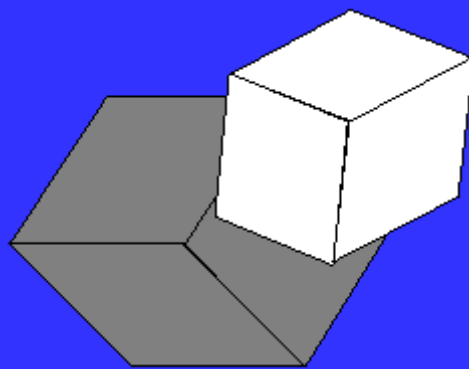


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## Papers Abstracts



**FORUM**

## ON FUZZY MINIMAL STRUCTURES

Moiz Ud Din Khan, Rifaqat Noreen

In this paper, we define fuzzy open (closed)  $M$ -set, fuzzy  $M$ - frontier, fuzzy  $M$ -semi frontier, fuzzy rarely  $M$ -set, and fuzzy rarely  $M$ -continuous functions in fuzzy minimal spaces. We will explore several interesting properties and characterizations of these newly defined notions. (pp. 1–15)

## A NOTE ON $S$ -ACTS AND BOUNDED LINEAR OPERATORS

Samira Najji Kadhim, Muna Jasim Mohammed Ali, Zainab Abed Atiya

In this work, the properties of the certain operator have been studied by looking at the associated  $S$ -act of this operator, and conversely. Some operators, for example such operator, one to one, onto operators have been looked. On the other hand, basic mathematical interpretation understanding of  $S$ -acts, such as faithful, finitely generated, singular, separated, torsion free and noetherian acts. We have found out the properties may be associated with  $S$ -act which has any of these properties. Let  $V$  be a inner product space over a field  $F$ ,  $T$  be a bounded operator on  $V$ , and let  $S = \{e^{x+y} | x, y \text{ are independent variables in } R\}$  be the semigroup. Define  $\theta : S \times V \rightarrow V$  by  $\theta(e^{x+y}, v) = e^{T+T^*}(v)$ . This function makes  $V$  a left  $S$ -act, denote by  $V_{T+T^*}$  and we call it the associated  $S$ -act of  $T + T^*$ . (pp. 16–22)

## APPLICATION OF RECURRENT NEURAL NETWORK USING MATLAB SIMULINK IN MEDICINE

Raja Das, Madhu Sudan Reddy

In this paper, a recurrent neural network (RNN) for finding the solution of linear programming problems is presented with better, spontaneous and fast converging. To achieve optimality in accuracy and also in computational effort, an algorithm is also presented. This paper covers the MATLAB Simulink modeling and simulative confirmation of such a recurrent neural network. Modeling and simulative results validate the theoretical analysis and efficiency of the recurrent neural network for finding the solution for linear programming problem. An application RNN in medicine has been presented to show the performance of the recurrent neural network. (pp. 23–30)

## GENERALIZED NUMERICAL RADIUS INEQUALITIES FOR $2 \times 2$ OPERATOR MATRICES

Watheq Bani-Domi

We prove some new generalized numerical radius inequalities for  $2 \times 2$  operator matrices, which improve and generalize an earlier numerical radius inequalities.

(pp. 31–38)

## A COMPARATIVE STUDY ON ACHROMATIC AND $B$ -CHROMATIC NUMBER OF CERTAIN GRAPHS

K.P. Thilagavathy, A. Santha

In this paper, we find the achromatic number of central graph of Crown graph and we discussed its structural properties. We compare the achromatic and  $b$ -chromatic number of central graph of Sunlet graph, central graph of web graph. Also we study the structural properties of the central graph of Sunlet graph.

(pp. 39–44)

## A WEAKER QUANTITATIVE CHARACTERIZATION OF THE SPORADIC SIMPLE GROUPS

Jinbao Li, Guiyun Chen

It is proved in this paper that all the sporadic simple groups can be characterized by their orders and one special conjugacy class sizes, such as largest conjugacy class sizes, and smallest conjugacy class sizes greater than 1.

(pp. 45–54)

## LAWVERE-TIERNEY SHEAVES, FACTORIZATION SYSTEMS, SECTIONS AND $j$ -ESSENTIAL MONOMORPHISMS IN A TOPOS

Zeinab Khanjanzadeh, Ali Madanshekaf

Let  $j$  be a Lawvere-Tierney topology (a topology, for short) on an arbitrary topos  $\mathcal{E}$ ,  $B$  an object of  $\mathcal{E}$ , and  $j_B = j \times 1_B$  the induced topology on the slice topos  $\mathcal{E}/B$ . In this manuscript, we analyze some properties of the pullback functor  $B^* : \mathcal{E} \rightarrow \mathcal{E}/B$  which are dealing with topologies. Then for the left cancellable class  $\mathcal{M}$  of all  $j$ -dense monomorphisms in a topos  $\mathcal{E}$ , we achieve some

necessary and sufficient conditions for that the pair  $(\mathcal{M}, \mathcal{M}^\perp)$  is a factorization system in  $\mathcal{E}$ , which is related to the factorization systems in slice topoi  $\mathcal{E}/B$ , where  $B$  ranges over the class of objects of  $\mathcal{E}$ . Among other things, we prove that an arrow  $f : X \rightarrow B$  in  $\mathcal{E}$  is a  $j_B$ -sheaf in  $\mathcal{E}/B$  whenever the graph of  $f$ , is a section in  $\mathcal{E}/B$  as well as the object of sections  $S(f)$  of  $f$ , is a  $j$ -sheaf in  $\mathcal{E}$ . Furthermore, we introduce a class of monomorphisms in  $\mathcal{E}$ , which we call each member of the class  $j$ -essential. Some equivalent forms and some of their properties are presented. Also, we prove that any presheaf in a presheaf topos has a maximal essential extension. Finally, some similarities and differences of the obtained result are discussed if we put a (productive) weak topology  $j$ , studied by some authors, instead of a topology. (pp. 55–72)

### HX-TYPE CHAOTIC (HYPERCHAOTIC) SYSTEM BASED ON FUZZY INFERENCE MODELING

Baojie Zhang, Hongxing Li, Zitian Li

In this paper, we investigate HX equations of autonomous system. The right-hand side of  $n$ -dimensional HX equations theoretically consists of  $2^n$  terms. HX equations seems to be more complicated than the original system. For autonomous chaotic(hyperchaotic) system with polynomial right-hand side, we obtain its HX equations by termwise modeling. We find not all coefficients are variable in HX equations. Even some HX equations are equal to the original chaotic (hyperchaotic) systems. Under some fuzzy partition, HX-type chaotic (hyperchaotic) system is defined if some of its coefficients is exactly variable. By adjusting fuzzy partition, a family of chaotic (hyperchaotic) systems can be defined. They are different but qualitatively similar. Numerical simulations are provided to verify the existence of HX-type chaotic (hyperchaotic) system. (pp. 73–88)

### THE $Q$ -CONJUGACY CHARACTER TABLE OF DIHEDRAL GROUPS

H. Shabani, A. R. Ashrafi, E. Haghi, M. Ghorbani

In a seminal paper published in 1998, Shinsaku Fujita introduced the concept of  $Q$ -conjugacy character table of a finite group. He applied this notion to solve some problems in combinatorial chemistry. In this paper, the  $Q$ -conjugacy character table of dihedral groups is computed in general. As a consequence,  $Q$ -conjugacy character table of molecules with point group symmetries  $D_3 \cong C_{3v} \cong Dih_6$ ,  $D_4 \cong D_{2d} \cong C_{4v} \cong Dih_8$ ,  $D_5 \cong C_{5v} \cong Dih_{10}$ ,  $D_{3d} \cong D_{3h} \cong D_6 \cong C_{6v} \cong Dih_{12}$ ,  $D_2 \cong C_{2v} \cong Z_2 \times Z_2 \cong Dih_2$ ,  $D_{4d} \cong Dih_{16}$ ,  $D_{5d} \cong D_{5h} \cong Dih_{20}$ ,  $D_{6d} \cong Dih_{24}$  are computed, where  $Z_2$  denotes a cyclic group of order 2 and  $Dih_n$  is the dihedral group of even order  $n$ . (pp. 89–96)

### $\alpha$ -NILPOTENT GROUPS DERIVED FROM HYPERGROUPS WITH $\xi^*$ -RELATION

E. Mohammadzadeh, F. Mohammadzadeh

This paper deals with hypergroups, as a generalization of classical groups. An important tool in the theory of hyperstructures is the fundamental relation, which brings us into the classical algebra. In this paper for an automorphism  $\alpha$  we introduce and study the construction of  $\alpha$ -nilpotent fundamental relation in hypergroups. We will characterize  $\alpha$ -nilpotent groups via strongly regular relations and several results on the topic are presented. (pp. 97–106)

### COMBINED EFFECT OF MAGNETISM AND ROUGHNESS ON A FERROFLUID SQUEEZE FILM IN POROUS TRUNCATED CONICAL PLATES: EFFECT OF VARIABLE BOUNDARY CONDITIONS

Hardik P. Patel, G.M. Deheri, R.M. Patel

This article aims to discuss the performance of a ferrofluid squeeze film between transversely rough porous truncated conical plates resorting to special type of boundary conditions depending on the magnetization parameter. Invoking the stochastic averaging model of Christensen and Tonder regarding the roughness characterization, the associated stochastically averaged Reynolds type equation is solved to get the pressure distribution, in turn, which gives the load carrying capacity. The results affirm that suitable boundary condition may help in scaling down the adverse effect of roughness to a large extent appropriately choosing the magnetization parameter. However, in the case of negatively skewed roughness the situation remains relatively better. It is also found that the absence of flow doesn't deter the bearing system from supporting certain amount of load, which is very much unlikely in the case of conventional lubricant based bearing system. (pp. 107–119)

### ON SOME PROPERTIES OF ROUGH APPROXIMATIONS OF SUBRINGS VIA COSETS

Y. Madhavi Reddy, P. Venkat Raman, E. Keshava Reddy

In 1982, Zdzislaw Pawlak introduced the theory of Rough sets to deal with the problems involving imperfect knowledge. This present research article studies some interesting properties of Rough approximations of subrings via an equivalence relation involving cosets of an ideal. In this present work, a ring structure is assigned to the universe set and a few results on the Rough approximations of subrings of the universe set are established. (pp. 120–127)

## ON GENERALIZED ZERO-DIVISOR GRAPH ASSOCIATED WITH A COMMUTATIVE RING

N. Jahanbakhsh Basharlou, M. J. Nikmehr, R. Nikandish

Let  $R$  be a commutative ring with identity, and let  $Z(R)$  be the set of zero-divisors of  $R$ . The generalized zero-divisor graph of  $R$  is defined as the graph  $\Gamma_g(R)$  with the vertex set  $Z(R)^* = Z(R) \setminus \{0\}$ , and two distinct vertices  $x$  and  $y$  are adjacent if and only if  $\text{ann}_R(x) + \text{ann}_R(y)$  is an essential ideal of  $R$ . It follows that each edge (path) of the zero-divisor graph  $\Gamma(R)$  is an edge (path) of  $\Gamma_g(R)$ . It is proved that  $\Gamma_g(R)$  is connected with diameter at most three and with girth at most four, if  $\Gamma_g(R)$  contains a cycle. Furthermore, all rings with the same generalized zero-divisor and zero-divisor graphs are characterized. Among other results, we show that the generalized zero-divisor graph associated with an Artinian ring is weakly perfect, i.e., its vertex chromatic number equals its clique number. (pp. 128–139)

## THE FIXED POINT OF MEROMORPHIC SOLUTIONS FOR DIFFERENCE RICCATI EQUATION

Chang-Wen Peng

In this paper, we mainly investigate some properties of the transcendental meromorphic solution  $f(z)$  for the difference Riccati equation

$$f(z+1) = \frac{P_1(z)f(z) + P_2(z)}{f(z) + P_3(z)},$$

where  $P_i(z)$  ( $i = 1, 2, 3$ ) are polynomials. And we obtain some estimates of exponents of convergence of fixed points and  $c$ -points of  $f(z)$  and its shift  $f(z+n)$ . (pp. 140–153)

## STRUCTURE OF $(w_1, w_2)$ -TEMPERED ULTRADISTRIBUTION USING SHORT-TIME FOURIER TRANSFORM

Hamed M. Obiedat, Ibraheem Abu-Falahah

We characterize the space  $\mathfrak{S}_{w_1, w_2}$  of test functions of  $(w_1, w_2)$ -tempered ultradistribution in terms of their short-time Fourier transform. As a result of this characterization and using Riesz representation theorem, we characterize the space  $(w_1, w_2)$ -tempered ultradistribution. (pp. 154–164)

## PARTIALLY BLIND SIGNATURE SCHEME BASED ON CHAOTIC MAPS AND FACTORING PROBLEMS

Nedal Tahat, E.S. Ismail, A.K. Alomari

Due to the importance of security and efficiency of electronic signatures schemes, there is an increase in interest among scholars to develop such schemes based on mathematical problems to be more secure and efficient. In this paper, we propose a scheme with a low computation cost based on both cryptographic and chaotic system characteristics. The security of the scheme depends upon the intractability of the factorization problem and discrete logarithm of Chebyshev polynomials. The performance comparison demonstrated that the proposed scheme has a lower communication cost than the existing schemes in the literature, such as the one proposed by Tahat et al. To the best of our knowledge, this is the first time a partially blind signature scheme based on chaotic maps and factoring problem has been proposed. (pp. 165–177)

## A STUDY ON PSEUDOORDERS IN ORDERED \*-SEMIHYPERGROUPS

Xinyang Feng, Jian Tang, Yanfeng Luo

In this paper, we study the pseudoorders on ordered \*-semihypergroups in detail. To begin with, we introduce the concept of pseudoorders on an ordered \*-semihypergroup, and investigate its related properties. Furthermore, the relationship between strongly regular equivalence relations and pseudoorders on an ordered \*-semihypergroup is established, and some homomorphism theorems of ordered \*-semi-hypergroups by pseudoorders are given. Finally, we investigate the direct product of ordered \*-semihypergroups, and study the pseudoorders on direct product of ordered \*-semihypergroups. (pp. 178–193)

## DIFFERENTIAL TRANSFORMATION METHOD FOR SOLVING HIGH ORDER FUZZY INITIAL VALUE PROBLEMS

A.F. Jameel, N.R. Anakira, M.M. Rashidi, A.K. Alomari, A. Saaban and M.A. Shakhathreh

In this paper, we develop and analyze the use of the Differential transformation method (DTM) to find the semi analytical solution for high order fuzzy initial value problems (FIVPs) involving ordinary differential equations. DTM allows for the solution of FIVPs to be calculated in the form of an infinite series by which the components will be simply computed. Also DTM will be constructed and formulated to obtain a semi-analytical solution of high order

FIVPs using the basic properties and definitions of fuzzy set theory. Numerical example involving high order linear FIVPs was solved to illustrate the capability of DTM in this regard. The results obtained by DTM have been compared with the exact solution in the form of figures and tables. (pp. 194–208)

### SOME PROPERTIES OF ZERO GRADATIONS ON SAMANTA FUZZY TOPOLOGICAL SPACES

Mohammad Abry, Jafar Zanjani

Considering the fuzzy topological spaces in the sense of Samanta, the notion of a zero gradation as a fuzzy topological invariant is introduced that might be the first basic step to develop a theory of dimension on the fuzzy topological spaces. Also, some critical properties and applications are established.

(pp. 209–219)

### MATHEMATICAL ANALYSIS OF AN AGE-STRUCTURED QUARANTINE/ISOLATION MODEL

Mohammad A. Safi

A new age-structured model for disease transmission, subject to the use of quarantine (of asymptomatic cases) and isolation (of individuals with disease symptoms) is presented and rigorously analyzed. The model is, first of all, shown to be properly-posed mathematically by formulating it as an abstract Cauchy problem. For the case where the contact rate is separable (i.e.,  $\beta(a, b) = \beta_1(a)\beta_2(b)$ ), rigorous analysis of the model reveals that it has a globally-asymptotically stable disease-free equilibrium whenever its associated reproduction number is less than unity. The model has a unique endemic equilibrium when the threshold quantity exceeds unity. The endemic equilibrium is shown to be locally stable whenever its associated reproduction number exceeds unity. Furthermore, it is shown that adding age-factor to the basic quarantine-isolation model in M. A. Safi and A. B. Gumel (*Discrete Contin. Dyn. Syst. Ser. B.* : 209-231, 2010) does not alter the qualitative dynamics of the autonomous system (with respect to the elimination or persistence of the disease). Numerical simulations show disease elimination whenever its associated reproduction number is less than unity and the disease will persist in this case whenever its associated reproduction number exceeds unity. (pp. 220–242)



## BANACH AND KANNAN CONTRACTIONS ON $S$ -METRIC SPACE

T. Phaneendra

Unique fixed points are obtained for Banach and Kannan contractions on an  $S$ -metric space. Also, the unique fixed points are shown to be  $S$ -contractive fixed points. (pp. 243–247)

## FIXED POINT THEOREMS FOR ONE AND TWO SELF-MAPS ON A $G$ -METRIC SPACE

T. Phaneendra

The proof of a recent result of Vats et al is presented, by employing the infimum property of nonnegative real numbers. Then the unique fixed point is shown to be the  $G$ -limit of all the orbits of the form  $x, fx, \dots, f^n x, \dots, x \in X$ . That is, the unique fixed point is a  $G$ -contractive fixed point. Further, a fixed point for a pair of self-maps is obtained as another application of the infimum property. (pp. 248–257)

## GENERALIZATIONS OF PRIME TERNARY SUBSEMIMODULES OF TERNARY SEMIMODULES

Malik Bataineh, Rashid Abu-Dawwas, Wurood Oteir

Let  $R$  be a commutative semiring with  $1 \neq 0$  and all semimodules are unital. Weakly prime ternary subsemimodules of ternary semimodules have been studied. In this paper we introduce the concept of almost prime ternary subsemimodule of a ternary semimodule over a ternary semiring as a new generalization of prime ternary subsemimodule. We will give some of its properties, characteristics and its relationship among other algebraic structures. Also we carry out this concept under multiplication ternary semimodules. (pp. 258–268)

### IMPACT OF HARVESTING, NOISE AND DIFFUSION ON THE DYNAMICS OF A FOOD CHAIN MODEL WITH RATIO-DEPENDENT FUNCTIONAL RESPONSE III

G. Basava Kumar, M.N. Srinivas

The current article is associated with impact of noise and harvesting of a three species eco system which consists of prey–predator–top predator with Holling classification III. The stability of the given system is checked at the interior steady state and bionomial steady states are also evaluated. Model formulation of the optimal harvesting policy is given and its solution is derived at interior steady state by using Pontryagins Maximum principle. We also examined the population intensities of variations at the positive steady state due to environmental attribute, we have also highlighted the diffusive steadiness of the structure along with some numerical simulations. (pp. 269–289)

### CHARACTERIZATIONS OF ORDERED SEMIHYPERGROUPS BASED ON ORDERED FUZZY POINTS

Jian Tang, Xiaolong Xin, Xiangyun Xie

In this paper, we introduce the concepts of quasi-prime and quasi-semiprime fuzzy left hyperideals of ordered semihypergroups, and investigate their related properties. Furthermore, we give some characterizations of strongly semisimple ordered semihypergroups in terms of ordered fuzzy points and fuzzy left hyperideals. Especially, we prove that an ordered semihypergroup  $S$  is strongly semisimple if and only if every fuzzy left hyperideal of  $S$  can be expressed as the intersection of all quasi-prime fuzzy left hyperideals of  $S$  containing it. (pp. 290–311)

### ON TOPOLOGICAL EFFECT ALGEBRAS

M.R. Rakhshani, R.A. Borzooei, G.R.

In this paper the notions of topological and paratopological effect algebras are defined and their properties are investigated. Then by considering the notion of uniformity space and using of Riesz ideals in effect algebras, a topology and a uniformity space on any effect algebra is obtained. Finally, by definition of an equivalence relation on the set of all Cauchy nets of an effect algebra, and getting a quotient structure by that set, a completion for above uniformity space is constructed. (pp. 312–325)

## A CERTAIN NEW FAMILIAR CLASS OF UNIVALENT ANALYTIC FUNCTIONS WITH VARYING ARGUMENT OF COEFFICIENTS INVOLVING CONVOLUTION

Tariq Al-Hawary

In this paper, we used the generalization of the modified-Hadamard products to obtain some interesting characterization theorems for certain general subclass of uniformly functions with positive coefficients. (pp. 326–333)

## SPECIAL HOOP ALGEBRAS

A. Namdar, R.A. Borzooei

Hoops are naturally ordered commutative residuated integral monoids, introduced by B. Bosbach in [5,6]. In this paper, we introduce the concepts of special hoop algebra and special filter in hoop algebras and study some properties of them. We establish relation between special hoops with other structures such as simple hoops, local hoops, locally finite hoops, perfect hoops, semi-De Morgan algebras and Boolean algebras. Then, by define the notion of special filter in bounded hoops, we study the relationship between special filters and implicative (positive implicative, maximal, obstinate) filters on bounded and special hoops. Finally, we investigated the properties of a quotient structure, when it is produced by a special filter. (pp. 334–349)

## ON SOME GENERATING FUNCTIONS FOR THE TWO-PARAMETERS ONE-VARIABLE SRIVASTAVA POLYNOMIALS

Ahmed Ali Atash, Salem Saleh Barahmah

In the present paper we prove a general theorems on generating functions involving the two-parameter one-variable Srivastava polynomials, Hermite and Laguerre polynomials of two variables. It is also shown how these theorems can be used to derive several bilateral generating functions (known or new) involving Hermite and Laguerre polynomials of two variables and other classical polynomials of one variable which are contained by the two-parameter one-variable Srivastava polynomials. (pp. 350–358)

### SOME CLASSES OF INVARIANT SUBMANIFOLDS OF $(LCS)_n$ -MANIFOLDS

S.K. Hui, V.N. Mishra, T. Pal, Vandana

The object of the present paper is to study the pseudoparallel, Ricci generalized pseudoparallel and  $\eta$ -parallel invariant submanifolds of  $(LCS)_n$ -manifolds and we obtained some equivalent conditions of invariant submanifolds of  $(LCS)_n$ -manifolds under which the submanifolds are totally geodesic. Among others we found the necessary and sufficient condition of the second fundamental form to be  $\eta$ -parallel in an invariant submanifold of a  $(LCS)_n$ -manifold. Finally an example of invariant submanifold of  $(LCS)_5$ -manifold is constructed.

(pp. 359–372)

### HAAR WAVELET COLLOCATION METHOD FOR SOLVING NONLINEAR KURAMOTO-SIVASHINSKY EQUATION

Inderdeep Singh, Sheo Kumar

A collocation method based on Haar wavelet is presented for solving numerical solution of fourth order nonlinear Kuramoto-Sivashinsky equation. Efficiency and accuracy of the present method has been established by comparing the numerical results with exact solutions.

(pp. 373–384)

### APPLICATION OF PARAMETER OPTIMIZATION ALGORITHM IN DIGITAL ARCHITECTURE DESIGN

Yunhua Zhu

In recent years, digital technology has been used in all aspects of peoples work and life more and more frequently and has exerted a certain impact on the design of buildings. More and more architectural design teams add mathematical logic and digital technology to the early stages of their design to better control the design work and implement the construction. Digital building design mainly includes parametric design and algorithm generation design, the former of which is mainly studied in this paper. As a modeling design algorithm which puts an emphasis on logic and reason, parametric design emphasizes the scientific nature of architectural design. In order to achieve the optimal design of the combination of building physics, institutional performance and parameterization, this paper applies the Dijkstra algorithm and the most energy efficient

scheme generation (MEESG) energy consumption prediction method to the optimization design of wiring and performance and achieves good results, suggesting that the parameter optimization algorithm has a good impetus to the design of digital building. (pp. 385–392)

## TOTAL AND CONNECTED DOMINATION IN CHEMICAL GRAPHS

Doostali Mojdeh, Mohammad Habibi, Leila Badakhshian

For a given graph  $G$  a subset  $D$  of the vertex-set  $V(G)$  of  $G$  is called a total dominating set if every vertex  $v \in V(G)$  is adjacent to at least one vertex of  $D$ . The total domination number  $\gamma_t(G)$  is the cardinality of the smallest total dominating set. Also  $D$  is called a connected dominating set if every vertex  $v \in V(G) - D$  is adjacent to at least one vertex in  $D$  and the induced subgraph  $\langle D \rangle$  is connected. The connected domination number  $\gamma_c(G)$  is the minimum cardinality taken over all connected dominating sets of  $G$ . In this paper, we determine the domination number, the total domination number and the connected domination number for some chemical graphs. (pp. 393–401)

## THE BORDA RULE COMPREHENSIVE EVALUATION METHOD BASED ON SPEARMAN RANK

Yili Tan, Xinghuo Wan, Meilin Zhang

In the perspectives of systematization and axiomatization, the improved Borda rule was used to assign the evaluation results to avoid getting the same value when evaluation objects were different. Then the ideal evaluation result which was determined by criteria was summed up. Finally the Spearman rank correlation coefficient of the ideal evaluation result and all commonly used comprehensive evaluation methods was calculated through comparison. According to the size of the correlation coefficient, each method was sorted based on rationality. Finally the rationality of these methods was simulated by numerical values. (pp. 402–409)

ON SUBCLASS OF MEROMORPHIC UNIVALENT FUNCTIONS  
DEFINED BY A LINEAR OPERATOR ASSOCIATED WITH  
 $\lambda$ -GENERALIZED HURWITZ-LERCH ZETA FUNCTION AND  
 $q$ -HYPERGEOMETRIC FUNCTION

K.A. Challab, M. Darus, F. Ghanim

In this article, a linear operator associated with the  $\lambda$ -generalized Hurwitz-Lerch zeta function and  $q$ -hypergeometric function by using the Hadamard product (or convolution) is defined by the authors, a different interesting properties of certain subclass of meromorphic univalent functions related to a linear operator in the punctured unit disk are introduced and investigated. The authors also consider some closely related (known or new) corollaries and consequences of the main results presented in this paper. (pp. 410–423)

THE REALIZATION OF RADIO FREQUENCY IDENTIFICATION  
HANDSET ANTENNA BASED ON INTERNET OF THINGS

Yi Hu, Linna Wang

Radio frequency identification (RFID) as an automatic identification technology is one of the core technologies of Internet of Things. RFID handset is known for its light and portable characteristics; therefore, reducing the volume of RFID antenna which is the forefront and important component of RFID system without influencing working distance is an important direction of its design. According to the rules of information exchange and communication based on Internet of Things, this study investigated the effects of antenna parameters on system performance using a passive RFID system. It was found that, when the read-write distance was fixed, the higher the frequency was, the larger the one-way space loss would be; for antennae with the same structure size, higher frequency could bring better gain effects; the polarization property of antennae was the same as the polarization property of electromagnetic waves; the matching of the polarization of receiving antenna and incoming wave could induce polarization loss. A kind of RFID near-field antenna which was suitable for near-distance reading was designed through analyzing the design principles of a kind of microstrip-fed monopole near-field antenna. Then the antenna was manufactured based on simulation using High Frequency Structure Simulator (HFSS). The actual test suggested that the front end of the antenna could read the tags which were 6 cm away and 3 to 4 tags at once, which was applicable to actual application environment. (pp. 424–433)

## COUPLED FIXED AND COINCIDENCE POINT THEOREMS FOR GENERALIZED CONTRACTIONS IN METRIC SPACES WITH A PARTIAL ORDER

K. Ravibabu, Ch. Srinivasa Rao, Ch. Raghavendra Naidu

In this paper, we establish results on the existence and uniqueness of coupled common fixed point theorems and coupled coincidence fixed point theorems for such non-linear contraction mappings having a mixed monotone property in partially ordered complete metric spaces with out using continuity. Our results generalize and extend the results of V. Lakshmikantham and L. Ćirić [13], Sintunavarat and Poom Kumam [16]. (pp. 434–450)

## ON THE $k$ -NORMAL ELEMENTS AND POLYNOMIALS OVER FINITE FIELDS

Mahmood Alizadeh, Mohammad Reza Darafsheh, Saeid Mehrabi

An element  $\alpha \in \mathbb{F}_{q^n}$  is normal over  $\mathbb{F}_q$  if the set  $\{\alpha, \alpha^q, \dots, \alpha^{q^{n-1}}\}$  is a basis of  $\mathbb{F}_{q^n}$  over  $\mathbb{F}_q$ . The  $k$ -normal elements over finite fields are defined and characterized by Huczynska, Mullen, Panario and Thomson (2013). For  $0 \leq k \leq n-1$ , the element  $\alpha \in \mathbb{F}_{q^n}$  is said to be a  $k$ -normal element if  $\gcd(x^n - 1, \sum_{i=0}^{n-1} \alpha^{q^i} x^{n-1-i})$  has degree  $k$ . It is well known that a 0-normal element is a normal element. So, the  $k$ -normal elements are a generalization of normal elements. By analogy with the case of normal polynomials, a monic irreducible polynomial of degree  $n$  is called a  $k$ -normal polynomial if its roots are  $k$ -normal elements of  $\mathbb{F}_{q^n}$  over  $\mathbb{F}_q$ . In this paper, a new characterization and construction of  $k$ -normal elements and polynomials over finite fields are given. (pp. 451–464)

## LEFT ALMOST POLYGROUPS

Naveed Yaqoob, Irina Cristea, Muhammad Gulistan, Shah Nawaz

In this introductory note we define the concept of left almost polygroups and provide several examples. We also discuss the quotient structure and isomorphism theorems for left almost polygroups. (pp. 465–474)

## ON PROPERTIES OF VARIOUS MORPHISMS IN THE CATEGORIES OF GENERAL KRASNER HYPERMODULES

H. Shojaei, R. Ameri, S. Hoskova-Mayerova

In a recent paper entitled *Pre-semihyperadditive Categories*, we introduced some categories in which for objects  $A$  and  $B$ , the class of all morphisms from  $A$  to  $B$  denoted by  $Mor(A, B)$ , admits an algebraic hyperstructures such as semihypergroup or hypergroup. Then after defining and fixing a general Krasner hyperring  $R$ , we introduced and studied the categories of general Krasner  $R$ -hypermultiples,  $R\mathcal{G}.mod$ ,  $R_s\mathcal{G}.mod$ ,  $R_w\mathcal{G}.mod$  and etc. In this paper we present some properties of multi-valued homomorphisms as morphisms of these categories and study various concepts related to these morphisms in connection with the fundamental relation of their domain or codomain. (pp. 475–484)

## A STUDY ON BIJECTIVE SOFT HEMIRINGS

Li Zhang, Jianming Zhan

The concept of bijective soft hemirings is firstly proposed and some of related properties are investigated. Especially the basic operations of soft bijective hemirings are discussed and some good examples are also given. We also define a bijective soft ideal and discuss the primary operations of bijective soft  $h$ -ideals ( $k$ -ideals, strong  $k$ -ideals) and idealistic soft hemirings. (pp. 485–497)

## ON THE FIBONACCI NUMBERS OF THE MOLECULAR GRAPHS OF SOME BENT PHENYLENES

Jaroslav Seibert, Libor Koudela

The Fibonacci number  $f(G)$  of a graph  $G = (V, E)$  is defined as the number of all subsets  $U$  of  $V$  such that no two vertices in  $U$  are adjacent. Phenylenes represent a class of condensed polycyclic conjugated compounds which have the molecular graph possessing both six-membered and four-membered circuits. In this paper we are concerned with special types of bent phenylenes expanding our previous results on the linear phenylenes. The explicit formulas for the Fibonacci numbers of the bent phenylenes are found as functions of the number  $n$  of hexagons in both mentioned branches of phenylene. (pp. 498–507)



### OFF-STEP DISCRETIZATION FOR SYSTEM OF NONLINEAR SINGULAR BOUNDARY VALUE PROBLEMS USING VARIABLE MESH

Arshad Khan, Sucheta Nayak, R.K. Mohanty

In this paper, we propose two generalized variable mesh schemes based on off-step points to solve the system of nonlinear singular two point boundary value problems . Theoretical analysis proves that the proposed methods have second, third and fourth order convergence. Both the methods are applicable to singular boundary value problems. Numerical results are also provided to show the accuracy and efficiency of the proposed methods. (pp. 508–529)

### SOME NEW RESULTS RELATED TO SUBGROUP COMMUTATIVITY DEGREES AND $p$ -COMMUNTATIVITY DEGREES OF FINITE GROUPS

A. Javadi, F. Fayazi, A. Gholami

In 1970 Gallagher introduced the concept of commutativity degree of finite groups. Then some authors extend this concept to some variety of groups. In this paper, we define commutativity degree and  $p$ -commutativity degree with respect to Burnside variety of groups and study their subgroups and some properties of finite groups in this variety.

(pp. 530–543)

### CHAPMAN-JOUGUET TRAVELLING WAVE FOR A TWO-STEPS REACTION SCHEME

Abdolrahman Razani

In this paper, a version of the Majda's model for a two-steps reaction is studied. Then by studying the stable and unstable manifolds of the resulted system of ordinary differential equations, the existence of a heteroclinic orbit is proved. (pp. 544–553)

## ZARISKI TOPOLOGY FOR SECOND SUBHYPERMODULES

Razieh Mahjoob, Vahid Ghaffari

The purpose of this paper is introduce the concept of second subhypermodules over commutative hyperring and topologize the collection of all second subhypermodules and also investigate the properties of this topological space.

(pp. 554–568)

## THE HECKE ALGEBRA $H(P_{\mathbb{Q}}, P_{\mathbb{Z}})$ AND ITS RELATION TO THE CROSSED PRODUCT $H(P_{\mathbb{Q}}^+, P_{\mathbb{Z}}) \times_{\beta} \{1, -1\}$

Mamoon Ahmed, Fida Moh'd

The algebra  $H(P_{\mathbb{Q}}^+, P_{\mathbb{Z}})$  arose in number theory has been studied by Bost and Connes in [2]. In [1] a related Hecke algebra  $H(P_{\mathbb{Q}}, P_{\mathbb{Z}})$  is considered wherein it is shown to be a universal  $*$ -algebra generated by the elements  $\{\mu_n : n \in \mathbb{N}^*\}$ ,  $\{e(r) : r \in \mathbb{Q}/\mathbb{Z}\}$  and an element  $u = \left[ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$ . The goal of this paper is to study the relationship between the Hecke algebra of Bost and Connes and the Hecke algebra  $H(P_{\mathbb{Q}}, P_{\mathbb{Z}})$ . By showing the existence of a  $*$ -automorphism  $\alpha$  of  $H(P_{\mathbb{Q}}^+, P_{\mathbb{Z}})$ , we construct a covariant representation  $(\iota, U)$  of  $H(P_{\mathbb{Q}}^+, P_{\mathbb{Z}}) \times_{\beta} \{1, -1\}$  on  $H(P_{\mathbb{Q}}, P_{\mathbb{Z}})$ . This leads to our main result that  $H(P_{\mathbb{Q}}, P_{\mathbb{Z}})$  is realized as the crossed product  $H(P_{\mathbb{Q}}^+, P_{\mathbb{Z}}) \times_{\beta} \{1, -1\}$ .

(pp. 569–578)

## SOME GENERALIZED SEQUENCE SPACES OF INVARIANT MEANS DEFINED BY IDEAL AND MODULUS FUNCTIONS IN $N$ -NORMED SPACES

Kuldip Raj, S.A. Mohiuddine, M. Ayman Mursaleen

In the present paper we introduce and study some generalized difference sequence spaces of invariant means defined by ideal and a sequence of modulus functions over  $n$ -normed space. We study some topological properties and prove some inclusion results between these spaces. Further, we also study some results on statistical convergence.

(pp. 579–595)

## PARAMETER ESTIMATION FOR A CLASS OF DIFFUSION PROCESS FROM DISCRETE OBSERVATION

Chao Wei

This paper is concerned with the parameter estimation problem for a class of diffusion process with drift coefficient  $\alpha X_t^{2\gamma-1}$  and diffusion coefficient  $\sigma X_t^\gamma$  from discrete observation. Euler-Maruyama scheme and iterative method are used to get the joint conditional probability density function. The maximum likelihood approach is applied for obtaining the parameter estimators and the explicit expressions of the error of estimation are given. The strong consistency of the estimators and asymptotic normality of the error of estimation are proved by using the law of large numbers for martingales, the strong law of large numbers and central-limit theorem. Hypothesis testing is made to verify the effectiveness of the estimation method used in this paper. (pp. 596–607)

## MORE PROPERTIES OF AN OPERATION ON SEMI-GENERALIZED OPEN SETS

Nazihah Ahmad, Baravan A. Asaad

The paper continues studying properties of an operation on  $\tau_{sg}$ . The notions of  $sg\gamma$ -generalized closed sets and some of its properties are investigated. It also introduces  $sg\text{-}\gamma\text{-}T_{\frac{1}{2}}$  space via  $sg\gamma$ -generalized closed set and  $sg\text{-}\gamma$ -closed set. Some basic characterization of  $sg\text{-}(\gamma, \beta)$ -irresolute functions with  $sg\text{-}\beta$ -closed graphs have been obtained. It studies the concept of  $sg\text{-}\gamma_0$ -closed space. Finally, it gives some properties of  $sg\text{-}\gamma^*$ -regular and  $sg\text{-}\gamma^*$ -normal spaces by using  $sg$ -open and  $sg$ -closed sets. (pp. 608–627)

## NORMAL EDGE-TRANSITIVE CAYLEY GRAPHS WHOSE ORDER ARE A PRODUCT OF THREE PRIMES

M. Ghorbani, M. Songhori, M. Rajabi Parsa

The Cayley graph  $X = Cay(G, S)$  on group  $G$  with respect to connection set  $S$  is normal edge-transitive, if  $N_{Aut(X)}(R(G))$  acts transitively on edge set. In this paper we determine the structure of automorphism group of all tetravalent normal edge-transitive Cayley graphs of order  $pqr$ . (pp. 628–635)

## A BAYESIAN METHOD TO FIT AN ARMA MODEL

Guochao Zhang, Qingming Gui, Changran Duan, Peng Zhao

The method of time series analysis is widely used in many fields of science, engineering, finance and economics etc, and fitting a time series model accurately is the important basis of time series analysis. Based on the Bayesian statistical theory, this paper presents a Bayesian method which can identify an ARMA (autoregressive moving-average) model and estimate the model parameters simultaneously. Firstly, in order to determine the orders of the ARMA model, an identification model with the recognition variables is constructed. Moreover, the problem of determining the orders of the ARMA model is transformed into two sets of hypothesis tests. By the principle of Bayesian hypothesis testing, it is suggested to solve the above hypothesis test problems by calculating the posterior probabilities of the hypotheses. However, due to the large number of the unknown parameters in the identification model, this paper proposes to obtain the samples by Gibbs sampling and then calculate the posterior probabilities of the hypotheses, the AR coefficients, the MA coefficients and the variance of the random errors to fit the ARMA model. Finally, in order to illustrate the good performance of the method proposed in this article, we design three simulation examples and compare the results of our method with two existing methods: RJMCMC method and EACF method. It can be found clearly that the method proposed in this paper has more accurate results for fitting an ARMA model.

(pp. 636–648)

## ROUGH SOFT $BCK$ -ALGEBRAS AND THEIR DECISION MAKING

Li Zhang, Xueling Ma, Jianming Zhan

Rough sets and soft sets are important tools to deal with uncertainties. In this paper, we apply rough soft set theory to  $BCK$ -algebras. The lower and upper rough soft  $BCK$ -algebras (ideals) are discussed. Finally, we establish a kind of decision making method for rough soft  $BCK$ -algebras. (pp. 649–659)

## ISO-ARRAYS AND CONDITIONAL COMMUNICATION ON $P$ SYSTEM

K. Bhuvaneswari, T. Kalyani

Among the theoretical applications of formal language theory, membrane computing is a new theoretical model. It is a cell-like structure in which regions are separated by membranes. It is introduced by G.H. Paun, called  $P$  system. This computational system can be used for generating string languages, two

dimensional picture languages, tiling patterns and tessellations. In this paper we introduced triangular tile pasting  $P$  system with conditional communication and explained with an example. Comparison result between triangular tile pasting system and triangular tile pasting  $P$  system has been given. The results on generating powers of iso-array grammars and triangular tile pasting  $P$  system are also examined in this paper. [\(pp. 660–671\)](#)

## A HIGH-ORDER ACCURACY EXPLICIT DIFFERENCE SCHEME WITH BRANCHING STABILITY FOR SOLVING FOUR-DIMENSIONAL PARABOLIC EQUATIONS

Yongqiang Zhan

A high-order explicit difference scheme for solving four-dimensional parabolic equations is given. The scheme is constructed by the method of undetermined coefficients, and appropriate parameter is chosen to endow the truncation error of schemes is  $O(\Delta t^4 + \Delta x^4)$ . And the new difference scheme is proved to be stable if  $r \leq \frac{1}{12}$  with the Fourier analysis method. Finally, the numerical experiment shows the numerical solutions of difference scheme and the exact solutions are matched and the difference scheme is effective. (pp. 672–682)

## HERMITE-HADAMARD TYPE INEQUALITIES FOR GENERALIZED $(s, m, \varphi)$ -PREINVEX GODUNOVA-LEVIN FUNCTIONS

Artion Kashuri, Rozana Liko

In the present paper, the notion of  $(m, \varphi)$ -invex set and generalized  $(s, m, \varphi)$ -preinvex Godunova-Levin function of second kind are introduced and some new integral inequalities involving generalized  $(s, m, \varphi)$ -preinvex Godunova-Levin function of second kind along with beta function are given. By using new identities for fractional integrals some new estimates on generalizations of Hermite-Hadamard type inequalities for generalized  $(s, m, \varphi)$ -preinvex Godunova-Levin functions of second kind via Riemann-Liouville fractional integral are established. At the end, some applications to special means are given.

(pp. 683–700)

## ALGEBRAIC HYPERSTRUCTURES AND SOCIAL RELATIONS

Sarka Hoskova-Mayerova, Antonio Maturo

The relations between the people of a certain set of individuals can be described, from a static point of view, through the arrays of Moreno. From a dynamic point of view, however, account must be taken of the coalitions formed between people. These coalitions can be specified or one or more alternatives can be selected while making decisions involving more decision-makers; further there are presented coordination strategies for more people in order to ensure the maximum utility in social problems modelling using cooperative games. This paper presents how algebraic hyperstructures can be a useful mathematical tool both for the study of social relations from a static point of view and for the

study of the social dynamics leading to the formation of coalitions. It shows that, surprisingly, many properties deemed significant only from an algebraic or geometric viewpoints in a set of individuals, deep meanings from the social point of view. (pp. 701–709)

## NOTE ON RELATIONS AMONG MULTIPLE ZETA(-STAR) VALUES

Masahiro Igarashi

In the present paper, we shall show that various relations among multiple zeta(-star) values and their multivariable extensions can be derived from the hypergeometric identities of G. E. Andrews, C. Krattenthaler and T. Rivoal. The results in the present paper give us various identities for multiple Hurwitz zeta values also. (pp. 710–756)

## LINEAR MAPS ON $M_n(\mathbb{C})$ PRESERVING INNER LOCAL SPECTRAL RADIUS ZERO

H. Benbouziane, M. Ech-Cherif El Kettani, A. M. Vadel

Let  $M_n(\mathbb{C})$  be the algebra of all complex  $n \times n$  matrices. Let  $x_0$  be a nonzero vector in  $\mathbb{C}^n$ . We show that a linear unital map  $\phi : M_n(\mathbb{C}) \rightarrow M_n(\mathbb{C})$  preserves the inner local spectral radius zero at  $x_0$ , if and only if there exists an invertible matrix  $A \in M_n(\mathbb{C})$  such that  $\phi(T) = ATA^{-1}$ . (pp. 757–763)

## AN EQUIVALENT DEFINITION OF A $\mathcal{C}$ -GROUP

Jianjun Liu, Guiyun Chen

In 1974, Fattahi [2] classified finite non-nilpotent groups in which every subgroup is either normal or abnormal. Surely, it is an interesting topic to define and classify a bigger class of groups containing those classified by Fattahi. In 2012, the first author etc defined a  $\mathcal{C}$ -group. A finite group  $G$  is said to be a  $\mathcal{C}$ -group if for each divisor  $d$  of the order of  $G$ ,  $G$  always contains a subgroup  $H$  of order  $d$  such that  $H$  is either normal or abnormal in  $G$ . The class of  $\mathcal{C}$ -groups is really a class of groups bigger than those classified by Fattahi. During the first author etc investigate the structure of of  $\mathcal{C}$ -groups, they found that some property cannot hold if 'normal' is changed into 'subnormal'. So the authors of this paper is motivated to find an equivalent definition of a  $\mathcal{C}$ -group, which can be described by 'subnormal' and 'abnormal'. A  $\mathcal{C}_1$ -group is defined , some good

properties of a  $\mathcal{C}_1$ -group are given, then it is proved that a  $\mathcal{C}_1$ -group is equivalent to a  $\mathcal{C}$ -group. At last, as an example of effectiveness of a  $\mathcal{C}_1$ -group, the necessary and sufficient conditions of a semi-product of a  $p$ -group and a  $p'$ -group to be a  $\mathcal{C}$ -group is given. (pp. 764–770)

#### STUDY OF EXTENDED WEYL $k$ -FRACTIONAL INTEGRAL VIA CHEBYSHEV INEQUALITIES

M.K. Azam, Fiza Zafar, M.A. Rehman, F. Ahmad, Shahid Qaisar

In this paper, we introduce the extended Weyl  $k$ -fractional integral and then present its results and some inequalities. These results and inequalities hold true for: (i)  $k$ -Weyl fractional integral when  $s \rightarrow 0$ ; (ii) Weyl fractional integral when  $s \rightarrow 0$  and  $k \rightarrow 1$ . (pp. 771–782)

#### MODIFICATION OF ACCURACY ESTIMATION USING STOCK MARKET DATA

S. Al Wadi

It is well known that the simplest way of estimation of statistical parameters is the method of least squares using linear functions. However, the problem with this method is in how to find out a linear observations. estimation accuracy is very important concept in many field such that; medicine, humanities, engineering, industry, economics and others since an unbiased vision is crucial in order to support the industry for decision making, also suitable data to obtain is, how estimates are completed, what factors encourage the choice of estimation methods and the current level of estimation accuracy. Therefore, this article purposes a novel technique in field of improving inference about population characteristic estimation, mathematical models were implemented in content of stock market data are collected from Amman stock exchange (ASE). Estimation accuracy directly will be implemented and Daubechies Wavelet transform (DWT) combined with interval estimation accuracy will be calculated also. As a result, the DWT combines with traditional estimation accuracy is better than traditional estimation accuracy directly. The results are implemented using (SPSS) and MATLAB. (pp. 783–792)



## HYPER QUASI-MV ALGEBRAS AND IDEALS

Wenjuan Chen, Bijan Davvaz

In this paper, we introduce hyper quasi-MV algebras as the generalizations of hyper MV-algebras and quasi-MV algebras. First we give the definition of hyper quasi-MV algebras and investigate some basic properties of hyper quasi-MV algebras. Second we introduce ideals and weak ideals in a hyper quasi-MV algebra. Especially, we study two types of (weak) ideals and discuss the relationship between them. We also present the dual notions of ideals and weak ideals in this paper. Finally, we show the properties of ideals and weak ideals under the homomorphism of hyper quasi-MV algebras. (pp. 793–809)

## ON AN EXTENSION OF THE DUBINS CONDITIONAL PROBABILITY AXIOMATIC TO COHERENT PROBABILITY OF FUZZY EVENTS

Fabrizio Maturo

An approach to the concept of fuzzy event as an extension of conditional event is introduced. The probability of fuzzy events is presented as an extension of the de Finetti's probability of conditional events and depends on a score function subjectively assigned by an expert. It is shown that the introduced fuzzy probability extends in a fuzzy ambit the conditions considered by Dubins for finitely additive conditional probability. Possible applications for decision-making under uncertainty are sketched.

(pp. 810–821)

## SANDWICH SETS AND CONGRUENCES IN COMPLETELY INVERSE $AG^{**}$ -GROUPOIDS

Waqar Khan, Kostaq Hila, Guiyun Chen

In this paper, we investigate sandwich set in a completely inverse  $AG^{**}$ -groupoid and make use of it for congruence pairs of a completely inverse  $AG^{**}$ -groupoid. We discuss completely inverse  $AG^{**}$ -groupoids in terms of partial order and show that the natural partial order is an equality relation in an  $AG$ -group. Further, we study idempotent-separating and idempotent-pure congruences on completely inverse  $AG^{**}$ -groupoids and show that the quotient structure for a maximum idempotent-separating congruence on an  $AG^{**}$ -groupoid is fundamental. Further, we characterize  $E$ -unitary completely inverse  $AG^{**}$ -groupoids and provide a condition for a compatibility relation to be transitive.

(pp. 822–838)

## SOME GENERALIZATION OF SUBPULLBACK FLAT

Pouyan Khamechi, Leila Nouri, Hossein Mohammadzadeh Saany

Bulman-Fleming et al. in [*Pullbacks and flatness properties of acts II*, Comm. Algebra, 29(2) (2001), 851-878] initiated the study of three flatness properties of right  $S$ -act  $A_S$  that can be described by means of when the functor  $A_S \otimes -$  preserves certain types of pullbacks. The present paper is extended these results to  $S$ -posets and is presented equivalences of them. Moreover, we show that flatness properties of  $S$ -posets that mentioned in this paper and in [A. Golchin, L. Nouri, *Subpullbacks and Po-flatness Properties of S-posets*, J. Sci. Islam. Repub. Iran, 25(4) (2014), 369-377], can be transferred to their coproducts, and vice versa. (pp. 839–852)