

## THE NONLINEAR ERGODIC THEOREMS IN BANACH SPACE

Shahram Saeidi

In this paper, we study nonlinear ergodic properties for an amenable semigroup of nonexpansive mappings in a Banach space in our version. Among other things, we prove the existence of an ergodic retraction onto the common fixed point set of a specific amenable semigroup of nonexpansive mappings.

## ON THE GROWTH SEQUENCES OF FREE PRODUCT OF $PSL(m, q)$

Ahmad Erfanian

Let  $G^n$  be the direct product of  $n$  copies of a given group  $G$ . The aim of this article is to study the growth sequence  $\{d(G^n)\}_{n \geq 1}$ , where  $d(G^n)$  is the minimum number of generators of  $G^n$ , and  $G = PSL(m, q) * PSL(m', q')$  is the free product of the projective special linear simple groups  $PSL(m, q)$  and  $PSL(m', q')$  with  $m, m', q, q' \geq 2$ . It is shown that  $d((PSL(m, q) * PSL(m', q'))^k) = 4$ , for every positive integer  $k$  such that  $1 \leq k \leq h(2, PSL(m, q))h(2, PSL(m', q'))$ , where  $h(2, G)$  is the maximum number  $n$  such that  $d(G^n) = 2$ . Moreover, we prove that  $d(G^{h(n, PSL(m, q))h(2, PSL(m', q'))}) \leq n + 2$ , for all  $n \geq 2$  and some examples are given.

## $\mathcal{E}$ -LATTICES

Marius Tărnăuceanu

The goal of this paper is to present a generalization of the lattice concept.

## APPROXIMATION OF FIXED POINT FOR $\varphi$ -HEMICONTRACTION AND $\varphi$ -STRONGLY QUASI-ACCETIVE SET-VALUED OPERATOR

Qinghai He, Fuchun Yang

Let  $X$  be a real uniformly smooth Banach space and let  $T : D(T) \subset X \rightarrow CB(X)$  be a  $\varphi$ -hemicontractive and upper semicontinuous mapping. Under suitable assumptions on the parameters  $\{\alpha_n\}$  and  $\{\beta_n\}$ , using Reich's inequality, we proved that the Mann and Ishikawa iteration processes for  $T$  converge strongly to the isolate fixed point  $q$  of  $T$ . Our results extend and generalize the corresponding ones that are already known.

## FRACTIONAL INTEGRAL OPERATORS WITH BESSEL TYPE KERNELS

Ahmad Al-Salman, Ali A. Al-Jarrah

In this paper, we establish the  $L^p$  boundedness of a class of fractional integral operators with rough kernels involving Bessel functions of the first kind. Our result generalizes Calderón-Zygmund's result in [4].

## FIXED POINT AND COINCIDENCE POINT THEOREMS FOR A BROAD CLASS OF MULTIMAPS

S.V.R. Naidu

Multimaps from a metric space into the collection of all its nonempty closed subsets are considered and fixed point and coincidence point theorems for such maps have been obtained under new contractive conditions not involving the extended Hausdorff metric.

## FUZZY TRANSFER MINIMAL CLOSED MULTIFUNCTIONS

**M. Alimohammady, M. Roohi**

This paper deals with fuzzy transfer m-closed multifunctions and fuzzy transfer m-closed family of fuzzy sets. We would established some properties of these fuzzy multifunctions and some results for  $KKM$  maps but in fuzzy case.

## FRACTIONS ON A LATTICE

**M. Hosseinyazdi, A. Hasankhani, M. Mashinchi**

In this paper we consider  $R$  as a distributive lattice. Let  $S$  be a non-empty subset of  $R$  which is complete meet-semilattice. We consider a set  $S^{-1}R$  as the equivalence classes of a defined equivalence relation on  $R \times S$ . Then it is shown that  $S^{-1}R$  is a pseudo-Boolean lattice (or a Heyting algebra), if  $R$  is so. More properties of  $S^{-1}R$  are proved. Finally we characterize finite pseudo-Boolean lattice by  $S^{-1}R$  for some  $S$  and  $R$ .

## ON FUZZY CLOSED, INVERTIBLE AND REFLEXIVE SUBSETS OF HYPERGROUPS

**R. Ameri, H. Hedayati**

The aim of this paper is the study of important algebraic fuzzy subsets of transposition hypergroups (non-commutative join spaces). In this regards, we first introduce notions of fuzzy closed, fuzzy normal, fuzzy reflexive and fuzzy invertible subsets of transposition hypergroups and, then we investigate the basic properties of these notions. Fuzzy quotient space of a transposition hypergroup, namely  $\mu$  with respect to an equivalence relation  $\theta$  has been constituted and it is shown that under the certain conditions the properties of fuzzy (resp. closed, normal, reflexive, invertible) hypergroups in the quotient spaces are preserved. Finally briefly we discuss on fuzzy products of two fuzzy hypergroups and obtain some related basic results.

## CHARACTERIZATIONS OF REGULAR HYPERRINGS

**A. Asokkumar, M. Velrajan**

In this paper, we study regularity in the sense of Von Neumann in hyperrings and give two characterizations for a hyperring to be regular.

## ON FLEXIBLE GRADED MODULES

**Mashhoor Refai, Fida A.M. Moh'd**

Let  $G$  be a group with identity  $e$  and  $R$  be an associative  $G$ -graded ring with unity 1. In this paper we study the Flexible graded modules and give some of their properties.

## TWO CHARACTERIZATIONS OF IDEALS OF OPERATORS ON HILBERT SPACE AND APPLICATIONS

**A. Taghavi**

Let  $\mathcal{H}$  be a Hilbert space with  $\dim \mathcal{H} > 1$  and  $\mathcal{B}(\mathcal{H})$  denote the algebras of all bounded linear operators on  $\mathcal{H}$ . We show that if  $\mathcal{N} \subset \mathcal{B}(\mathcal{H})$  is a (linear) subspace and  $TA + AT \in \mathcal{N}$ , or  $T^*A + A^*T \in \mathcal{N}$ , for all  $A \in \mathcal{N}$  and  $T \in \mathcal{B}(\mathcal{H})$ , then  $\mathcal{N}$  is a two-sided ideal in  $\mathcal{B}(\mathcal{H})$ . Also if  $\rho$  is a seminorm satisfying  $\rho(T^2) \leq \rho(T)^2$  or  $\rho(T^*T) \leq \rho(T)^2$  for all  $T \in \mathcal{B}(\mathcal{H})$ , then  $\ker \rho$  is a two-sided ideal and  $\rho$  is a constant multiple of a submultiplicative seminorm. In fact, we show that second cases is true for any  $*$ -algebra  $\mathcal{A}$ .

## CONVERGENCE THEOREMS FOR THE ISHIKAWA ITERATIVE PROCESS ASSOCIATED WITH A PAIR OF STRONGLY PSEUDO-CONTRACTIVE OPERATORS

Ljubomir B. Ćirić, Jeong Sheok Ume

Osilike [17] have proved that two assumptions in the main theorem of Zhou [22] are contradictory. In this note we give a correction to the main result of Zhou [22]. We modify the definition of the Ishikawa iteration process with errors and prove strongly convergence theorems for such iteration processes, associated with a pair of strongly pseudo-contractive multi-valued operators. While the main theorem is formulated for a pair of point-to-set operators, its point-to-point analogue for a single operator is also a new result.

## ON CERTAIN NEW REPRESENTATIONS OF MOCK-THETA FUNCTIONS

Remy Y. Denis, S.N. Singh, S.P. Singh

In this paper, making use of a known series identity and certain summations of truncated series, an attempt has been made to establish new representations for mock-theta functions in term of basic hypergeometric series with several bases.

## NEAREST POINTS IN $2k$ -INNER PRODUCT SPACES

Mohammad Darvishzadeh, Mohammad Sal Moslehian

In this paper, we establish the existence of nearest points in  $2k$ -Hilbert spaces and prove that for each closed subspace  $M$  of a  $2k$ -Hilbert space  $X$ ,  $X = M \oplus M^\perp$ .

## MOTION OF TUBULAR SURFACES

H.N. Abd-Ellah

The present paper intends to investigate the kinematics of a particular type of linear Weingarten surfaces, namely tubular surfaces, in terms of their intrinsic geometric formulas. The evolution equations for the local frame, the first and the second fundamental quantities for the motion are established. The mean curvature flow is studied. Thus the evolution equations of the curvatures are obtained. Two examples of tubular surfaces and their motions are considered and plotted.

## MINIMAL $H_v$ -VECTOR SPACES

M. Alimohammady, M. Roohi

In this note by considering the notions of hyper-vector space and linear minimal space, as a generalization of vector space and topological vector space we introduce the notion of minimal hyper-vector space. Further, some related basic results which are compatible by the general case are given. In particular, it is shown that in a minimal hyper-vector space, the assignment  $x \mapsto t_0x + x_0$  is minimal lower semi-continuous.

## TOPOLOGICAL GENERALIZED RINGS

M.R. Molaei, H.M. Mohammadinejad, H. Shariati

In this paper topological Generalized rings as a generalization of topological rings are considered. Topological M-rings are studied. Two methods for constructing new topological M-rings are deduced.

## **TWO-STEP PREDICTOR-CORRECTOR METHODS FOR SOLVING FUZZY DIFFERENTIAL EQUATIONS**

**Mahmoud Mohseni Moghadam, Mohammad Taghi Khodadad**

In this paper a particular numerical algorithm of 2-step predictor-corrector methods for solving fuzzy ordinary differential equations is presented. We will prove that the algorithm converges to the exact solution as the step size goes to zero. Also the validity of the algorithm is illustrated by solving some examples.

## **FUZZY HARDLY OPEN FUNCTIONS**

**G. Palani Chetty, G. Balasubramanian**

In this paper the concepts of fuzzy hardly open functions and fuzzy nearly feebly open functions are introduced and some interesting properties of these functions are investigated besides giving some characterizations of these functions.

## **ROBUST STABILIZATION FOR UNCERTAIN LINEAR DESCRIPTOR SYSTEMS VIA PROPORTIONAL AND DERIVATIVE STATE FEEDBACK**

**Baowei Wu, Hui Tian**

In this paper, the problem of robust stability is studied for a class of descriptor systems with  $A$ -matrix having uncertainties, whose nominal systems are normalizable. The equivalent existence condition of proportional and derivative (P-D) state feedback is given by linear matrix inequality (LMI), which results in the optimization problem. Furthermore, for the system with uncertainties in matrices  $E, A$ , a necessary and sufficient condition for robust quadratic stabilization will be obtained by introducing a fictitious input and a useful lemma. Finally, on the basis of equivalent conditions, two algorithms for robust stabilizing controller are derived by solving the corresponding LMIs, and two numerical examples are presented to illustrate the efficiency of these methods.

## **INVERTIBLE FUZZY TOPOLOGICAL SPACES**

**V. Seenivasan, G. Balasubramanian**

In this paper we introduce a new class of fuzzy topological spaces called invertible fuzzy topological spaces. We discuss and study several interesting properties of this newly introduced fuzzy topological space besides giving some characterizations of these spaces.

## **COMMON FIXED POINTS THEOREMS FOR TWO HYBRID PAIRS OF NONSELF MAPPINGS**

**M. Imdad, Ladlay Khan**

Some results on coincidence and common fixed points for two hybrid pairs of compatible as well as pointwise  $R$ -weakly commuting mappings satisfying a generalized contraction type condition on a complete metrically convex metric spaces are proved which generalize relevant results due to Khan et al. [16], Assad [5], Assad and Kirk [4], Ahmad and Imdad [1], [2], Ahmad and Khan [3] and several others.

**ON THE CLASSES OF HEREDITARILY  $\ell_p$  BANACH SPACES  
AND ASYMPTOTICALLY ISOMETRIC COPIES OF  $c_0$  AND  $\ell_p$  IN THE SPACES**

**P. Azimi, A.A. Ledari**

We shall continue the study of the classes of hereditarily  $\ell_p$  Banach sequence spaces. These spaces were presented by Hagler and the first named author. The constructed spaces were denoted by  $X_{\alpha,p}$ . We shall consider three cases. In case (i) we show that any  $X_{\alpha,1}$  is isomorphic to  $\ell_1$  and in case (ii) any  $X_{\alpha,p}$  is isomorphic to  $c_0$ . In case (iii) We survey first the results of J. Hagler and the first named author on the structure of subspaces of  $X_{\alpha,1}$  and  $X_{\alpha,p}$  and then show that in this case the dual of any  $X_{\alpha,p}$  is nonseparable, and observe that  $c_0$  is asymptotically isometric to a quotient space of the predual of  $X_{\alpha,1}$ . We show that the closed unit ball of  $X_{\alpha,p}$  is weak\*-closed convex hull of its extreme points.

**ON NON STRICT CONVEXITY OF NORM IN BANACH SPACES**

**A.K. Mirmostafae**

In this paper, a non-strictly convexifiable space is exhibited. More precisely, we will give a direct proof of the fact that  $X/c_0$ , where  $X$  is a subspace of  $\ell^\infty$  which does not contain a copy of  $\ell^\infty$ , can not be renormed in strictly convex manner. This improves Bourgain method in [1].

**ORDER-PRESERVING TRANSFORMATION SEMIGROUPS WHOSE BI-IDEALS  
AND QUASI-IDEALS COINCIDE**

**Yupaporn Kemprasit**

Let  $\mathcal{BQ}$  denote the class of all semigroups whose bi-ideals and quasi-ideals coincide. It is known that  $\mathcal{BQ}$  contains all regular semigroups. However, a semigroup in  $\mathcal{BQ}$  need not be regular. For an interval  $X$  in  $\mathbb{R}$ , the order-preserving full transformation semigroup on  $X$ ,  $OT(X)$ , is known to be regular if and only if  $X$  is closed and bounded. Hence if  $X$  is a closed and bounded interval in  $\mathbb{R}$ , then  $OT(X) \in \mathcal{BQ}$ . The purpose of this paper is to show that for a nontrivial interval  $X$  in a subfield  $F$  of  $\mathbb{R}$ ,  $OT(X) \in \mathcal{BQ}$  if and only if  $F = \mathbb{R}$  and  $X$  is closed and bounded. Consequently, this condition is also necessary and sufficient for the regularity of  $OT(X)$ .