A CONNECTION BETWEEN CATEGORIES OF (FUZZY) MULTIALGEBRAS AND (FUZZY) ALGEBRAS

R. Ameri[1]

T. Nozari

Department of Mathematics
Faculty of Basic Science
University of Mazandaran
Babolsar
Iran
e-mail: ameri@umz.ac.ir

Abstract. The purpose of this paper is study the relationship between the categories of fuzzy multialgebras and crisp algebras. In this regards first we briefly study the categories of multialgebras and fuzzy multialgebras and then by using the fundamental relation of multialgebras we construct a functor from category of fuzzy multialgebras to the category of fuzzy algebras and hence. Hence it can be derived a fuzzy algebra from every fuzzy multialgebras through the fundamental relation.

Keywords: universal algebra, multialgebra, fuzzy multialgebra, fundamental relation, fuzzy set, homomorphism.

1. Introduction

Several aspects of homomorphisms, subalgebras and subdirect decompositions of multialgebras also called hyperalgebra were developed for special cases in [13], [14] by Picket and in [9] by Hansoul. In [17], D. Schweigert studied the congruences of multialgebras and the exponentiations of universal hyperalgebras. Ameri and Zahedi introduced the notion of hyperalgebraic systems in [2]. Ameri et.al. in [3] introduced congruence of multialgebras. The notion of direct product, identities and fundamental relation of multialgebras in [10], [11] and [12] introduced and studied by Pelea.

In this paper, we follow [2] and [3] to study the relationship between the category of fuzzy multialgebras and category of fuzzy algebras. The paper is organized in four sections. In Section 2, we gather the definitions and basic properties of multialgebras and fuzzy algebras that will be used in the next sections. In Section 3, the category of multialgebras are briefly discussed. In Section 4, first the category of fuzzy multialgebras are investigated and then, we use the fundamental

[1]The first author has been supported in part by Fuzzy System Research Center, University of Sistan and Bluchestan, Zahedan, Iran

relation to assigning to every fuzzy multialgebra a fuzzy algebra. Finally, we construct a functor, which is called the fundamental functor from category of fuzzy multialgebras to the category of fuzzy algebras.

2. Preliminaries

In this section, we present some definitions and simple properties of multialgebras which will be used in the next section. In the sequel, H is a fixed nonvoid set, $P^*(H)$ is the family of all nonvoid subsets of H, and for a positive integer n we denote for H^n the set of n-tuples over H (for more see [6] and [7]).

For a positive integer n, an n-ary *hyperoperation* β on H is a function $\beta : H^n \to P^*(H)$. We say that n is the *arity* of β. A subset S of H is *closed* under the n-ary hyperoperation β if $(x_1, \ldots, x_n) \in S^n$ implies that $\beta(x_1, \ldots, x_n) \subseteq S$. A nullary hyperoperation on H is just an element of $P^*(H)$; i.e., a nonvoid subset of H.

A *hyperalgebraic system* or a *multialgebra* $\langle H, (\beta_i, \mid i \in I)\rangle$ is the set H with together a collection $(\beta_i, \mid i \in I)$ of hyperoperations on H.

A subset S of a multialgebra $\mathbb{H} = \langle H, (\beta_i, \mid i \in I)\rangle$ is a *submultialgebra* of $\mathbb{H}$ if S is closed under each hyperoperation β_i, for all $i \in I$, that is $\beta_i(a_1, ..., a_n) \subseteq S$, whenever $(a_1, ..., a_n) \in S^n$. The *type* of $\mathbb{H}$ is the map from I into the set $\mathbb{N}^*$ of nonnegative integers assigning to each $i \in I$ the arity of β_i. Two multialgebras of the same type are similar.

For $n > 0$, we extend an n-ary hyperoperation β on H to an n-ary operation $\overline{\beta}$ on $P^*(H)$ by setting for all $A_1, ..., A_n \in P^*(H)$

$$\overline{\beta}(A_1, ..., A_n) = \bigcup\{\beta(a_1, ..., a_n) | a_i \in A_i (i = 1, ..., n)\}$$

It is easy to see that $\overline{\mathbb{H}}\langle P^*(H), (\overline{\beta}_i, \mid i \in I)\rangle$ is an algebra of the same type of $\mathbb{H}$. Whenever possible we write a instead of the the singleton $\{a\}$; e.g. for a binary hyperoperation $\circ$ and $a, b, c \in H$ we write $a \circ (b \circ c)$ for $\{a\} \circ (\{b\} \circ \{c\}) = \bigcup\{a \circ u | u \in b \circ c\}$.

Example 2.1.

(i) A *hypergroupoid* is a multialgebra of type (2), that is a set H together with a (binary) hyperoperation $\circ$. A hypergroupoid $(H, \circ)$, which is associative, that is $x \circ (y \circ z) = (x \circ y) \circ z$ for all $x, y, z \in H$ is called a *semihypergroup*.

(ii) A *hypergroup* is a semihypergroup such that for all $x \in H$ we have $x \circ H = H = H \circ x$ (called the *reproduction axiom*).

An element e in a hypergroup $\mathbb{H} = (H, \circ)$ is called an *identity* of $\mathbb{H}$ if, for all $x \in H$, $x \in (e \circ x) \cap (x \circ e)$.

(iii) A *polygroup* (or multigroup) is a semihypergroup $\mathbb{H} = (H, \circ)$ with $e \in H$ such that for all $x, y \in H$.

(iv) $e \circ x = x = x \circ e$.

(v) there exists a unique element, $x^{-1} \in H$ such that

$$e \in (x \circ x^{-1}) \cap (x^{-1} \circ x), x \in \bigcap_{z \in x \circ y} (z \circ y^{-1}), y \in \bigcap_{z \in x \circ y} (x^{-1} \circ z).$$

In fact, a polygroup is a multialgebra of type $(2, 1, 0)$.

Definition 2.2. Let $\mathbb{H} = \langle H, (\beta_i, \mid i \in I) \rangle$ and $\overline{\mathbb{H}} = \langle \overline{H}, (\overline{\beta}_i, \mid i \in I) \rangle$ be two similar multialgebras. A map h from $\mathbb{H}$ into $\overline{\mathbb{H}}$ is called a

(i) A *homomorphism* if for every $i \in I$ and all $(a_1, ..., a_{n_i}) \in H^{n_i}$ we have that $h(\beta_i((a_1, ..., a_{n_i})) \subseteq \overline{\beta}_i(h(a_1), ..., h(a_{n_i}))$;

(ii) a *good homomorphism* if for every $i \in I$ and all $(a_1, ..., a_{n_i}) \in H^{n_i}$ we have that $h(\beta_i((a_1, ..., a_{n_i})) = \overline{\beta}_i(h(a_1), ..., h(a_{n_i}))$.

Definition 2.3. A *universal algebra* or *algebra* whose hyperoperations are singleton valued (i.e. $|\beta(a_1, ..., a_n)| = 1$ for all $a_1, ..., a_n \in H$) viewed as maps from H^n into H and called operation.

Definition 2.4. Let X be a nonempty set. A fuzzy subset μ of X is a function

$$\mu : X \to [0, 1].$$

Let μ and υ be two fuzzy subset of X, we say that μ is contained in υ, if

$$\mu(x) \leq \upsilon(x), \forall x \in X.$$

If μ_i be a collection of fuzzy subsets of X, then we define the fuzzy subset $\bigcap_{i \in I} \mu_i$ by:

$$\left(\bigcap_{i \in I} \mu_i \right)(x) = \inf_{i \in I} \{\mu_i(x)\}, \quad \forall x \in X.$$

Definition 2.5. Let μ be a fuzzy subset of X and $t \in [0, 1]$. The set $A_t = \{x \in X \mid A(x) \geq t\}$ is called a level subset of X.

3. The category of multialgebras

Definition 3.1. Let $\mathbb{H}$ be a multialgebra. Define inductively a relation ϵ^* on $\mathbb{H}$ as follows:

Set $\epsilon_0 = \{(x, x) \mid x \in H\}$. If $n \geq 0$ and ϵ_n has been defined set

$$\epsilon_{n+1} = \epsilon_n \cup \{\beta_i(a_1, ..., a_{n_i}) \times \beta_i(b_1, ..., b_{n_i}) : i \in I \text{ and } (a_j, b_j) \in \epsilon_n \text{ for all } j = 1, ..., n_i\}$$

and set $\epsilon = \bigcup_{n=0}^{\propto} \epsilon_n$. Finally, set ϵ^* be the transitive closure of ϵ (here, as usual the transitive closure of ϵ is the set of all $(a,b) \in H^2$ such that there exists $k \geq 0$ and $a = a_0, a_1, ..., a_k = b$ with $(a_j, a_{j+1}) \in \epsilon$ for all $j = 0, ..., k-1$). A direct induction shows that ϵ_n is an equivalence relation on $\mathbb{H}$ for all $n \geq 0$ and hence ϵ^* is an equivalence relation on $\mathbb{H}$. Denote by H/ϵ^* the set of blocks (also called equivalence classes) of ϵ^*. Let $i \in I$ and $(a_j, b_j) \in \epsilon^*$ for all $j = 1, ..., n_i$. Then there exist $m_j \geq 0$ such that $(a_j, b_j) \in \epsilon_{m_j}$ for all $j = 1, ..., n_i$. Let $m = max(m_1, ..., m_{n_i})$. By the definition of ϵ^* clearly $(\beta_i(a_1, ..., a_{n_i}), \beta_i(b_1, ..., b_{n_i})) \in \epsilon_{m+1} \subseteq \epsilon^*$. This shows that for arbitrary blocks $B_1, ..., B_{n_i}$ of ϵ^* the set $\beta_i(B_1, ..., B_{n_i})$ is included in a block B of ϵ^*. It follows that $\mathbb{H}/\epsilon^* = (H/\epsilon^*, (\beta_i : i \in I))$ is a universal algebra. It can be verified that ϵ^* is the least equivalence relation such that $\mathbb{H}/\epsilon^*$ is an universal algebra.

Remark 3.2. Consider a hyperalgebra $\mathbb{H}$. The smallest equivalence relation such that the factor algebra $\mathbb{H}/\epsilon^*$ is an algebra is called the fundamental relation of $\mathbb{H}$ (for more see [11]). In [11], Pelea introduced and studied the fundamental relation of a multialgebra based on term functions. In this paper we present a different approach of [11] to introduce the fundamental relation of a hyperalgebra.

Theorem 3.3. *The relation ϵ^* is the fundamental relation on A.*

Definition 3.4. Let $\mathbb{H}_j = \langle H_j, (\beta_{ji} : i \in I)\rangle, (j \in J)$ be a nonvoid family of similar multialgebras. Set $H = \bigcup_{j \in J} H_j$ and denote by $X = \prod_{j \in J} H_j$ the set of maps $f : J \to H$ such that $f(j) \in H_j$ for all $j \in J$. For $i \in I$ and $g_1, ..., g_{n_i} \in X$ define $g = \beta'_i(g_1, ..., g_{n_i})$ by setting $g(j) = \beta_{ji}(g_1(j), ..., g_{n_i}(j))$ for all $j \in J$. Clearly g is a nonvoid subset of X and so β'_i is a hyperoperation on X. Therefore, $\prod_{j \in J} \mathbb{H}_j = \mathbb{X} = \langle X, (\beta'_i : i \in I)\rangle$ is a multialgebra.

In this section, we briefly introduce the category of multialgebras.

Definition 3.5. The category $\mathcal{MA}$ of multialgebras, is defined as follows:

(i) The objects of $\mathcal{MA}$ are the multialgebras;

(ii) For objects A, B of $\mathcal{MA}$, of the same type, the set $Hom(A, B)$ of morphism from A to B, is the set all homomorphism from A to B;

(iii) The composition gf of morphisms $f : A \to B$ and $g : B \to C$ is defined by setting $(gf)(x) = g(f(x))$ for all $x \in A$.

(iv) For any object A, the morphism $1_A : A \to A$ $(x \mapsto x)$, is the identity morphism.

Note that in the part (ii) of above if we replace $Hom(A, B)$ with $Hom_g(A, B)$, the set of all good homomorphisms, we will obtain a new category, which it is denoted by $\mathcal{MA}_g$. In fact, $\mathcal{MA}_g$ is a subcategory of $\mathcal{MA}$, which is not full.

We denote by $\mathcal{A}$ the category of (universal) algebras .

Lemma 3.6. *Let $\mathbb{M} = \langle M, (\beta_i : i \in I)\rangle$ and $\mathbb{N} = \langle N, (\beta'_i : i \in I)\rangle$ be two similar multialgebras and f be a good homomorphism from $\mathbb{M}$ into $\mathbb{N}$. Let ϵ^* and ϵ'^* be the fundamental relations of $\mathbb{M}$ and $\mathbb{N}$ respectively. Then*

(i) *$f([x]_{\epsilon^*}) \subseteq [f(x)]_{\epsilon'^*}$; for all $x \in M$;*

(ii) *the map $f^* : M/\epsilon^* \to N/\epsilon'^*$ by setting $f^*([x]_{\epsilon^*}) = [f(x)]_{\epsilon'^*}$ for all $x \in M$ is a homomorphism of the algebra M/ϵ^* into the algebra N/ϵ'^* (where $[x]_{\epsilon^*}$ and $[f(x)]_{\epsilon'^*}$ are the blocks of M/ϵ^* and N/ϵ'^* containing x and $f(x)$).*

Proof. (i) We proof by induction on $n \geq 0$ that $f(\epsilon_n) \subseteq \epsilon^*_n$. Let $n = 0$. Then, the blocks of ϵ_0 are the singletons of M and their image consists of certain singletons of N. Suppose $n \geq 0$ and $f(\epsilon_n) \subseteq \epsilon'_n$, we show that $f(\epsilon_{n+1}) \subseteq \epsilon'_{n+1}$ (where ϵ'_{n+1} denotes the corresponding relation on N). Let $(a, b) \in \epsilon_{n+1} \setminus \epsilon_n$. Then, there exist $i \in I$ and $(a_j, b_j) \in \epsilon_n (j = 1, ..., n_i)$ such that $a = \beta_i(a_1, ..., a_{n_i})$ and $b = \beta_i(b_1, ..., b_{n_i})$. Clearly, $f(a) = \beta'_i(f(a_1), ..., f(a_{n_i}))$, $f(b) = \beta'_i(f(b_1), ..., f(b_{n_i}))$ and so $(f(a), f(b)) \in \epsilon'_{n+1}$. This proves that $f(\epsilon_{n+1}) \subseteq \epsilon'_{n+1}$. By the definition of transitive closure, we obtain that $f(\epsilon^*) \subseteq \epsilon'^*$, which proves the theorem.

(ii) Straightforward.

Theorem 3.7. *The mapping $F : \mathcal{MA} \to \mathcal{A}$ defined by $F(M) = M/\epsilon^*$ and $F(f) = f^*$ is a functor.*

Proof. It is an immediate consequence of Lemma 3.6.

Theorem 3.8. *Let $\mathbb{H}_j (j \in J)$ be a family of similar multialgebras $\mathbb{H}_j$ with fundamental relations ϵ_j*. Then the fundamental relation ϵ^* on $\prod_{j\in J} \mathbb{H}_j$ consists of (f, g) with $f, g \in \prod_{j\in J} \mathbb{H}_j$ satisfying $(f(j), g(j)) \in \epsilon_j*$ for all $j \in J$.*

Proof. By induction on $n \geq 0$, one proves the result for ϵ_n.

Remark 3.9. The theorem guarantees the existence of the fundamental relation for special multialgebras such as, semihypergoups, hypergroups, hyperrings, hypermodules, hypervector spaces.

Corollary 3.10. *Let $f : \mathbb{M} \to \mathbb{N}$ be a morphism in $\mathcal{MA}$ and let φ_M and φ_N denote the canonical projections of $\mathbb{M}$ and $\mathbb{N}$ into $\mathbb{M}/\epsilon^*$ and $\mathbb{N}/\epsilon'^*$, respectively. Then the following diagram is commutative:*

$$\begin{array}{ccc} M & \xrightarrow{\quad f \quad} & N \\ {\scriptstyle \varphi_M}\downarrow & & \downarrow{\scriptstyle \varphi_N} \\ M/\varepsilon^* & \xrightarrow{\quad f^* \quad} & N/\varepsilon^* \end{array}$$

4. Category of fuzzy multialgebras

Definition 4.1. Let $\mathbb{H}$=$\langle H, (\beta_i : i \in I)\rangle$ be a multialgebra and μ be a fuzzy subset of H. We say that μ is a fuzzy multialgebra in symbol $\mu <_{FMA} \mathbb{H}$ if

(i) for every $i \in I$, such that the arity n_i of β_i is positive, for all $a_1, ..., a_{n_i} \in H$ and every $z \in \beta_i(a_1, ..., a_{n_i})$ we have $\mu(z) \geq \mu(a_1) \wedge ... \wedge \mu(a_{n_i})$;

(ii) if there exist a nullary β_i then: the image in μ has a greatest element m and $\mu(z) = m$ for every $z \in \beta_j$ with $n_j = 0$. Denote by $FMA(\mathbb{H})$, the set of fuzzy multialgebras of $\mathbb{H}$.

For a (universal) algebra in the above definition, the condition in (i) reduces to

$$\mu(\beta_i(a_1, ..., a_{n_i})) \geq \mu(a_1) \wedge ... \wedge \mu(a_{n_i}),$$

and the condition (ii) became $\mu(\beta) = m$.

Denote by $FA(\mathbb{H})$, the set of all fuzzy algebras of $\mathbb{H}$.

Definition 4.2. The category of fuzzy multialgebras of a given type τ denoted by $\mathcal{FMA}_\tau$, is defined as follows:

(i) The objects of $\mathcal{FMA}$ are the fuzzy multialgebras;

(ii) For the objects $\mu <_{FMA} \mathbb{M}$ and $\mu' <_{FMA} \mathbb{N}$ a morphism is a homomorphism f, from $\mathbb{M}$ to $\mathbb{N}$, such that $\mu(z) \geq t \Rightarrow \mu'(f(z)) \geq t$, for all $z \in M$ and $t \in L$;

(iii) the composition of morphisms is the composition of homomorphisms and the identity morphism is the identity selfmap.

Clearly, fuzzy algebras (considered as singleton valued multialgebras) form a subcategory of $\mathcal{FMA}_\tau$, we denote this subcategory by $\mathcal{FA}$.

Definition 4.3. Let $\mathbb{H}$=$\langle H, (\beta_i : i \in I)\rangle$ be an algebra and μ be a fuzzy subalgebra of H. Define the following hyperoperations on H by

$$\beta_i^* : H \times ... \times H \longrightarrow P^*(H)$$
$$\beta_i^*(x_1, ...x_{n_i}) = \{t \mid \mu(t) = \mu(\beta_i(x_1, ..., x_{n_i}))\}, \text{ for every } i \in I$$

then $\mathbb{H} =< H, (\beta_i^* : i \in I) >$ is a multialgebra and μ is a fuzzy multialgebra.

Proof. We must prove that for every $i \in I$ and all $z \in \beta_i^*(a_1, ..., a_{n_i})$

$$\mu(z) \geq \mu(a_1) \wedge ... \wedge \mu(a_{n_i}).$$

But if $z \in \beta_i^*(a_1, ..., a_{n_i})$ then $\mu(z) = \mu(\beta_i(a_1, ..., a_{n_i})) \geq \mu(a_1) \wedge ... \wedge (a_{n_i})$ (since μ is a fuzzy subalgebra). ∎

Theorem 4.4. *Let $\mathbb{H}$ be a multialgebra and μ be a fuzzy subset of H. Then, μ is a fuzzy submultialgebra if and only if for every t in $[0, 1]$, the level subset μ_t be a submultialgebra of $\mathbb{H}$.*

Proof. ($\Rightarrow$) Let μ be a fuzzy multialgebra of $\mathbb{H}$. Let $t \in [0,1]$, $i \in I$ and $a_1, ..., a_{n_i} \in \mu_t$. To prove that $\beta_i(a_1, ..., a_{n_i}) \subseteq \mu_t$, let $z \in \beta_i(a_1, ..., a_{n_i})$ be arbitrary . Then $\mu(a_1) \wedge ... \wedge \mu(a_{n_i}) \leq \mu(z)$, and hence we have $\mu(a_1) \geq t, ..., \mu(a_{n_i}) \geq t$, thus $\mu(z) \geq \mu(a_1) \wedge ... \wedge \mu(a_{n_i}) \geq t$ proving $z \in \mu_t$.

($\Leftarrow$) Suppose that μ_t, for every $0 \leq t \leq 1$, is a submultialgebra of $\mathbb{H}$. Let $i \in I$ and $a_1, ..., a_{n_i} \in H$. Set $t = \mu(a_1) \wedge ... \wedge \mu(a_{n_i})$. Now, clearly, $\mu(a_i) \geq t$, hence, $a_i \in \mu_t$ for all $i = 1, ..., n_i$. As μ_t is a submultialgebra of $\mathbb{H}$, clearly $\beta_i(a_1, ..., a_{n_i}) \subseteq \mu_t$ proving $\mu(z) \geq t = \mu(a_1) \wedge ... \wedge \mu(a_{n_i})$ for all $z \in \beta_i(a_1, ..., a_{n_i})$. ∎

Lemma 4.5. *Let $\mathbb{H}=\langle H, (\beta_i : i \in I)\rangle$ be a multialgebra and μ be a fuzzy submultialgebra of H. If $0 \leq t_1 < t_2 \leq 1$, then $\mu_{t_1} = \mu_{t_2}$ if and only if there is no x in H such that $t_1 \leq \mu(x) \leq t_2$.*

Proof. Straightforward. ∎

Let μ be a fuzzy multialgebra on $\mathbb{H}$ and α^* be the fundamental relation on $\mathbb{H}$. Define μ^* on H/α^*, such that $\mu^* = w(\mu)$, where $w : \mathbb{H} \to \mathbb{H}/\alpha^*$ is the natural homomorphism.

Lemma 4.6. *Let $\mu <_{FMA} \mathbb{H}$ and let ϵ^* be the fundamental relation on $\mathbb{H}$. Define $\mu^* : H/\epsilon^* \to [0,1]$ by setting $\mu^*(B) = \vee\{\mu(b) : b \in B\}$, then μ^* is a fuzzy algebra on $\mathbb{H}/\alpha^*$.*

Proof. Let $i \in I$ and $B_1, ..., B_{n_i}$ be blocks of ϵ^*. Then $\beta_i(B_1, ..., B_{n_i}) \subseteq B$ for some block B of ϵ^*. Now,

$$\begin{aligned}
\mu^*(B_1) \wedge ... \wedge \mu^*(B_{n_i}) &= \bigwedge_{i=1}^{n_i} \left(\bigvee_{t_j \in B_i} \mu(t_j) \right) \\
&= \bigvee \{\mu(t_1) \wedge ... \wedge \mu(t_{n_i}) : t_1 \in B_1, ..., t_{n_i} \in B_{n_i}\} \\
&\leq \bigvee \{\mu(z) : z \in \beta_i(t_1, ..., t_{n_i}), t_1 \in B_1, ..., t_{n_i} \in B_{n_i}\} \leq \mu^*(B).
\end{aligned}$$ ∎

Lemma 4.7. *Let f be a homomorphism of $\mu <_{FMA} \mathbb{H}$ into $\mu' <_{FMA} \mathbb{H}'$ and let ϵ^* and ϵ'^* be the fundamental relations of $\mathbb{H}$ and $\mathbb{H}'$. Define $f^* : H/\epsilon^* \to H'/\epsilon'^*$ by setting $f^*(B) = B'$ where B' is the block of ϵ'^* containing $f(B)$. Then f^* is a homomorphism from the fuzzy algebra $\mu^* <_{FA} \mathbb{H}/\epsilon^*$ to $\mu'^* <_{FA} \mathbb{H}'/\epsilon'^*$.*

Proof. It follows from Lemma 3.2 and the definitions. ∎

The next result follows immediately from Lemmas 4.6 and 4.7.

Theorem 4.8. *The map F: $\mathcal{FMA} \longrightarrow \mathcal{FA}$ defined by $F(\mu) = \mu^*$ and $F(f) = f^*$ is a functor.*

Acknowledgement. This research was supported partially by the Center of Excellence: "Fuzzy Systems and their Applications, Shahid Bahonar University of Kerman, Iran" and the "Research Center on Algebraic Hyperstructure and Fuzzy Mathematics, Babolsar, Iran".

References

[1] Ameri, R., *On categories of hypergroups and hypermodules,* Italian Journal of Pure and Applied Mathematics, vol. 6 (2003), 121-132.

[2] Ameri, R., Zahedi, M.M., *Hyperalgebraic system,* Italian Journal of Pure and Applied Mathematics, vol. 6 (1999), 21-32.

[3] Ameri, R., Rosenberg, I.G., *Congruences of Multialgebras*, Multivalued Logic and Soft Computing (to appear).

[4] Breaz, S., Pelea, C., *Multialgebra and Term Functions over the Algebra of the Nonvoid Subsets*, Mathematica, 43 (66) (2001), 143-149.

[5] Burris, S., Sankappanavar, H.P., *A course in universal algebra*, Springer Verlag, 1981.

[6] Corsini, P., *Prolegomena of hypergroup theory*, Aviani Editore, 1993.

[7] Corsini, P., Leoreanu, L., *Applications of hyperstructure theory,* Kluwer, Dordrecht, 2003.

[8] Gratzer, G., *Universal algebra*, 2nd edition, Springer Verlag, 1970.

[9] Hansoul, G.E., *A Simultaneous Characterization of Subalgebras and Conditional Subalgebras of a Multialgebra*, Bull. Soc. Roy. Science, Liege, 50 (1981), 16-19.

[10] Pelea, C., *On the direct product of multialgebras*, Studia Univ. Babes-Bolyai, Mathematica, vol. XLVIII (2003), 93-98.

[11] Pelea, C., *On the fundamental relation of a multialgebra*, Italian Journal of Pure and Applied Mathematics, vol. 10 (2001), 141-146.

[12] Pelea, C., *Identities and multialgebras*, Italian Journal of Pure and Applied Mathematics, vol. 21 (1967), 327-342.

[13] Pickett, H.E., *Subdirect Representations of Relational Systems*, Fund. Math., 56 (1964), 223-240.

[14] Pickett, H.E., *Homomorphism and subalgebras of multialgebras*, Pacific J. Math, vol. 10 (2001), 141-146.

[15] Rosenfeld, A., *Fuzzy groups*, J. Math. Anal. Appl., 35 (1971), 512-517.

[16] Šlapal, J., *On Exponentiation of Universal Hyperalgebras*, Algebra Universalis, vol. 44, nos. 1-2 (2000), 187-193.

[17] Schweigert, D., *Congruence Relations of Multialgebras* , Discrete Mathematics, 53 (1985), 249-253.

Accepted: 15.02.2009