

## THE PERIODIC PROBLEM FOR A SPHERICAL WAVE EQUATION

Abdou Kouider Ben–Naoum

This note is devoted to the existence of solutions of the periodic problem

$$u_{tt} - \Delta_n u = g(t, x, u), \quad t \in S^1, \quad x \in S^n,$$

where  $S^n$  is the  $n$ -dimensional sphere,  $\Delta_n$  denotes the Laplace–Beltrami operator on  $S^n$  and  $g(t, x, u)$  satisfies the Carathéodory conditions for  $L^2(S^1 \times S^n, \mathbb{R})$ . The positive part of the spectrum of the spherical wave operator is made of eigenvalues of finite multiplicity. Consequently, those problems fall into the scope of the results on the dual least action principle. We shall use this technique to obtain some existence results.

## INTEGRAL REPRESENTATIONS OF NEW FAMILIES OF POLYNOMIALS

G. Dattoli, S. Lorenzutta, D. Sacchetti

Si utilizza una rappresentazione integrale, introdotta di recente, per esprimere famiglie di polinomi non convenzionali in termini di funzioni elementari. Si considerano polinomi di Bessel, Miller–Lee e Sylvester e si dimostra che essi possono essere considerati casi particolari di forme più generali qui discusse in dettaglio. Lo stesso metodo viene utilizzato per generalizzare polinomi di Tchebycheff, Legendre e Laguerre e si discute infine il loro legame con i polinomi di Bateman.

We use a recently introduced integral representation to express families of non standard polynomials in terms of elementary functions. We consider Bessel, Miller–Lee and Sylvester polynomials and show that they can be viewed as particular cases of more general forms, which are discussed with some detail. We extend the same method to generalized forms of Tchebycheff, Legendre and Laguerre polynomials and discuss their link with those of the Bateman type.

## CUBICHE RIGATE DI $PG(5, q)$ E FIBRAZIONI DEI PIANI DI ANDRÉ GENERALIZZATI FINITI DI ORDINE $q^3$

Alessandro Basile, Paolo Brutti

Questa nota consiste di due parti. Nella prima si studiano certe ipersuperficie algebriche di ordine tre dello spazio proiettivo  $PG(5, q)$ . Esse vengono chiamate "cubiche rigate" in quanto possiedono tre schiere di piani. nella seconda parte si mostra che ogni fibrazione, associata ad un piano di André generalizzato finito di ordine  $q^3$ , si ottiene da una fibrazione regolare di  $PG(5, q)$  scambiando, in un insieme di cubiche rigate disgiunte, la prima schiera di ciascuna con una delle altre due.

## ON DIRECTED GRAPHS OF HYPERVECTOR SPACES

Mashhour I. Mohammed

The aim of this paper is to introduce the notion of directed graphs of hypervector spaces and to extend the results of Ribenboim [2], on vector spaces to hypervector spaces.

## A COMMENT ON THE BEREZIN TRANSFORM OF CERTAIN OPERATORS

S.M. Vaezpour

Let  $A$  be a bounded operator on a Hilbert space of analytic functions on  $\mathbf{D}$ . The Berezin transform of  $A$  is defined by  $\tilde{A}(z) = \langle Ak_z, k_z \rangle$  for every  $z$  in  $\mathbf{D}$  where  $k_z$  is the normalized reproducing kernel. In this paper we characterize those operators for which their Berezin transforms are multiplicative or commutative.

**ON NONCYCLIC VECTORS FOR CERTAIN UNILATERAL WEIGHTED SHIFTS**  
**K. Hedayatian**

In this note it is shown that the singular inner function with unit point mass at 1 is not a cyclic vector for certain unilateral weighted shifts.

**ON THE TOPOLOGICAL PROPERTIES OF SOME COHOMOGENEITY ONE MANIFOLDS OF NON POSITIVE CURVATURE**  
**S.M.B. Kashani**

In this paper we study some non positively curved (especially flat) Riemannian manifolds acted on by a closed Lie group of isometries with principal orbits of codimension one. Among other results, it is proved that the orbit space of flat cohomogeneity one manifolds can not be  $[0, 1]$  and the singular orbit (if it exists) is a totally geodesic submanifold.

**SOME INEQUALITIES FOR THE CSISZÁR  $\phi$ -DIVERGENCE WHEN  $\phi$  IS AN  $L$ -LIPSCHITZIAN FUNCTION AND APPLICATIONS**  
**S.S. Dragomir**

Some inequalities of Jessen's type for vector valued Lipschitzian functions and applications for the discrete Csiszár  $\phi$ -divergence are given.

**NONLINEAR PROGRAMMING APPROACH USING GOAL ATTAINMENT METHOD**  
**Reza Farshadnia**

A multi-criteria optimization approach, based on the goal attainment method is proposed for the design of compensators for single-input, single-output control systems. An illustrative example demonstrates that this method offers the designers an oerdere technique for the handling of a variety of design objectives or specifications.

**IDENTITIES AND MULTIALGEBRAS**  
**Cosmin Pelea**

This paper deals with multialgebras. An important instrument in this paper is the fundamental relation of a multialgebra, which can bring us into the class of the universal algebras. In the first part of the article we will see that the fundamental structure of a multialgebra verifies the identities of the given multialgebra. When trying to obtain multistructures that verify (even in a weak manner) the identities of their fundamental structure we get a new class of multialgebras. In the particular case of the semihypergroups these multialgebras are the complete semihypergroups.

**TRANSPOSITION HYPERGROUPS FORMED BY TRANSFORMATION OPERATORS ON RINGS OF DIFFERENTIABLE FUNCTIONS**  
**Jan Chvalina, Ludmila Chvalinová**

Using the standard functorial assignment of semihypergroups to quasi-ordered semigroups we are presenting constructions of non-commutative transposition hypergroups, i.e. non-commutative join spaces on quasi-ordered groups and monoids of certain transformation operators acting on rings of continuously differentiable functions. In the case of functions of the class  $C^1$  there is established relationship between compactness of the definition interval  $J$  of considered functions and reproducibility of corresponding hyperoperations for transformation operators on  $C^1(J)$ .

**PIANI BOOLEANI, I**  
**Sandro Rajola, Maria Scafati Tallini**

We define boolean plane the geometric structure where the points are the ordered pairs of subsets of a non-empty set  $S$  and the lines are the ordered pairs  $(x, y)$  such that  $ax + by + c = 0$ , where  $a, b, c, x, y$  are subsets of  $S$ , the sum is the symmetric difference and the product is the intersection. In this context we define and study the lines, the triangles and the collineations. Moreover we define a metric and the circles studying their properties. The results are quite unexpected compared with the classical euclidean case.

**THE STRUCTURE OF THE  $(P, Q)$ -SUPERLATTICE AND ORDER RELATED PROPERTIES**

This paper is dedicated to the memory of K. Serafimidis  
**K. Serafimidis, Ath. Kehagias, M. Konstantinidou**

In a previous work we have introduced the  $(P, Q)$ -superlattice, a hyperstructure of the form  $(L, \overset{P}{\vee}, \overset{Q}{\wedge})$ . Here  $(L, \vee, \wedge)$  is a lattice and the hyperoperations  $\overset{P}{\vee}, \overset{Q}{\wedge}$  are defined by  $a \overset{P}{\vee} b \doteq a \vee b \vee P$ ,  $a \overset{Q}{\wedge} b \doteq a \wedge b \wedge Q$ ; when the sets  $P, Q \subseteq L$  satisfy appropriate conditions  $(L, \overset{P}{\vee}, \overset{Q}{\wedge})$  is a superlattice. In this work we continue the investigation of  $(P, Q)$ -superlattice and consider the structure of the sets  $a \overset{P}{\vee} b, a \overset{Q}{\wedge} b$  as well as some “order-like” relationships between such sets.

**SOME REPRESENTATION RESULTS FOR  $(P, Q)$ -SUPERLATTICES**

This paper is dedicated to the memory of K. Serafimidis  
**K. Serafimidis, Ath. Kehagias**

On the the lattice  $(L, \vee, \wedge)$  we construct the *hyperoperations*  $\overset{P}{\vee}, \overset{Q}{\wedge}$  as follows:  $a \overset{P}{\vee} b = a \vee b \vee P$ ,  $a \overset{Q}{\wedge} b = a \wedge b \wedge Q$ . If the sets  $P, Q \subseteq L$  satisfy appropriate conditions, then  $(L, \overset{P}{\vee}, \overset{Q}{\wedge})$  is a *superlattice*. In this paper we give two *representation* results for  $(P, Q)$ -superlattices. The first result is an analog of the [1] representation result. The second result gives necessary and sufficient conditions for a general superlattice to be expressed as a  $(P, Q)$ -superlattice; this condition is expressed in terms of associativity of  $\overset{P}{\vee}$  with  $\vee$  and of  $\overset{Q}{\wedge}$  with  $\wedge$ .

**ON THE PRESENTATION RANK OF FINITE SIMPLE GROUPS**

**Ahmad Erfanian**

The paper is devoted to study a problem on the presentation rank of finite non-abelian simple groups. The problem states that for every non-abelian simple groups  $A$  and  $B$  with  $|A| \leq |B|$ , whether  $pr(A^n) \geq pr(B^n)$  for all  $n \geq 1$ , where  $pr(A^n)$  is the presentation rank of  $A^n$  and  $A^n$  is the direct product of  $n$  copies of a group  $A$ . The validity of the problem is shown for some types of finite simple groups when  $n$  is large enough. Moreover, some counterexamples are given for small values of  $n$ , in final section.

## **DISTRIBUTIVITY AND IM-LATTICES**

**Grigore Călugăreanu, Reyadh R. Khazal**

Notice that in the diamond (i.e., the 5-element nondistributive lattice) the intersection of the (three) maximal elements is not irredundant, and a lattice is not distributive if it contains a diamond. Hence, connections between the distributivity of a lattice and the irredundancy of the intersection of its family of maximal elements seem plausible.

In this paper, the authors prove that, under natural hypothesis, distributivity is equivalent with certain conditions on maximal elements. Applications to the distributivity of the lattice of all ideals of a semiprimitive ring with identity are given.

## **A DECOMPOSITION OF STRONG MAPS**

**Talal A. Al-Hawary**

The aim of this paper is to introduce the notions of OFF-sets and OFF-strong maps. Together with the notion of prestrong, we obtain a decomposition of strong maps: a map  $f : M_1 \rightarrow M_2$ , where  $M_1$  is a loopless closure matroid, is strong if and only if it is both prestrong and OFF-strong. This decomposition is non-trivial.

## **COMMON FIXED POINTS AS BEST APPROXIMANTS**

**M. Imdad, Q.H. Khan**

Motivated by a common fixed point theorem of Jeong and Rhoades we derive a common fixed point theorem in complete metric spaces which is used to prove a common fixed point theorem via best approximation. In process results due to Brosowski, Singh, Hicks-Humphries, Shahzad are generalized and improved either partially or completely.

## **A VEBLÉN TYPE AXIOM FOR GEOMETRIC SPACES AND PARTICULAR HYPERGROUPS**

**Domenico Lenzi**

In this paper we introduce a generalization of projective spaces: the pseudo-projective spaces. These are geometric spaces fulfilling an analogous of the celebrated Veblen's axiom of linear spaces. Considering the algebraic counterpart of these concepts, it turns out that a geometric space is pseudo-projective if and only if it is a hypergroup with respect to a suitable hyperoperation. This is a generalization of a result of G. Tallini. Some other characterizations and some properties of pseudo-projective spaces are also given. The obtained results reduce to theorems for projective spaces, when the geometric spaces are linear.

## **ON HYPERRING OF POLYNOMIALS**

**B. Davvaz, A. Koushky**

The aim of this paper is to construct a hyperring of polynomials over a hyperring. Then we state and prove some properties of hyperring of polynomials.

## **CATEGORY OF $OCU_2$ -POLYGROUPS**

**M.M. Zahedi, S.Sh. Mousavi, S.N. Hosseini**

In this paper first we give the notion of  $OCU_2$ -Polygroup. Then we construct the category  $OCU_2$ -PGrp of  $OCU_2$ -Polygroups. Also we derive a group from a given  $OCU_2$ -Polygroup, and show that an  $OCU_2$ -PGrp has free objects over the category of groups. Moreover the induced adjoint is a geometric morphism. Finally we discuss about projective objects, injective objects, embeddings, and quotient morphisms in  $OCU_2$ -PGrp.