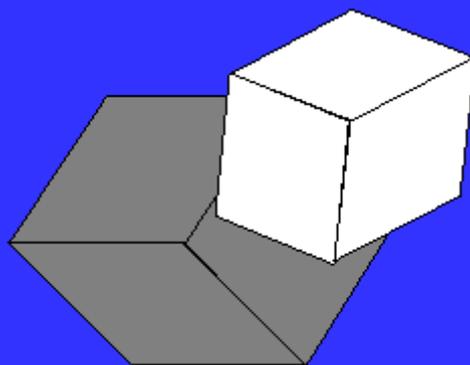


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## Papers Abstracts



**FORUM**

**AN UNCONDITIONALLY STABLE FINITE DIFFERENCE SCHEME FOR EQUATIONS OF CONSERVATION LAW FORM**

Saeid Abbasbandy, Ahmad Shirzadi

This study presents a numerical scheme for solving one dimensional equations of conservation law form. The Saul'yev's finite difference techniques are used to compute the solution. Although the resulting difference equation do not appear explicit, a suitable use of the equation make it explicit. It is shown that this explicit scheme is unconditionally stable. A numerical example is presented to demonstrate the accuracy and efficiency of the proposed computational procedure. (pp. 1-4)

**AN AUTOMATIC SCHEME ON THE HOMOTOPY ANALYSIS METHOD FOR SOLVING NONLINEAR ALGEBRAIC EQUATIONS**

Safwan Al-Shara', Fadi Awawdeh, S. Abbasbandy

In this paper, an automatic scheme coupled with homotopy analysis method is presented for solving nonlinear algebraic equations. The experimental results show the potential and limitations of the new method and imply directions for future work. (pp. 5-14)

**A NEW INTEGRAL TRANSFORM**

B.G. Sidharth

Using Bauer's expansion and properties of spherical Bessel and Legendre functions, we deduce a new transform and briefly indicate its use. (pp. 15-18)

**DISJOINT  $J$ -CLASS OPERATORS**

Abdelaziz Tajmouati, Mohammed El Berrag

In this paper, we first introduce the notion of disjoint extended limit set for a tuple of bounded linear operators on a separable Banach space  $X$ , and we extend some results from a single operator to a tuple of sequences of operators. (pp. 19-28)

**EQ-ALGEBRAS WITH PSEUDO PRE-VALUATIONS**

Yongwei Yang, Xiaolong Xin

The concepts of (positive implicative, implicative) pseudo pre-valuations and strong pseudo pre-valuations are introduced and some related characterizations are studied. The relationships among positive implicative pseudo pre-valuations, implicative pseudo pre-valuations and pseudo pre-valuations are investigated, and conditions for a real-valued function to be a pseudo pre-valuation are also discussed. By using a congruence relation induced via a pseudo valuation, we construct a quotient structure and prove certain isomorphism theorems. (pp. 29-48)

**A NOTE ON COMPLETENESS OF THE HAUSDORFF FUZZY METRIC SPACES**

Changqing Li, Yanlan Zhang, Kexiu Zhang

In this paper, completeness and completableness of the Hausdorff fuzzy metric spaces on the family of nonempty finite sets are explored. Also, necessary and sufficient conditions for the Hausdorff fuzzy metric spaces on the family of nonempty compact sets to be complete are found. (pp. 49-58)

## ON NEARLY *CAP*-EMBEDDED SUBGROUPS OF FINITE GROUPS

Yong Xu, Guiyun Chen

We introduce a new subgroup embedding property of a finite group called nearly *CAP*-embedded subgroup. Using this subgroup property, we determine the structure of finite groups with some nearly *CAP*-embedded subgroups of Sylow subgroups. Our results unify and generalize some recent theorems on  $p$ -nilpotency and supersolvability of finite groups. (pp. 59-68)

## SOURCE TERM IDENTIFICATION IN SEMIDIFFERENTIAL EQUATIONS

Mohammad F. Al-Jamal, E.A. Rawashdeh

In this paper we propose a numerical method for the source term identification in semidifferential equations from noisy data. Our method employ a mollification technique to stabilize (regularize) the inverse solution. We prove convergence results for both the continuous and discretized problems. Numerical examples are provided to validate the effectiveness of the proposed approach. (pp. 69-76)

## REGULAR MULTIPLICATIVE TERNARY HYPERRING

Md. Salim, T.K. Dutta, K.P. Shum

Regular multiplicative ternary hyperring are introduced and considered. Some properties of regular multiplicative ternary hyperring are studied. Several characterization theorems of the above ternary hyperrings in terms of its hyperideals are obtained. In addition, regular hyperideals in a multiplicative ternary hyperring are particularly considered and investigated. Finally, we explore the relationships between the regular multiplicative ternary hyperrings and the hyperideals of a multiplicative ternary hyperring. (pp. 77-88)

## NUMERICAL TREATMENT OF NEUTRAL FRACTIONAL VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

E.A. Rawashdeh

This paper deals with the numerical solution of fractional integro-differential equations with infinite delay. We applied the continuous spline collocation methods to approximate the solution. A new technique for evaluating the Caputo fractional derivative of the spline polynomials based on the Lagrange coefficients is obtained. Some numerical examples are provided to test the convergence of the method. (pp. 89-96)

## A BRIEF COMPARISON OF $G$ -CONTRACTION CONDITIONS AND A GENERALIZED FIXED POINT THEOREM

T. Phaneendra, S. Saravanan

Let  $(X, G)$  be a  $G$ -metric space and  $f$  denote a self-map on  $X$ . A brief comparison of some  $G$ -contraction conditions is made, and a new generalized fixed point theorem is obtained by employing a wider inequality. (pp. 97-104)

## EXISTENCE SOLUTION FOR WEIGHTED $p(x)$ -LAPLACIAN EQUATION

S.R. Mousaviankhatir, M. Alimohammady, H. Jafari

This paper deals with the existence solution for the following type of boundary value problems:

$$\begin{cases} \Delta \left( |x|^{p(x)} |\Delta u|^{p(x)-2} \Delta u \right) = \lambda |u|^{q(x)-2} u, & \text{in } \Omega, \\ u = \Delta u = 0, & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega$  is a smooth bounded domain in  $\mathbb{R}^N$ . It is established for a negative  $\lambda$ , there exists at least one weak solution. Our approach relies on the variable exponent theory of generalized Lebesgue-Sobolev spaces and a variant of the Mountain Pass theorem. (pp. 105-112)

## ON REVERSE WEIGHTED ARITHMETIC-GEOMETRIC MEAN INEQUALITIES FOR TWO POSITIVE OPERATORS

Jianming Xue

Let  $A, B$  be positive operators on a Hilbert space with  $0 < m \leq A, B \leq M$ . Then for every positive unital linear map  $\Phi$ ,

$$\begin{aligned}(A\nabla_{\mu}B)^2 &\leq \left[\frac{(M+m)^2}{4Mm}\right]^2 (A\sharp_{\mu}B)^2, & 0 \leq \mu \leq 1, \\ \Phi^2(A\nabla_{\mu}B) &\leq \left[\frac{(M+m)^2}{4Mm}\right]^2 \Phi^2(A\sharp_{\mu}B), & 0 \leq \mu \leq 1, \\ \Phi^2(A\nabla_{\mu}B) &\leq \left[\frac{(M+m)^2}{4Mm}\right]^2 [\Phi(A)\sharp_{\mu}\Phi(B)]^2, & 0 \leq \mu \leq 1.\end{aligned}$$

(pp. 113-116)

## SOME INCLUSION PROPERTIES OF STARLIKE AND CONVEX FUNCTIONS ASSOCIATED WITH HOHLOV OPERATOR. II

M. Kasthuri, K. Vijaya, K. Uma

A new subclass  $\mathbf{K}(\lambda, \alpha)$  involving Hohlov Operator is introduced and some inclusion relations and distortion bounds are obtained for  $f \in \mathbf{K}(\lambda, \alpha)$ . (pp. 117-126)

## BOUBAKER PIVOTAL ITERATION SCHEME (BPIS)

Amjed Zraiqat

In this paper, we present a numerical scheme for the solution of fourth-order boundary value problems with two-point boundary conditions. The Boubaker Pivotal Iteration Scheme (BPIS) is applied to construct the numerical solution. This approach provides the solution in the form of analytical function and not at grid points. Some examples are displayed to demonstrate the computational efficiency of the method. (pp. 127-138)

## THE STABILITY AND HOPF BIFURCATION OF THE DENGUE FEVER MODEL WITH TIME DELAY

Guan Jinlan, Wu Lizhi, Chen Minna, Dong Xiaoqian, Tang Huiping, Chen Zhiqi

This paper studied the dengue fever model with time delay. This paper divided the time delay into four cases: (1)  $\tau_1 = \tau_2 = 0$ , (2)  $\tau_1 = \tau$ ,  $\tau_2 = 0$ , (3)  $\tau_1 = 0$ ,  $\tau_2 = \tau$ , (4)  $\tau_1 = \tau$ ,  $\tau_2 = \tau$ , and studied the stability and Hopf bifurcation of the model on these three cases. At the end of this paper, we simulated the dengue model with time delay by using Matlab software, and gained the numerical condition of this model which appearing periodic solutions and Hopf bifurcation. On the first case  $\tau_1 = \tau$ ,  $\tau_2 = 0$ , the time delay threshold is  $\tau_0 = 0.6155$ ; on the second case  $\tau_1 = 0$ ,  $\tau_2 = \tau$ , the time delay threshold is  $\tau_0 = 0.0490$ ; on the third case  $\tau_1 = \tau$ ,  $\tau_2 = \tau$ , the time delay threshold is  $\tau_0 = 3.5454$ . (pp. 139-156)

## ENTROPY OF QUANTUM DYNAMICAL SYSTEMS WITH INFINITE PARTITIONS

Abolfazl Ebrahimzadeh, Zahra Eslami Giski

In this paper, the concepts of the entropy and relative entropy on quantum logic with countable partitions are defined and some ergodic properties of quantum dynamical systems are investigated. Finally, we show that the entropy is invariant under isomorphic relation. (pp. 157-164)

## FURTHER PROPERTIES OF THE GENERALIZATION OF PRIMAL SUPERIDEALS

Ameer Jaber, Moh'D Yasein

In [7], the author studied several generalizations of primal superideals of a commutative super-ring  $R$ . This paper is devoted to study further properties of  $\phi$ -primal superideals of  $R$ , where  $\phi : \mathfrak{J}(R) \rightarrow \mathfrak{J}(R) \cup \{\emptyset\}$  is any function and  $\mathfrak{J}(R)$  is the set of all proper superideals of  $R$ . In particular, if  $J \in \mathfrak{J}(R)$  then there is a one-to-one correspondence between  $\phi$ - $P^I$ -primal superideals  $I$  of  $R$  with  $J \subseteq \phi(I)$  and  $\phi$ - $J$ - $P^I$ -primal superideals of  $R/J$ . Moreover, for a multiplicatively closed subset  $S$  of  $h(R)$ , if  $I - \phi(I) = \rho^{-1}(I_S) - \rho^{-1}(\phi_S(I_S))$  for any  $I \in \mathfrak{J}(R)$ , then there is a one-to-one correspondence between  $\phi$ - $P^I$ -primal superideals  $I$  of  $R$  and  $\phi_S$ - $P_S^I$ -primal superideals  $I_S$  of  $R_S$  with  $P^I \cap S = \emptyset$ .

(pp. 165-172)

## COUNT AND CRYPTOGRAPHIC PROPERTIES OF GENERALIZED SYMMETRIC BOOLEAN FUNCTIONS

Shashi Kant Pandey, P.R. Mishra, B.K. Dass

Boolean functions with symmetry have been the object of interest to the researchers. With their concise representation and ease of computation, they offer themselves as cut above the rest candidates for the filtering and exploration of optimal Boolean functions. Generalized Boolean functions have been explored as a number of trade-offs in usual Boolean hinder, the process of selecting good Boolean functions required for a specific application. Therefore, it would be interesting to investigate symmetry in Boolean functions in a generalized scenario. We look into three different symmetries in generalized Boolean function according to different parameter of symmetry and present enumeration formulae. We also present an exhaustive construction of bent and balanced symmetric generalized functions (in form of ANF) on smaller domains.

(pp. 173-182)

## RESULTS ON PRIME IDEALS IN PMV-ALGEBRAS AND MV-MODULES

S. Saidi Goraghani, R. A. Borzooei

In this paper, by considering the notions of  $MV$ -modules and  $PMV$ -algebras, we study  $\cdot$ -prime ideals in  $PMV$ -algebras, prime  $A$ -ideals in  $MV$ -modules and investigate some properties on them. Also, we present the definitions of radical of a  $\cdot$ -ideal in  $PMV$ -algebras, radical of an  $A$ -ideal in  $MV$ -modules and verify some properties on them. Finally, we state a method to obtain the radical of a  $\cdot$ -ideal in  $PMV$ -algebras.

(pp. 183-196)

## EOQ MODELS FOR NONINSTANTANEOUS/INSTANTANEOUS DETERIORATING ITEMS WITH CUBIC DEMAND RATE UNDER INFLATION AND PERMISSIBLE DELAY IN PAYMENTS

K. Rangarajan, K. Karthikeyan

In this article, an attempt is made to develop two inventory models for non- instantaneous deteriorating items and two inventory models for instantaneous deteriorating items with linear deterioration rate and cubic demand rate. That is, the demand rate is a piecewise cubic function of time under inflation and permissible delay in payments. This model supports in minimizing the total inventory cost by finding an optimal replenishment policy; where shortages are allowed, partially backlogged and completely backlogged cases are considered. The backlogging rate is variable and dependent on the waiting time for the next replenishment. Numerical examples are given to establish the analytical results. Sensitivity analysis of the optimal solution with respect to major parameters is carried out and the effects are discussed in detail.

(pp. 197-218)

## ON THE LINEAR QUADRATIC DYNAMIC OPTIMIZATION PROBLEMS WITH FIXED-LEVELS CONTROL FUNCTIONS

Vadim Azhmyakov, Luz Adriana Guzman Trujillo

This paper deals with a constrained LQ-type optimal control problem (OCP) in the presence of fixed levels input restrictions. We consider control processes governed by linear differential equations with a priori known control switching structure. The set of admissible inputs reflects some important natural engineering applications and moreover, can also be interpreted as a result of a quantization procedure applied to the original dynamic system. We propose a novel implementable algorithm that makes it possible to calculate a (numerically consistent) approximative solution to the constrained LQ-type OCPs under consideration. Our contribution mainly discusses theoretic aspects of the proposed solution scheme and contains an illustrative numerical example. (pp. 219-236)

## FALLING FUZZY GÖDEL IDEALS OF $BL$ -ALGEBRAS

Biao Long Meng, Xiao Long Xin

The notions of falling fuzzy (resp. Gödel, Boolean, implicative) ideals of a  $BL$ -algebra are introduced based on the theory of falling shadows and fuzzy sets. Several characterizations and relations of these notions are studied. Finally we apply the concept of falling fuzzy inference relations to ideal theory of  $BL$ -algebras and obtain some related results. (pp. 237-258)

## FUZZY $p$ -IDEALS IN $MV$ -ALGEBRAS

F. Forouzesh

In this paper, we introduce the notion of  $p$ -ideals of  $MV$ -algebras and characterization of  $p$ -ideals is given. Also, we show that  $p$ -ideals equivalent to Boolean ideals in  $MV$ -algebras. In addition, we introduce the notion of fuzzy  $p$ -ideals of an  $MV$ -algebra and show that in any  $MV$ -algebra, the concept of fuzzy  $p$ -ideals is equivalent to fuzzy Boolean ideals and fuzzy implicative ideals. Also, several characterizations of these fuzzy ideals are given and prove that extension theorem of fuzzy  $p$ -ideals. Furthermore, we describe the transfer principle for fuzzy  $p$ -ideals in terms of level subsets. Finally, by using the notions of maximal and normal fuzzy  $p$ -ideals, we show that under certain conditions a fuzzy  $p$ -ideal is two valued and takes the values 0 and 1. (pp. 259-273)

## MODULES WHOSE PRIMARY-LIKE SPECTRA WITH THE ZARISKI-LIKE TOPOLOGY ARE NOETHERIAN SPACES

Hosein Fazaeli Moghimi, Fatemeh Rashedi

Let  $R$  be a commutative ring with identity and  $M$  be a unital  $R$ -module. The primary-like spectrum  $Spec_L(M)$  is the collection of all primary-like submodules  $Q$  of  $M$  such that  $M/Q$  is a primeful  $R$ -module. The Zariski-like topology on  $Spec_L(M)$ , denoted  $\mathcal{T}$ , is described by taking the set  $\eta = \{\nu(N) \mid N \text{ is a submodule of } M\}$  as the set of closed sets of  $Spec_L(M)$ , where  $\nu(N) = \{Q \in Spec_L(M) \mid \sqrt{(N : M)} \subseteq \sqrt{(Q : M)}\}$ . We establish necessary and sufficient conditions for topological space  $(Spec_L(M), \mathcal{T})$  to be a Noetherian space. We show that if  $M$  is a finitely generated  $R$ -module and  $|Spec_L(M)| < \infty$ , then the combinatorial dimension of  $(Spec_L(M), \mathcal{T})$  and the Krull dimension of  $R/Ann(M)$  are equal. In particular, for the Noetherian space  $(Spec_L(M), \mathcal{T})$  of zero combinatorial dimension the set of irreducible components is finite, and its elements have the form  $\nu(pM)$  for some minimal prime ideal  $p \supseteq Ann(M)$ . (pp. 274-288)

## ON THE EXISTENCE OF CATEGORICAL UNIVERSAL COVERINGS

Ali Pakdaman, Hamid Torabi, Behrooz Mashayakhy

In this paper, we study necessary and sufficient conditions for the existence of categorical universal coverings using open covers of a given space  $X$ . As some applications, first we show that a first countable Peano space has a categorical universal covering or has an uncountable fundamental group. Second, we prove that the one point union  $X_1 \vee X_2 = \frac{X_1 \cup X_2}{x_1 \sim x_2}$  has a categorical universal covering if and only if both  $X_1$  and  $X_2$  have categorical universal coverings. (pp. 289-300)

## PSEUDO SEMI $b$ -FREDHOLM AND GENERALIZED DRAZIN INVERTIBLE OPERATORS THROUGH LOCALIZED SVEP

Abdelaziz Tajmouati, Mohammed Karmouni, Mbark Abkari

In this paper, we define and study the pseudo upper and lower semi B-Fredholm of bounded operators in a Banach space. In particular, we prove equality up to  $S(T)$  between the left generalized Drazin spectrum and the pseudo upper semi B-Fredholm spectrum,  $S(T)$  is the set where  $T$  fails to have the SVEP. Also, we prove that both of the left and the right generalized Drazin operators are invariant under additive commuting power finite rank perturbations and some perturbations for the pseudo upper and lower semi B-Fredholm operators are given. As applications, we investigate some classes of operators as the supercyclic and multiplier operators. (pp. 301-314)

## INJECTIVITY OF $G$ -NOMINAL SETS

Khadijeh Keshvardoost

In this paper, we consider injectivity for nominal sets,  $G$ -sets for a subgroup  $G$  of the group  $\text{Perm}(\mathbb{D})$  of permutations on an infinite countable set  $\mathbb{D}$ , with finitely supported elements. Also, we study injectivity of nominal sets with respect to monomorphisms whose domains or codomains are single-orbit. Furthermore, we define the category of  $G_C$ -sets, for finite subsets  $C$  of  $\mathbb{D}$ , where  $G_C$  is the set of elements of  $G$  fixing the elements of  $C$ , and study injectivity in such categories. (pp. 315-328)

## THE STRUCTURE OF FINITE GROUPS WITH $c^*$ -NORMAL SUBGROUPS

Ping Kang

Let  $H$  be a subgroup of a finite group  $G$ .  $H$  is said to be  $c^*$ -normal in  $G$  if there exists a normal subgroup  $K$  of  $G$  such that  $G = HK$  and  $H \cap K$  is  $s$ -quasinormally embedded in  $G$ . We fix in every non-cyclic Sylow subgroup  $P$  of  $G$  some subgroup  $D$  satisfying  $1 < |D| < |P|$  and study the structure of  $G$  under the assumption that every subgroup  $H$  of  $P$  with  $|H| = |D|$  is  $c^*$ -normal in  $G$ . Some recent results are generalized and unified. (pp. 329-338)

## SUBSPACE MIXING AND UNIVERSALITY CRITERION FOR A SEQUENCE OF OPERATORS

A. Tajmouati, M. Amouch, M.R.F. Alhomidi Zakariya, M. Abkari

Let  $B(X)$  denote the algebra of all bounded linear operators on an infinite-dimensional separable complex Banach space  $X$  and  $M$  be a nonzero subspace of  $X$ . We will characterize properties of being  $d-M$  mixing for a  $N \geq 2$  sequence  $T_{1,j}, T_{2,j}, \dots, T_{N,j}$  of operators in  $B(X)$ . Also, we will give necessary and sufficient conditions for a  $N \geq 2$  sequence  $T_{1,j}, T_{2,j}, \dots, T_{N,j}$  of operators in  $B(X)$  to satisfy  $d-M$  universality criterion in terms of  $d-M$  topologically transitivity of this sequence. (pp. 339-346)

## NOTES ON YOUNG AND ARITHMETIC-GEOMETRIC MEAN INEQUALITIES FOR MATRICES

Yanqiu Wu

In this short note, we prove that the conjecture of singular value Young inequality holds when  $j = n$ . Meanwhile, we also present a refinement of the arithmetic-geometric mean inequality for unitarily invariant norms. (pp. 347-350)

## FINITE GROUPS HAVING EXACTLY 42 ELEMENTS OF MAXIMAL ORDER

Zhangjia Han, Longhui Zhang

Let  $G$  be a finite group,  $M(G)$  denotes the number of elements of maximal order of  $G$ . In this note a finite group  $G$  with  $M(G) = 42$  is determined. (pp. 351-354)

## ON REVERSE AM-GM INEQUALITIES FOR $n$ OPERATORS

Xingkai Hu, Ying Sun

In this paper, we generalize some operator inequalities due to Fu and He [Linear Multilinear Algebra, 63 (2015), 571-577] as follows: Let  $A_i$  ( $i = 1, \dots, n$ ) be positive operators on a Hilbert space with  $0 < m \leq A_i \leq M$ . Then for every positive unital linear map  $\Phi$ ,

$$\Phi^{2p} \left( \frac{A_1 + \dots + A_n}{n} \right) \leq \left[ \frac{(M+m)^{2p}}{4M^p m^p} \right]^2 \Phi^{2p}[G(A_1, \dots, A_n)], \quad 1 \leq p < \infty,$$

and

$$\Phi^{2p} \left( \frac{A_1 + \dots + A_n}{n} \right) \leq \left[ \frac{(M+m)^{2p}}{4M^p m^p} \right]^2 G^{2p}[\Phi(A_1), \dots, \Phi(A_n)], \quad 1 \leq p < \infty,$$

where  $G(A_1, \dots, A_n)$  is Ando-Li-Mathias geometric mean.

(pp. 355-360)

## CHARACTERISTIC SEMIMODULES

Barbora Batíková, Tomáš Kepka, Petr Nĕmec

In the paper, a particular class of semimodules typical for additively idempotent semirings possessing at least two right multiplicatively absorbing elements is investigated.

(pp. 361-376)

## THE IMPACT OF NONLINEAR INCIDENCE RATE AND REMOVABLE STORAGE MEDIA ON VIRAL PREVALENCE

Shaoheng Wang, Yuming Feng

In this paper, a new computer virus propagation model is proposed by introducing a generalized nonlinear incidence rate into the generic *SLBRS* model. Theoretical analysis and numerical simulations show that, under some moderate conditions, the proposed model admits a globally asymptotically stable viral equilibrium. The impact of nonlinear incidence rate and removable storage media on viral prevalence is also illustrated.

(pp. 377-386)

## ON EXTENSIONS OF $k$ -SUBADDITIVE LATTICE GROUP-VALUED CAPACITIES

G. Barbieri, A. Boccutto

We prove some theorems on extension of lattice group-valued  $k$ -subadditive monotone set functions, continuous and  $(s)$ -bounded with respect to a single regulator. Furthermore, we pose some open problems.

(pp. 387-408)

## ON $\Gamma$ -BIACTS AND THEIR GREEN'S RELATIONS

A.R. Shabani, H. Rasouli

A well-known generalization of a semigroup  $S$  is called the  $\Gamma$ -semigroup. We generalize the notion of biacts over semigroups to  $\Gamma$ -biacts over  $\Gamma$ -semigroups. Green's relations on semigroups and biacts play an important role in these theories. In this paper, we study Green's relations on  $\Gamma$ -biacts.

(pp. 409-418)

## THE ANALYSIS OF A VARIABLE-VISCOSITY FLUID FLOW BETWEEN PARALLEL POROUS PLATES WITH NON-UNIFORM WALL TEMPERATURE

A.R. Hassan, R. Maritz

This paper examined the effectiveness of porosity on a variable-viscosity channel flow with non-uniform wall temperature. The flow is considered to be a steady, incompressible and the fluid viscosity varies linearly with temperature. The analytic expressions are obtained seeking asymptotic solutions for fluid velocity and temperature and these expressions are used to derive and obtain solutions for entropy generation rate and Bejan numbers with variations in other physical parameters present in the fluid flow.

(pp. 419-430)

## EXISTENCE AND UNIQUENESS SOLUTION OF A BOUNDARY VALUE PROBLEMS FOR INTEGRO-DIFFERENTIAL EQUATION WITH PARAMETER

R.N. Butris, A.Sh. Rafeeq

In this paper, we investigate the existence and uniqueness of the solution to a boundary value problem for integro-differential equation with parameter by using Schauder's fixed point theorem. (pp. 431-440)

## ON HOMOLOGICAL PROPERTIES OF SOME $H_v$ -STRUCTURES

Hülya Inceboz, Bijan Davvaz

The main aim of this note is to investigate the fundamental homological properties of various module derivations for  $H_v$ -structures and get some functorial relations for these derivation sets as a continuous line of [6]. (pp. 441-454)

## AN ANALYTICAL APPROXIMATION TECHNIQUE FOR THE DUFFING OSCILLATOR BASED ON THE ENERGY BALANCE METHOD

Md. Alal Hosen, M.S.H. Chowdhury, Mohammad Yeakub Ali, Ahmad Faris Ismail

In this paper, an analytical approximation technique has been presented of obtaining higher-order approximate solutions for highly nonlinear Duffing oscillator based on the energy balance method (EBM). Higher-order approximate natural frequencies have been obtained in a novel analytical way. The accuracy of the solution method is evaluated within an error analysis. It is highly remarkable that using the presented technique, the approximation solutions produce desired results even for large oscillation as compared with the exact ones. Moreover, the solution method yields much better results than existing solutions after using a suitable truncation formula. The presented technique is applied to well-known Duffing oscillator to illustrate its novelty, reliability and wider applicability. (pp.455-466)

## OD-CHARACTERIZABILITY OF THE SYMMETRIC GROUP $S_{27}$

G.R. Rezaeezadeh, M. R. Darafsheh

Let  $G$  be a finite group with degree pattern  $D(G)$ . A finite group  $G$  is called  $k$ -fold OD-characterizable if there are exactly  $k$  non-isomorphic groups  $H$  such that  $|G| = |H|$  and  $D(G) = D(H)$ . In this paper our purpose is to correct an earlier paper and prove that the symmetric group on 27 letters is 38-OD-characterizable. (pp. 467-472)

## A NOTE ON EXTRINSIC FRAME HOMOGENEITY OF HYPERQUADRICS

Parviz Ahmadi, Masoud Hasani

Let  $p_i$  and  $q_i$  belong to a hyperquadric  $Q$  and  $(e_{1_i}, \dots, e_{n_i})$  and  $(f_{1_i}, \dots, f_{n_i})$  be orthonormal frames in  $T_{p_i}Q$  and  $T_{q_i}Q$ , respectively, where  $1 \leq i \leq m$ . We study sufficient and necessary conditions for existence of an isometry  $\varphi : \mathbb{R}_\nu^{n+1} \rightarrow \mathbb{R}_\nu^{n+1}$  such that  $\varphi(Q) \subset Q$ ,  $\varphi(p_i) = q_i$  and  $d\varphi(e_{j_i}) = f_{j_i}$ . (pp. 473-476)

## SOME GEOMETRIC AGGREGATION OPERATORS BASED ON PICTURE FUZZY SETS AND THEIR APPLICATION IN MULTIPLE ATTRIBUTE DECISION MAKING

Chunyang Wang, Xiaoqiang Zhou, Huonian Tu, Shengda Tao

In this paper, we investigate the multiple attribute decision making (MADM) problems with picture fuzzy information. Firstly, concepts and some operational laws of picture fuzzy sets are introduced. Then, we develop some picture fuzzy geometric operators and discuss their basic properties. Next, we apply the proposed operators to deal with multiple attribute decision making problems under picture fuzzy environment. Finally, an illustrative example is given to demonstrate the practicality and effectiveness of our proposed method. (pp. 477-492)

## ON CHARACTERIZATIONS OF BL-ALGEBRAS VIA IMPLICATIVE IDEALS

Yongwei Yang, Xiaolong Xin

In the paper, we introduce the concept of implicative ideals in BL-algebras by the pseudo implication operation and show some characterizations of ideals. We prove that implicative ideals coincide with Boolean ideals through analyzing the characterizations of implicative ideals. Finally, we consider the concepts of maximal ideals and investigate the relationships among the introduced ideals. (pp. 493-506)

## ON SOME PROPERTIES OF $\phi$ -MULTIPLIERS

A. Tajmouati, A. El Bakkali, M.B. Mohamed Ahmed

In this paper, we investigate some new properties of  $\phi$ -multipliers studied recently by M.Adib and A.Riazi on faithful Banach algebras. Specially we justifies the existence of Helgason-Wang function for a  $\phi$ -multipliers and give some characterizations. As corollary we obtain some results for classical multipliers. (pp. 507-516)

## NEW RESULTS ON FIXED POINTS FOR AN INFINITE SEQUENCE OF MAPPINGS IN $G$ -METRIC SPACE

Sh. Al-Sharif, A. Alahmari, M. Al-Khaleel, A. Salem

In this paper, we prove new results on coincidence and common fixed points for a sequence of mappings satisfying generalized  $(\Psi - \Phi)$  contractive conditions in  $G$ -metric space. Also we investigate the existence of common fixed point for a sequence of mappings satisfying the almost generalized cyclic weak contractive condition in  $G$ -metric space. An example supporting our results is included. (pp. 517-540)

## APPROXIMATION OF CONTINUOUS FUNCTIONS BY VALLEE-POUSSIN'S SUMS

Rateb Al-Btoush, Al-Oushoush Nizar Kh. Kh.

Let  $V_{n,m}^{(\alpha,\beta)}(f;x) = \frac{1}{m+1} \sum_{k=n}^{n+m} S_k^{(\alpha,\beta)}(f;x)$  be the Vallee-Poussin's partial sums of Fourier-Jacobi series. In this paper, we study the deviations of  $V_{n,m}^{(\alpha,\beta)}(f;x)$  on  $[-1, 1]$  for continuous function  $f(x)$ . (pp. 541-552)

## MATCHING EXTENSION IN COMPLEMENTARY PRISM OF REGULAR GRAPHS

Pongthep Janseana, Nawarat Ananchuen

Let  $\overline{G}$  denote the complement of a simple graph  $G$ . The complementary prism of  $G$ , denoted by  $G\overline{G}$ , is obtained by taking a copy of  $G$  and a copy of  $\overline{G}$  and then adding a perfect matching that joins corresponding vertices. A connected graph  $G$  of order at least  $2k + 2$  is  $k$ -extendable if for every matching  $M$  of size  $k$  in  $G$ , there is a perfect matching in  $G$  containing all edges of  $M$ . In this paper, we establish some sufficient conditions for the complementary prism of regular graphs to be 2-extendable. (pp. 553-564)

## STATISTICAL INFERENCE IN PRINCIPAL COMPONENT ANALYSIS BASED ON STATISTICAL THEORY

Yan Liu

Principal component analysis is a diversified statistical method, while statistical inference is the major research subject in modern statistics, whose theories and methods have comprised the core content of mathematical statistics. According to the relevant knowledge of statistical theory and based on a large sample size, this study explored the statistical inference problem when population followed normal distribution. Besides, statistical methods were applied to further analyze the statistical inference problems in principle component analysis under the condition of population with non-normal

distribution or small sample size. First, principal component analysis was performed on parameter estimation and hypothesis testing on the condition that population followed multivariate normal distribution. Then under the condition of complex distribution of population, simulated sampling statistical inference method, i.e., Bootstrap method, was used to do interval estimation and discuss over other statistical inference problems of the characteristic values of the correlation coefficient matrixes in principle component analysis, and then the defects of Bootstrap method were adjusted using Bayesian theory. (pp. 565-574)

### **SOME RESULTS ON SLOWLY CHANGING FUNCTION ORIENTED RELATIVE ORDER, RELATIVE TYPE AND RELATIVE WEAK TYPE OF DIFFERENTIAL MONOMIALS**

Sanjib Kumar Datta, Tanmay Biswas, Debasmita Dutta

In this paper, we establish the relationship between the relative  $L$ -order (relative  $L^*$ -order), relative  $L$ -type (relative  $L^*$ -type) and relative  $L$ -weak type (relative  $L^*$ -weak type) of a transcendental meromorphic function  $f$  with respect to a transcendental entire function  $g$  and that of monomial generated by the meromorphic  $f$  and entire  $g$ . (pp. 575-582)

### **A NOVEL STUDY OF SOFT SETS OVER N-ARY SEMIGROUPS**

Qiumei Wang, Xiaowu Zhou, Jianming Zhan

In this paper, we show that the regular  $n$ -ary semigroups can be described by using idealistic soft  $n$ -ary semigroups. The relationships between regular  $n$ -ary semigroups and soft regular  $n$ -ary semigroups are also discussed. Finally, we introduce quotient  $n$ -ary semigroups via soft congruence relations and establish some homomorphisms and related properties with respect to soft congruence relations. (pp. 583-594)

### **REPRESENTATIONS OF POLYGROUPS**

R. Ameri, R.A. Borzooei, K. Ghadimi

The purpose of this paper is the study of representation of polygroups based on hyperspaces (hyper-vector spaces). In this regards we introduce and study representation and weak representation of a given polygroup. In particular, we study irreducible and weak irreducible representation of polygroups and obtain some basic properties of them. (pp. 595-610)

### **ON $H_3(p)$ HANKEL DETERMINANT FOR CERTAIN SUBCLASS OF $p$ -VALENT FUNCTIONS**

A.A. Amourah, Feras Yousef, Tariq Al-Hawary, M. Darus

The aim of this paper is to obtain an upper bound to the  $H_3(p)$  Hankel determinant for certain subclass of  $p$ -valent functions. To do so, we obtain best possible bounds for the functionals  $|a_{p+3} - a_{p+1}a_{p+2}|$  and  $|a_{p+2} - a_{p+1}^2|$ , then using known upper bound for the functional  $|a_{p+1}a_{p+3} - a_{p+2}^2|$  we obtain the required sharp upper bound to the  $H_3(p)$  Hankel determinant. (pp. 611-618)

### **FILTER THEORY OF PSEUDO HOOP-ALGEBRAS**

S.Z. Alavi, R.A. Borzooei, M. Aaly Kologani

In this paper, by considering the notion of pseudo hoop-algebras, which introduced by G. Georgescu et al. in [11] under the name of residuated integral monoids, and pseudo MV-algebras, pseudo Wajsberg-algebras and pseudo-BL algebras arise as particular cases of them, we introduce the notions of some types of filters ((positive) implicative filters, fantastic filters, associative filters) in pseudo hoop-algebras and to investigate their properties. Several characterizations of (positive) implicative, fantastic and associative filters are derived. Finally, the relations among these filters are investigated. (pp. 619-632)

## WEAK CLOSURE OPERATIONS WITH SPECIAL TYPES IN LOWER BCK-SEMILATTICES

Hashem Bordbar, Mohammad Mehdi Zahedi, Young Bae Jun

The notions of (strong) quasi prime mapping on the set of all ideals,  $t$ -type weak closure operation, and tender (resp., naive, sheer, feeble tender) weak closure operation are introduced, and their relations and properties are investigated. Conditions for a weak closure operation to be of  $t$ -type are provided. Given a weak closure operation, conditions for the new weak closure operation to be of  $t$ -type and to be a naive (sheer, feeble tender) weak closure operation are considered. We show that the new weak closure operation is the smallest tender weak closure operation containing the given weak closure operation. (pp. 633-644)

## ON A GROUP OF THE FORM $2^{10} : (U_5(2) : 2)$

Ayoub B. M. Basheer, Jamshid Moori

The full automorphism group  $U_5(2):2$  of the special unitary group  $U_5(2)$  has a 10-dimensional absolutely irreducible module over  $GF(2)$ . Hence a split extension of the form  $\overline{G} = 2^{10} : (U_5(2) : 2)$  does exist. In this paper we first determine the conjugacy classes of  $\overline{G}$  using the coset analysis technique. The structures of the inertia factor groups were determined. These are the groups  $U_5(2):2$ ,  $2_1^{1+6} : ((3^{1+2}:8):2)$  and  $O_5(2):2$ . We then determine the Fischer matrices and apply the Clifford-Fischer theory to compute the ordinary character table of  $\overline{G}$ . The Fischer matrices  $\mathcal{F}_i$  of  $\overline{G}$  are all  $\mathbb{Z}$ -valued, with sizes range between 1 and 5. The full character table of  $\overline{G}$ , which is  $109 \times 109$   $\mathbb{C}$ -valued matrix is available in the PhD Thesis [1] of the first author, which could be accessed online. (pp. 645-658)

## ORTHOGONAL-BASED HYBRID BLOCK METHOD FOR SOLVING GENERAL SECOND ORDER INITIAL VALUE PROBLEMS

E.O. Adeyefa

The direct integration of general second order initial value problems is considered in this paper. We employ a new class of orthogonal polynomials constructed as basis function to develop One Step Hybrid Block Method (OSHBM) adopting collocation technique. We present the recursive formula of the class of polynomials constructed and give analysis of the basic properties of OSBHM as findings show that the method is accurate and convergent. (pp. 659-672)

## A NOVEL VIEW OF ROUGH SOFT SEMIGROUPS BASED ON FUZZY IDEALS

Qiumei Wang, Jianming Zhan

By using a special  $t$ -level relation  $U(\mu, t) = \{(x, y) \in S \times S | (\mu(x) \wedge \mu(y)) \vee Id_S(x, y) \geq t\}$  based on a fuzzy ideal  $\mu$  of a semigroup  $S$ , which is a congruence relation, we study the roughness of soft semigroups under this special ideal of  $S$ , such as rough soft subsemigroups, rough soft ideals and rough soft prime ideals. (pp. 673-686)

## SOLVING A CLASS OF BOUNDARY VALUE PROBLEMS IN STRUCTURAL ENGINEERING AND FLUID MECHANICS USING HOMOTOPY PERTURBATION AND ADOMIAN DECOMPOSITION METHODS

H. Hosseinzadeh, H. Jafari, M.R. Gholami, D.D. Ganji

In this article, the performance of two analytical methods known as the homotopy perturbation method (HPM) and Adomian decomposition method (ADM) on solving linear and nonlinear boundary value problems structural engineering and fluid mechanics are compared. In order to compare these mathematical models, various problems in inelastic and viscoelastic flows, deformation of beams, and plate deflection theory are chosen. In addition, the results of these two methods are compared with exact solutions to evaluate the precision and accuracy of these numerical methods. (pp. 687-698)

## APPROXIMATE SOLUTIONS TO THE GENERALIZED TIME-FRACTIONAL ITO SYSTEM

H.M. Jaradat, Imad Jaradat, Marwan Alquran, M.M.M. Jaradat, Zead Mustafa, Khedr M. Abohassan, Ra'ed Abdelkarim

In this work, we consider the time-fractional version of the well-known integrable Ito system to study the effect of its fractional index "memory index". A modified approach of a relatively new method called residual power series (RPS) is applied to construct an analytical solution for the fractional system. For purpose of comparison, we derive one of the classical Ito system solitary wave solution using Tanh method. (pp. 699-710)