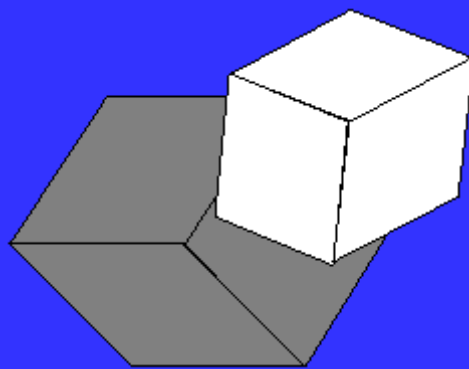


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## Papers Abstracts



**FORUM**

**A NOTE ON DIMENSION OF WEAK HYPERVECTOR SPACES**

A. Taghavi, R. Hosseinzadeh

In this paper, we study the dimension of weak hypervector spaces. First, we define linearly dependence and independence of vectors and also basis of weak hypervector spaces and then prove some results in this field. Finally, we consider weak subhypervector spaces of such spaces. (pp. 7-14)

**GENERALIZED EXPONENTIAL OPERATORS  
AND DIFFERENCE EQUATIONS**

Mohammad Asif, Anju Gupta

The present paper deals with the generalization of exponential operators used by Dattoli and Levi for translation and diffusive operator which were utilized to establish analytical solutions of difference and integral equations. The generalization of their technique is expected to cover wide range of such utilization. (pp. 15-34)

**ANALYSIS OF BLOOD FLOW THROUGH AN ARTERY  
WITH MILD STENOSIS: A TWO-LAYERED MODEL**

Bijendra Singh, Padma Joshi, B.K. Joshi

In this paper, a model of blood flow through a constricted arterial segment has been considered. We have proposed a trapezium shaped geometry of mild axisymmetric stenosis. The flow of blood with artery has been represented by a two-layered model consisting of a core layer and a peripheral layer. It has been observed that the resistance to flow and wall shear stress increase as the peripheral layer viscosity increases. The results are compared graphically with those of previous investigators. (pp. 35-44)

**THE EXPLICIT EXPRESSION OF THE DRAZIN INVERSE  
OF SUMS OF TWO MATRICES AND ITS APPLICATION**

Xiaoji Liu, Liang Xu, Yaoming Yu

In this paper, we give explicit expressions of  $(P \pm Q)_d$  of two matrices  $P$  and  $Q$ , in terms of  $P, Q, P_d$  and  $Q_d$ , under the condition that  $PQ = P^2$ , and apply the result to finding an explicit representation for the Drazin inverse of a  $2 \times 2$  block matrix  $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$  under some conditions. (pp. 45-62)

## **SUPER PRINCIPAL FIBER BUNDLE WITH SUPER ACTION**

**M.R. Farhangdoost**

We introduce super action for supermanifolds to devoted to principal fiber bundle with structural generalized Lie groups. We present a product super fiber bundle, also we extend the coordinate bundle in the sense of Steenrod to show that super coordinate bundles are equivalent if and only if their super actions agree. (pp. 63-70)

## **INTUITIONISTIC FUZZY $\alpha$ -IRRESOLUTE FUNCTIONS**

**V. Seenivasan, R. Renuka**

In this paper the concept of intuitionistic fuzzy  $\alpha$ -irresolute functions are introduced and studied. Besides giving characterizations of these functions, several interesting properties of these functions are also given. We also study relationship between this function with other existing functions. (pp. 71-80)

## **INVERTIBLE ELEMENTS IN BCK-ALGEBRAS**

**Olivier A. Heubo-Kwegna, Jean B. Nganou**

In this article, we introduce the notion of cyclic BCK-algebra and study some of its main properties. In addition, we obtain a structure theorem for bounded commutative of finite order and use it to prove a Lagrange-like theorem for the above class of algebras. Finally, we use the notion of invertible elements to obtain a new characterization of implicative BCK-algebras and study the intersection of all maximal ideals of bounded BCK-algebras. (pp. 81-92)

## **A NEW CHARACTERIZATION OF SIMPLETIC GROUP $S_8(2)$**

**Yanxiong Yan, Naiqing Song, Yuming Feng**

It is a well-known fact that characters of a finite group can give important information of the group's structure. Also it was proved by chen [1] that a finite simple group can be uniquely determined by its character table. In this paper, the authors attempt to investigate how to characterize a finite almost simple group by using less information of its character table, and successfully characterize the simplectic group  $S_8(2)$  by its order and at most two irreducible character degrees of its character table. (pp. 93-100)

## **ON CHARACTERISTIC SUBGRAPH OF A GRAPH**

**Zahra Yarahmadi, Ali Reza Ashrafi**

A subgraph  $H$  of a graph  $G$  is called characteristic if  $\varphi(H) = H$ , for each automorphism  $\varphi \in Aut(G)$ . In this paper, the main properties of this new concept in algebraic graph theory are presented. (pp. 93-86)

## MINIMUM COMPLEXITY AND LOW-WEIGHT NORMAL POLYNOMIALS OVER FINITE FIELDS

**M. Alizadeh, F. Hormozi-nejad**

In this paper, by using some algorithms, the distribution of the complexity of normal polynomials over finite fields of characteristic three with degree extensions up to 16 is provided. Also, the current results on the smallest known complexity for the remaining degree extensions up to 300 by using a combination of theorems and known exact values are given. In what follows, by using some algorithms, a table of normal trinomials and pentanomials with minimum complexity among all normal trinomials and pentanomials, respectively over  $\mathbb{F}_3$ , with their complexities for each degree  $n$  with  $3^n \leq 10^{50}$  is presented. Also, either normal trinomials or pentanomials with minimum weight over  $\mathbb{F}_3$ , for each  $n$ ,  $106 \leq n \leq 300$  are listed. (pp. 106-122)

## CYCLIC HYPERGROUPS WHICH ARE INDUCED BY THE CHARACTER OF SOME FINITE GROUPS

**Sara Sekhavatizadeh, Mohamad Mehdi Zahedi, Ali Iranmanesh**

Let  $G$  be a finite group and  $\hat{G}$  be the set of all irreducible characters of  $G$ . In this paper, the hypergroups obtained from the character table  $\hat{G}$  are considered. Moreover, we show that  $\hat{S}_n$  for  $n \geq 3$  and  $\hat{A}_n$  for  $n \geq 4$  are single-power cyclic hypergroups and  $\hat{D}_{2n}$  is cyclic with finite period. (pp. 123-132)

## A NOTE ON NON-FRAGMENTABLE SUBSPACE OF $\ell_\infty^c(\Gamma)$

**F. Heydari, D. Behmardi**

In this paper we consider  $\ell_\infty^c(\Gamma)$  where  $\Gamma$  is uncountable and introduce subspaces  $\{A_{\mathcal{P}}\}_{\mathcal{P} \in \Sigma}$  of  $\ell_\infty^c(\Gamma)$  which are fragmented by a metric that generates the discrete topology but  $A = \bigcup_{\mathcal{P} \in \Sigma} A_{\mathcal{P}}$  is not countable unions of fragmentable subspaces.

(pp. 133-138)

## ON NEW INEQUALITIES OF HERMITE-HADAMARD TYPE FOR GENERALIZED CONVEX FUNCTIONS

**Shahid Qaisar, Chuanjiang He, Sabir Hussain**

In this article, we obtain some inequalities of Hermite-Hadamard type for functions whose third derivatives absolute values are  $\phi$ -convex,  $\log \phi$ -convex and quasi- $\phi$ -convex. (pp. 139-148)

## REGULAR SUB-SEQUENTIALLY DENSE INJECTIVE IN THE CATEGORY OF $S$ -POSETS

**M. Haddadi, Gh. Moghaddasi**

Sequentially dense monomorphisms were first introduced and studied by Giuli for projection algebras and followed by Ebrahimi, Mahmoudi, Moghaddasi and Shahbaz for  $S$ -acts. In this paper we use the notion of sub-Cauchy sequences and introduce the class of regular sub-sequentially dense monomorphisms for  $S$ -posets, denoted by  $\mathcal{M}_s$ . We investigate the properties of the class  $\mathcal{M}_s$  and study  $\mathcal{M}_s$ -injectivity of  $S$ -posets. Further, we find  $\mathcal{M}_s$ -injective hull of  $S$ -posets over some kind of semigroups. (pp. 149-160)

## STRONG CONVERGENCE THEOREMS FOR FIXED POINT PROBLEMS AND EQUILIBRIUM PROBLEMS WITH APPLICATIONS

**Huan-Chun Wu, Cao-Zong Cheng, Wen-Jing Han**

In this paper, we present a new iterative algorithm for finding a common element of the set of fixed points of a nonexpansive mapping and the set of solutions of an equilibrium problem, and we prove the strong convergence theorems in Hilbert spaces. We also apply our results to the convex minimization and variational inequality problems. Our results extend and improve some recent results of Cai, Tang and Liu [Cai, Y., Tang, Y. and Liu, L.: *Iterative algorithms for minimum-norm fixed point of nonexpansive mapping in Hilbert space*. Fixed Point Theory Appl., 2012:49 (2012)] and others. (pp. 161-174)

## NEW EXTENDED $(G'/G)$ -EXPANSION METHOD FOR TRAVELING WAVE SOLUTIONS OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS (NPDEs) IN MATHEMATICAL PHYSICS

**J. Harun-Or-Roshid, M. F. Hoque, M. Ali Akbar**

The new extended  $(G'/G)$ -expansion method is proposed to construct abundant exact traveling wave solutions involving free parameters to the nonlinear partial differential equations (NPDEs) in mathematical physics. We highlight the power of the new extended  $(G'/G)$ -expansion method in providing generalized solitary wave solutions of different physical structures applying it in the right-handed noncommutative burgers and the  $(1+1)$ -dimensional compound KdVB equations. By this application, we enhanced new traveling wave solutions of the equations which can be used to exploit some practical physical and mechanical phenomena. Moreover, when the parameters are replaced by special values, the well-known solitary wave solutions of the equation rediscovered from the traveling waves that may imply some physical meaningful results in fluid mechanics, gas dynamics, traffic flow, nonlinear dispersion and dissipation effects. (pp. 175-190)

## REVERSE MAGIC STRENGTH OF FESTOON TREES

**S. Sharief Basha, K. Madhusudhan Reddy**

In this paper, we prove that the reverse super edge-magic strength of some different festoon trees. (pp. 191-200)

## ON HYPER PSEUDO $MV$ -ALGEBRAS

**R.A. Borzooei, O. Zahiri**

In this paper, we introduce the notion of hyper pseudo  $MV$ -algebra as a generalization of pseudo  $MV$ -algebra and hyper  $MV$ -algebra. Then we investigate some properties of this structure and attempt to construct a hyper pseudo  $MV$ -algebra from a  $l$ -group. Finally, we proved some theorems about filters, ideals, congruence relations, in hyper pseudo  $MV$ -algebras. (pp. 201-224)

## MERGING STATES IN DETERMINISTIC FUZZY FINITE TREE AUTOMATA BASED ON FUZZY SIMILARITY MEASURES

**Somaye Moghari, Mohammad Mehdi Zahedi, Reza Ameri**

This paper presents a contribution to the problem of measuring fuzzy similarity of states and merging them in a Deterministic Fuzzy Finite Tree Automaton (DFFTA). The main question is: how to merge some states of a complete and reduced DFFTA such that the languages of original automaton and minimized one be similar but not necessarily equal? In order to solving this problem, we generalize the concepts of distance and similarity measures between fuzzy sets to distance and similarity measures between states of DFFTA. Then, we define the notions of normal DFFTA and introduce an efficient algorithm (polynomial order of time complexity) for discovering similar state sets of a DFFTA. (pp. 225-240)

## ON AN INERTIA FACTOR GROUP OF $2^8:O_8^+(2)$

**Jamshid Moori, Thekiso Seretlo**

The group  $\overline{G} = 2^6:A_8$  is an inertia factor group of  $2^8:O_8^+(2)$ . As an inertia factor group, our group  $\overline{G}$  plays an essential role in the construction of the character table of  $2^8:O_8^+(2)$ . In this paper we look at two ways of constructing  $\overline{G}$ . In the first method, we use combinatorics and the natural action of  $A_8$  on  $W \cong 2^6$ . In the second method, we use a computational method to construct  $\overline{G} = N:A_8$  inside  $O_8^+(2)$ . We show that  $A_8$  acts irreducibly on both  $W$  and  $N$  and we prove that the two groups are indeed isomorphic. (pp. 241-254)

## CONVERGENCE OF LAGRANGE-HERMITE INTERPOLATION

**Swarnima Bahadur, Manisha Shukla**

In this paper, we consider explicit representations and convergence of Lagrange-Hermite Interpolation on two disjoint set of nodes, which are obtained by projecting vertically the zeros of  $(1-x^2)P_n^{(\alpha,\beta)}(x)$  and  $(1-x^2)P_n^{(\alpha,\beta)'}(x)$  respectively on the unit circle, where  $P_n^{(\alpha,\beta)}(x)$  stands for Jacobi polynomials. (pp. 255-262)

## ASYMMETRIC CLOPEN SETS IN THE BITOPOLOGICAL SPACES Irakli Dochviri, Takashi Noiri

In the paper the behavior of clopen sets in bitopological spaces and some properties of generalized objects (e.g.,  $(i, j)$ -quasi components and  $(i, j)$ -clopen compact subsets) are investigated. By using asymmetric clopen sets we introduce new classes of  $(i, j)$ -clopen irresolute and  $(i, j)$ -weakly clopen-continuous maps. Also, some their relations to  $p$ -ultra-Hausdorff bitopological structures are established. Characterizations and a preserving theorem of pairwise connected spaces are obtained. (pp. 263-272)

## A REFINEMENT ON THE GROWTH FACTOR IN GAUSSIAN ELIMINATION FOR ACCRETIVE-DISSIPATIVE MATRICES Junjian Yang

In this note, we give a refinement of the growth factor in Gaussian elimination for accretive-dissipative matrix  $A$  which is due to Lin [Calcolo, 2013, DOI 10.1007/s10092-013-0089-1]. (pp. 273-278)

## ON A SPECIAL CLASS OF FINITE $p$ -GROUPS OF MAXIMAL CLASS Haibo Xue, Heng Lv, Guiyun Chen

In this paper, we study the finite  $p$ -group  $G$  of maximal class in which every nonabelian subgroup  $H$  satisfies  $C_G(H) = Z(H)$ . We prove here that a finite  $p$ -group  $G$  of maximal class is metabelian if every nonabelian subgroup  $H$  satisfies  $C_G(H) = Z(H)$ , furthermore, if  $p \neq 3$ ,  $|G| \geq p^{2p}$ , then there is an abelian subgroup of index  $p$  in  $G$ . (pp. 279-284)

## CONJUGACY CLASS SIZES OF SUBGROUPS AND THE STRUCTURE OF FINITE GROUPS Zhangjia Han, Huaguo Shi

The authors investigate the influences of conjugacy class sizes of subgroups of a finite groups  $G$  on the structure of  $G$ . Some sufficient conditions for a finite group to be  $p$ -nilpotent,  $p$ -solvable and supersolvable are obtained. (pp. 285-292)

## AVERAGE $D$ -DISTANCE BETWEEN VERTICES OF A GRAPH D. Reddy Babu, P.L.N. Varma

The  $D$ -distance between vertices of a graph  $G$  is obtained by considering the path lengths and as well as the degrees of vertices present on the path. The average  $D$ -distance between the vertices of a connected graph is the average of the  $D$ -distances between all pairs of vertices of the graph. In this article we study the average  $D$ -distance between the vertices of a graph. (pp. 293-298)

**A NOTE ON HERMITE-HADAMARD INEQUALITIES  
FOR PRODUCTS OF CONVEX FUNCTIONS  
VIA RIEMANN-LIOUVILLE FRACTIONAL INTEGRALS**

**Feixiang Chen**

In this paper, we obtain some new Hermite-Hadamard type inequalities for products of convex functions via Riemann-Liouville integrals. We conclude that Our methods considered here may be a stimulant for further investigations concerning Hermite-Hadamard type inequalities for products of various kinds of convex functions involving Riemann-Liouville fractional integrals. (pp. 299-306)

**THE HOMOGENEOUS BALANCE METHOD  
AND ITS APPLICATIONS FOR FINDING THE EXACT SOLUTIONS  
FOR NONLINEAR EVOLUTION EQUATIONS**

**Elsayed M.E. Zayed, Khaled A.E. Alurrfi**

In this article, we apply the homogeneous balance method to find the exact solutions of some nonlinear evolution equations in mathematical physics, namely, the Kaup-Kupershmidt equation, the Ito equation, the Caudrey-Dodd-Gibbon equation, the Lax equation and the Sawada-Kotera equation. These equations have wide applications in quantum mechanics and non linear optics. The efficiency of this method for constructing these exact solutions is demonstrated. (pp. 307-318)

**SOME STRUCTURAL PROPERTIES OF HYPER KS-SEMIGROUPS**  
**Bijan Davvaz, Ann Leslie O. Vicedo, Jocelyn P. Vilela**

This study introduces a new class of algebra related to hyper BCK-algebras and semihypergroups, called hyper KS-semigroups. It presents some characterizations of a hyper KS-semigroup with respect to its hyper subKS-semigroups, hyper KS-ideals, and reflexive hyper KS-ideals and their relationships. A quotient structure is constructed from a hyper KS-semigroup via a reflexive hyper KS-ideal and some properties are established. This paper also shows some properties of hyper KS-semigroups homomorphism and specifically, the three isomorphism theorems for hyper KS-semigroups are proved. Moreover, this paper shows that the hyper product of any nonempty finite family of hyper KS-semigroups is also a hyper KS-semigroup and investigates some related properties. (pp. 319-332)

**WEAK OPEN SETS ON SIMPLE EXTENSION IDEAL  
TOPOLOGICAL SPACE**

**W. Al-Omeri, M.S.Md. Noorani, A. Al-Omari, Ahmad AL-Omari**

In this paper we intend to introduce a new class of sets known as  $e-\mathcal{I}^+$ -open sets, defined in the light of simple extension topology and ideal topology. This set is investigated and found to be a weaker form of  $e-\mathcal{I}$ -open sets. We have also generalized this concept and studied its properties. (pp. 333-344)



## EXISTENCE AND UNIQUENESS THEOREM FOR A SOLUTION OF FUZZY IMPULSIVE DIFFERENTIAL EQUATIONS

**R. Ramesh, S. Vengataasalam**

In this paper, we prove the existence and uniqueness of a solution of the fuzzy impulsive differential equation  $x'(t) = f(t, x(t)), x(t_0) = x_0, \Delta x(t_k) = I_k(x(t_k))$  by using the method of successive approximation. We also consider the  $\epsilon$ -approximate solution for the above fuzzy impulsive differential equation. (pp. 345-358)

## ON $L$ -FUZZY TOPOLOGICAL TM-SUBSYSTEM

**M. Annalakshmi, M. Chandramouleeswaran**

Recently, in 2010, Tamilarasi and Megalai introduced a new class of algebras known as TM-algebras. In this paper, we discuss the notion of an  $L$ -fuzzy topological TM-subsystem. (pp. 359-368)

## MIDPOINT DERIVATIVE-BASED TRAPEZOID RULE FOR THE RIEMANN-STIELTJES INTEGRAL

**Weijing Zhao, Zhaoning Zhang, Zhijian Ye**

In this paper, the midpoint derivative-based trapezoid rule for the Riemann-Stieltjes integral is presented which uses derivative value at the midpoint. This kind of quadrature rule obtains an increase of two orders of precision over the trapezoid rule for the Riemann-Stieltjes integral and the error term is investigated. At last, the rationality of the generalization of midpoint derivative-based trapezoid rule for Riemann-Stieltjes integral is demonstrated. (pp. 369-376)

## NEW CHARACTERIZATIONS OF SOLUBILITY OF FINITE GROUPS

**Jinbao Li, Wujie Shi, Guiyun Chen, Dapeng Yu**

A subgroup  $H$  of a group  $G$  is said to be  $S$ -supplemented in  $G$  if there exists a subgroup  $T$  of  $G$  such that  $G = HT$  and  $H \cap T \leq H_{sG}$ , where  $H_{sG}$  denotes the subgroup of  $H$  generated by all those subgroups of  $H$  which are  $S$ -permutable in  $G$ . In this paper, two new characterizations of solubility of finite groups are presented in terms of  $S$ -supplemented subgroups of primes power orders, where primes belong to  $\{3, 5\}$ . In particular, a counterexample is given to show that the conjecture, proposed by Heliel at the end of [A.A. Heliel, *A note on  $c$ -supplemented subgroups of finite groups*, Comm. Algebra, 42 (2014), 1650-1656] and related to  $c$ -supplemented subgroups of primes power orders, is negative. (pp. 377-382)

**THE TRIPARTITE RAMSEY NUMBERS  $r_t(C_4; 2)$  AND  $r_t(C_4; 3)$**   
**S. Buada, D. Samana, V. Longani**

The  $k$ -colored tripartite Ramsey numbers  $r_t(G; k)$  is the smallest positive integer  $n$  such that any  $k$ -coloring of lines of a complete tripartite graph  $K_{n,n,n}$  there always exists a monochromatic subgraph isomorphic to  $G$ . When  $G$  is  $C_4$  it is known, but unpublished in a journal, that  $r_t(C_4; 2) = 3$ . In this paper we simplify the proof of  $r_t(C_4; 2) = 3$  and show the new result that  $r_t(C_4; 3) = 7$ . (pp. 383-400)

**$\Delta$ -CONVERGENCE THEOREM FOR TOTAL ASYMPTOTICALLY NONEXPANSIVE MAPPING IN UNIFORMLY CONVEX HYPERBOLIC SPACES**

**Zhanfei Zuo, Yi Huang, Xiaochun Chen, Feixiang Chen, Zhengwen Tu**

Recently, Chang, et al introduce the concept of total asymptotically nonexpansive mapping which contain the asymptotically nonexpansive mapping. The purpose of the paper is to analyze a three-step iterative scheme for total asymptotically nonexpansive mapping in uniformly convex hyperbolic spaces. Meanwhile, we obtain a  $\Delta$ -convergence theorem of the three-step iterative scheme for total asymptotically nonexpansive mapping in CAT(0) spaces. Ours results obtained in this paper extend and improve some previous known results. (pp. 401-410)

**QUOTIENT RINGS VIA FUZZY CONGRUENCE RELATIONS**

**Xiaowu Zhou, Dajing Xiang, Jianming Zhan**

This paper aims to introduce fuzzy congruence relations over rings and give constructions of quotient rings induced by fuzzy congruence relations. The Fuzzy First, Second and Third Isomorphism Theorems of rings are established. Finally, we investigate the relationships between fuzzy ideals and fuzzy congruence relations on rings. (pp. 411-424)

**PROPERTIES OF HYPERIDEALS IN ORDERED SEMIHYPERGROUPS**

**Thawhat Changphas, Bijan Davvaz**

An ordered semihypergroup is a semihypergroup  $(S, \circ)$  together with a partial order  $\leq$  on  $S$  such that the monotone condition holds, i.e., for all  $x, y, a \in S$ , if  $x \leq y$ , then for all  $u \in x \circ a$  there exists  $v \in y \circ a$  such that  $u \leq v$ , and similarly, for all  $u' \in a \circ x$  there exists  $v' \in a \circ y$  such that  $u' \leq v'$ . Indeed, the concept of ordered semihypergroups is a generalization of the concept of ordered semigroups. In this paper, we study some aspects of hyperideals of ordered semihypergroups. We give a necessary and sufficient condition of a subset of Cartesian product of two ordered semihypergroups to be a prime hyperideal. Also, we study right simple element ordered semihypergroups containing right simple elements. (pp. 425-432)

## PRIME SUBMODULES IN EXTENDED *BCK*-MODULE

**R.A. Borzooei, S. Saidi Goraghani**

In this paper, by considering the notion of *BCK*-module, we define the concept of extended *BCK*-module which is a generalization of *BCK*-module and we state and prove some related results. Specially, we define the notions of prime submodule and torsion free module and we investigate some important results. Finally, we define the concept of radical of any submodule in extended *BCK*-modules and we characterize the elements of it. (pp. 433-448)

## OD-CHARACTERIZATION OF ALTERNATING GROUP OF DEGREE $p + 3$

**Yanxiong Yan, Yu Zeng, Haijing Xu, Guiyun Chen**

Let  $A_{p+3}$  be the alternating group of degree  $p + 3$ , where  $p$  is a prime,  $p + 4$  is a composite number,  $p + 6$  is a prime and  $7 \neq p \in \pi(1000!)$ . In the present paper, we prove that  $A_{p+3}$  is *OD*-characterizable by using the classification theorem of finite simple groups and Magma soft of computational group theory. This new method is introduced in order to deal with the subtle changes of the prime graph of a group in the discussion of its *OD*-characterization, which might occur. As a consequence of this theorem not only generalizes the result in [1] (Hoseini, A.A. and Moghaddamfar, A.R., *Frontiers of Mathematics in China*, 5 (3), 2010) but also gives a positive answer to a conjecture in [2] (Shi, W.J., *Contemporary Math.*, 82, 1989). (pp. 449-460)

## PARTITIONED FRAMES IN DISCRETE BAK SNEPPEN MODELS

**Livio Clemente Piccinini, Maria Antonietta Lepellere,  
Ting Fa Margherita Chang, Luca Iseppi**

In this paper, we wish to present some simplified cases of discrete Bak-Sneppen models in which explicit computations via Markov chains are possible, hence reaching a better understanding of some rather hidden phenomena of the general case: in particular "avalanches" can be read in terms of mean waiting times and in terms of transitions between structures. The simple models allow us to introduce new frames that do not seem to have been considered in the previous literature, namely the case of partitioned Bak-Sneppen frames, that appear more realistic from the point of view of speed of evolution and do not present a unique criticality level, but a staircase tending towards a final equilibrium level, cadenced by an increasing sequence of footholds. The introduction summarizes Bak-Sneppen models, starting from the central model due to Bak and Sneppen, and recalls their use in applied sciences. The first section gives the general frame of models where locality and globality coexist, the second section shows the simplest case of a matching between locality and globality, that will become exemplar in the most complex frames of Bak-Sneppen processes. The main quantitative theorems are stated and proved in the third section and finally the fourth section presents

examples that illustrate the more sophisticated points of our paper and the use (and limits) of experimental results, while the fifth section considers real world situations where Bak Sneppen partitioned schemes can be tailored to represent the core of their evolution. (pp. 461-488)

**EXISTENCE OF THREE SOLUTIONS FOR NONLOCAL ELLIPTIC SYSTEM OF  $(p_1, \dots, p_n)$ -KIRCHHOFF TYPE**

**Ming-Lei Fang, Shan Yue, Chun Li, Hao-Xiang Wang**

In this paper, we establish the existence of at least three solutions to a Dirichlet boundary problem involving the  $(p_1, \dots, p_n)$ -Kirchhoff type systems. Our technical approach is mainly based on the general three critical points theorem obtained by Ricceri. (pp. 489-500)