \cap (Q + SQS + SSQSS) \cap SSQ.$$

Let $d \in D$. Then $d \in QSS$, $d \in SSQ$ and

$$d = q + \Sigma s_i' q_i' s_i + \Sigma p_i' p_i'' q_i'' p_i''' p_i'''' \tag{2.1}$$

for $s_i', s_i, p_i', p_i'', p_i''', p_i'''' \in S$ and $q, q_i', q_i'' \in Q$.

Since $q \in Q \subseteq QSS$ and QSS is a $k-$ideal of S, therefore

$$\Sigma s_i' q_i' s_i + \Sigma p_i' p_i'' q_i'' p_i''' p_i'''' \in QSS.$$

Similarly,

$$\Sigma s_i' q_i' s_i + \Sigma p_i' p_i'' q_i'' p_i''' p_i'''' \in SSQ.$$

Therefore,

$$\Sigma s_i' q_i' s_i + \Sigma p_i' p_i'' q_i'' p_i''' p_i'''' \in QSS \cap (SQS + SSQSS) \cap SSQ \subseteq Q.$$

So,

$$\Sigma s_i' q_i' s_i + \Sigma p_i' p_i'' q_i'' p_i''' p_i'''' \in Q,$$

which implies that $d = q + \Sigma s_i' q_i' s_i + \Sigma p_i' p_i'' q_i'' p_i''' p_i'''' \in Q$. Thus $D \subseteq Q$. Hence $D = Q$. ■

Definition 2.7. An additive subsemigroup Q of a ternary semiring S is called a quasi $k-$ideal of Sif $\overline{QSS} \cap \overline{(SQS + SSQSS)} \cap \overline{SSQ} \subseteq Q$. Clearly, every quasi $k-$ideal is a quasi-ideal of S.

It is easy to see that if R be right $k-$ideal, M be lateral $k-$ideal and L be left $k-$ideal of S, then $Q = R \cap M \cap L$ is a quasi $k-$ideal of S, because $\overline{(R \cap M \cap L)SS} \cap \overline{SS(R \cap M \cap L)SS} \cap \overline{SS(R \cap M \cap L)} = \overline{RSS} \cap \overline{SSMSS} \cap \overline{SSL} = \overline{R} \cap \overline{M} \cap \overline{L} = R \cap M \cap L$.

Lemma 2.8. *Let S be a ternary semiring and $A, B, C \subseteq S$, then*

$$\overline{ABC} = \overline{\overline{A}\,\overline{B}\,\overline{C}}.$$

Proof. Since $A \subseteq \overline{A}$, $B \subseteq \overline{B}$ and $C \subseteq \overline{C}$, therefore $ABC \subseteq \overline{A}\,\overline{B}\,\overline{C}$. Hence, $\overline{ABC} \subseteq \overline{\overline{A}\,\overline{B}\,\overline{C}}$. Again, let $x \in \overline{A}$, $y \in \overline{B}$ and $z \in \overline{C}$. Then there exist $a_1, a_2 \in A$, $b_1, b_2 \in B$ and $c_1, c_2 \in C$ such that $x + a_1 = a_2$, $y + b_1 = b_2$ and $z + c_1 = c_2$. Now,

$$\begin{aligned}
& xyz + a_2b_2c_1 + a_2b_1c_2 + a_1b_2c_2 + a_1b_1c_1 \\
&\quad = xyz + (x + a_1)(y + b_1)c_1 + a_2b_1c_2 + a_1b_2c_2 + a_1b_1c_1 \\
&\quad = xyz + xyc_1 + xb_1c_1 + a_1yc_1 + a_1b_1c_1 + a_2b_1c_2 \\
&\quad + a_1b_2c_2 + a_1b_1c_1 \\
&\quad = xyc_2 + xb_1c_1 + a_1yc_1 + a_1b_1c_1 + a_2b_1c_2 \\
&\quad + a_1b_2c_2 + a_1b_1c_1 \\
&\quad = xyc_2 + xb_1c_1 + a_1yc_1 + a_1b_1c_1 + (x + a_1)b_1c_2 \\
&\quad + a_1b_2c_2 + a_1b_1c_1 \\
&\quad = x(y + b_1)c_2 + xb_1c_1 + a_1(y + b_1)c_1 + a_1b_1c_2 \\
&\quad + a_1b_2c_2 + a_1b_1c_1 \\
&\quad = xb_2c_2 + (x + a_1)b_1c_1 + a_1b_2c_1 + a_1b_1c_2 + a_1b_2c_2 \\
&\quad = (x + a_1)b_2c_2 + a_2b_1c_1 + a_1b_2c_1 + a_1b_1c_2 \\
&\quad = a_2b_2c_2 + a_2b_1c_1 + a_1b_2c_1 + a_1b_1c_2.
\end{aligned}$$

As $a_ib_jc_k(i, j, k = 1, 2) \in ABC$, therefore we can prove that $xyz \in \overline{ABC}$, for $x \in \overline{A}, y \in \overline{B}, z \in \overline{C}$. Suppose $t \in \overline{A}\,\overline{B}\,\overline{C}$. Then $t = \sum_{finite} a_ib_ic_i$ for some $a_i \in \overline{A}$, $b_i \in \overline{B}$ and $c_i \in \overline{C}$. Thus $t \in \overline{ABC}$, i.e.$\overline{A}\,\overline{B}\,\overline{C} \subseteq \overline{ABC}$. Therefore $\overline{\overline{A}\,\overline{B}\,\overline{C}} \subseteq \overline{\overline{ABC}} = \overline{ABC}$. Hence $\overline{\overline{A}\,\overline{B}\,\overline{C}} = \overline{ABC}$. ■

Definition 2.9. [4] Let S be a ternary semiring. Then S is called $k-$regular if for each $a \in S$ there exist $x, y \in S$ such that $a + axa = aya$.

Theorem 2.10. *If a ternary semiring S is $k-$regular, then every quasi $k-$ideal Q of S is of the form $Q = \overline{QSQSQ} = \overline{QSS} \cap \overline{SQS + SSQSS} \cap \overline{SSQ}$.*

Proof. Let Q be a quasi $k-$ideal of S. Then $\overline{QSS} \cap \overline{SQS + SSQSS} \cap \overline{SSQ} \subseteq Q$. Let $a \in Q$ and S is $k-$regular, then there exist $x, y \in S$ such that $a + axa = aya$. This implies that $axa + axaxa = ayaxa$. Since $axaxa, ayaxa \in QSQSQ$, therefore $axa \in \overline{QSQSQ}$. Similarly, $aya \in \overline{QSQSQ}$. Therefore $a \in \overline{\overline{QSQSQ}} = \overline{QSQSQ}$ (as $\overline{QSQSQ}$ is $k-$closed). Therefore $Q \subseteq \overline{QSQSQ}$. Again $QSQSQ \subseteq Q(SSS)S \subseteq QSS$ and $QSQSQ \subseteq SSQ$ and $QSQSQ \subseteq SSQSS$ which shows that $\overline{QSQSQ} \subseteq \overline{QSS}$, $\overline{QSQSQ} \subseteq \overline{SSQ}$ and $\overline{QSQSQ} \subseteq \overline{SQS + SSQSS}$ (as $0 \subseteq SQS$). Thus we have $Q \subseteq \overline{QSQSQ} \subseteq \overline{QSS} \cap \overline{SQS + SSQSS} \cap \overline{SSQ} \subseteq Q$ (as Q is a quasi $k-$ideal of S). Hence $Q = \overline{QSQSQ} = \overline{QSS} \cap \overline{SQS + SSQSS} \cap \overline{SSQ}$. ■

Theorem 2.11. *A ternary semiring S is $k-$regular if and only if $R \cap M \cap L = \overline{RML}$ holds for each right $k-$ideal R, lateral $k-$ideal M, and left $k-$ideal L of S.*

Proof. Since R is right $k-$ ideal, therefore $RML \subseteq RSS \subseteq R$ which shows that $\overline{RML} \subseteq \overline{R} = R$. Again, since M is a lateral $k-$ideal of S, then $RML \subseteq SMS \subseteq M$ and so $\overline{RML} \subseteq \overline{M} = M$. Similarly, we obtain that $\overline{RML} \subseteq L$. Therefore we have $RML \subseteq R \cap M \cap L$.

Again, suppose that $a \in R \cap M \cap L$. Since S is $k-$regular therefore there exist $x, y \in S$ such that $a + axa = aya$. This implies that $axa + a(xax)a = a(yax)a$. Since $a(xax)a, a(yax)a \in RML$ therefore $axa \in \overline{RML}$. Similarly, $aya \in \overline{RML}$. Therefore $a \in \overline{\overline{RML}} = \overline{RML}$(as $\overline{RMS}$ is $k-$closed. Hence $R \cap M \cap L \subseteq \overline{RML}$. Thus $R \cap M \cap L = \overline{RML}$.

Conversely, let $a \in S$. Then principal right $k-$ideal generated by a of S is given by $\overline{aSS + Z_0^+ a}$. Now,

$$\begin{aligned} \overline{aSS + Z_0^+ a} &= \overline{aSS + Z_0^+ a} \cap S \cap S \\ &= \overline{\overline{aSS + Z_0^+ a}\,\overline{S}\,\overline{S}} \text{ (as } S \text{ is itself (lateral, left) } k-\text{ideal of } S) \\ &= \overline{(aSS + Z_0^+ a)SS} \text{ (by Lemma 2.8)} \\ &= \overline{aSS}. \end{aligned}$$

Also, $a = a0S + 1.a \in aSS + Z_0^+ a \subseteq \overline{aSS + Z_0^+ a} = \overline{aSS}$. Therefore $a \in \overline{aSS}$. Similarly, it can easily be shown that $a \in \overline{SSa}$ and $a \in \overline{(SaS + SSaSS)}$.

Therefore, we have

$$\begin{aligned} a \in \overline{aSS} \cap \overline{(SaS + SSaSS)} \cap \overline{SSa} &= \overline{\overline{aSS}\,\overline{(SaS + SSaSS)}\,\overline{SSa}} \\ &= \overline{aSS(SaS + SSaSS)SSa} = \overline{aSa}. \end{aligned}$$

Therefore there exist $x, y \in S$ such that $a + axa = aya$. Thus S is $k-$regular. ∎

Definition 2.12. A ternary subsemiring B of a ternary semiring S is called a bi $k-$ideal of of S if $\overline{BSBSB} \subseteq B$.

Result 2.13. Every quasi $k-$ideal of a ternary semiring is a bi $k-$ideal of S.

Proof. It is obvious by Theorem 2.10. ∎

Result 2.14. Let S be a $k-$regular ternary semiring. Then every bi $k-$ideal B of a ternary semiring is a quasi $k-$ideal of S if $\overline{B} = B$ and $\overline{BSB} \subseteq B$.

Proof. Let B be a bi $k-$ideal of S. Then

$$\begin{aligned}
&\overline{BSS} \cap \overline{(SBS + SSBSS)} \cap \overline{SSB} \\
&\quad = \overline{\overline{BSS}\,\overline{(SBS + SSBSS)}\,\overline{SSB}} \quad \text{(by Theorem 2.11)} \\
&\quad = \overline{BSS(SBS + SSBSS)SSB} \quad \text{(by Lemma 2.8)} \\
&\quad = \overline{B(SSS)B(SSS)B + B(SSS)SB(SSS)SB} \\
&\quad \subseteq \overline{BSBSB + BSSBSSB} \\
&\quad \subseteq \overline{\overline{BSBSB} + \overline{B(SSS)SSB}} \\
&\quad \subseteq \overline{B + \overline{BSB}} \quad \text{(as B is a bi } k\text{−ideal of } S) \\
&\quad \subseteq \overline{B + B} \quad \text{(by hypothesis).} \\
&\quad \subseteq \overline{B} = B \quad \text{(by hypothesis).}
\end{aligned}$$

Hence B is a quasi $k-$ideal of S. ∎

Theorem 2.15. *The following conditions in a ternary semiring S are equivalent*

(i) *S is $k-$regular.*

(ii) *for every bi $k-$ideal B of S, $B = \overline{BSBSB}$.*

(iii) *for every quasi $k-$ideal Q of S, $Q = \overline{QSQSQ}$.*

Proof. (i) ⇒ (ii) Let $a \in B$. Since S is $k-$regular, then there exist $x, y \in S$ such that $a+axa = aya$. This implies that $axa+axaxa = ayaxa$. Since $axaxa, ayaxa \in BSBSB$, therefore $axa \in \overline{BSBSB}$.

Similarly, $aya \in \overline{BSBSB}$. Therefore, $a \in \overline{BSBSB}$. Thus, $B \subseteq \overline{BSBSB}$. Since B is bi $k-$ideal of S therefore $\overline{BSBSB} \subseteq B$. Hence $B = \overline{BSBSB}$.

(ii) ⇒ (iii) By Result 2.13.

(iii) ⇒ (i) Let the condition (iii) holds. Suppose R be right $k-$ideal, M be lateral $k-$ ideal and L be left $k-$ideal of S, then $Q = R\cap M\cap L$ is a quasi$k-$ideal of S. By hypothesis, we have

$$\begin{aligned}
R \cap M \cap L &= \overline{(R \cap M \cap L)S(R \cap M \cap L)S(R \cap M \cap L)} \\
&\subseteq \overline{RSMSL} \subseteq \overline{RML}.
\end{aligned}$$

But $\overline{RML} \subseteq R \cap M \cap L$. Therefore, $R \cap M \cap L = \overline{RML}$. Therefore, by Theorem 2.11, we have S is $k-$regular. ∎

Theorem 2.16. *Let S be a ternary semiring. Then the following are equivalent*

(i) *S is $k-$regular.*

(ii) *$B \cap M = \overline{BMBMB}$ for every lateral $k-$ideal M and bi$k-$ideal B of S.*

(iii) *$Q \cap M = \overline{QMQMQ}$ for every lateral $k-$ideal M and quasi$k-$ideal Q of S.*

Proof. (i) $\Rightarrow$ (ii) Let $a \in B \cap M$. Then $a \in B$ and $a \in M$. Since S is $k-$regular then there exist $x, y \in S$ such that $a + axa = aya$. This implies that $axa + axaxa = ayaxa$. Also, $axaxa + axaxaxa = ayaxaxa$ and $axaxaxa + axaxaxaxa = ayaxaxaxa$. Since $axaxaxaxa, ayaxaxaxa \in BSMSBSMSB \subseteq BMBMB$, therefore by property of $k-$closure it is easy to show that $a \in \overline{BMBMB}$. Therefore, $B \cap M \subseteq \overline{BMBMB}$.

Again, as $BMBMB \subseteq BSBSB$, therefore $\overline{BMBMB} \subseteq \overline{BSBSB} \subseteq B$ (as B is bi $k-$ideal of S). Therefore, $\overline{BMBMB} \subseteq B$. Also, $BMBMB \subseteq SM(SSS) \subseteq M, \overline{BMBMB} \subseteq \overline{M} = M$ (as M is lateral $k-$ideal of S). Whence $\overline{BMBMB} \subseteq B \cap M$. Therefore, $B \cap M = \overline{BMBMB}$.

(ii) $\Rightarrow$ (iii) By Result 2.13, (iii) holds.

(iii) $\Rightarrow$ (i) Since S is itself a lateral $k-$ideal of S, therefore $Q = Q \cap S = \overline{QSQSQ}$. Therefore, by the above theorem the result holds. ∎

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