

The following papers were presented at the **International Algebra Conference, Iași, October 14–17, 1998**, hosted by the Faculty of Mathematics of the "Al.I. Cuza" University of Iași, Romania.

The papers cover topics such as commutative algebra, fuzzy algebra, hyperstructures, noncommutative algebra.

The organizers wish to thank all the participants for their enthusiastic support throughout the Conference.

Special thanks are addressed to the Editors of the **Italian Journal of Pure and Applied Mathematics** for making possible the publication of these papers in excellent conditions.

The Organizing Committee

Piergiulio Corsini

Ion D. Ion

Ioan Tofan

### **BAICA'S EUCLIDEAN SOLUTION OF FERMAT'S LAST THEOREM (FLT)**

**Malvina Baica**

**Abstract.** The main interest of this paper is to put together all the information of the author's work on (FLT) that was given in her seven published papers. Also, we will answer some additional questions received after the publication of her paper titled *More Explanation about Baica's Proof of Fermat's Last Theorem* [4].

### **BINARY RELATIONS AND HYPERGROUPOIDS**

**Piergiulio Corsini**

**Abstract.** One associates a partial hyperoperation  $\langle \circ_R \rangle$  to every binary relation  $R$  and one determines conditions so that  $\langle \circ_R \rangle$  is defined everywhere.

### **EMBEDDING OF NONASSOCIATIVE ALGEBRAS**

**Ilie Burdujan**

It is well known that the lack of the associativity is as a serious shortcoming in the study of a nonassociative algebra. That is why one tries to embed every nonassociative algebra into an associative one (see Def.1) so that both operations of these algebras be intimately connected with each other. That is mainly done in order to use the well-developed structure theory of associative algebras in the structure theory for nonassociative algebras. Of course, the deviations from the associativity will be a natural presence in this context.

The aim of this Note is to prove the existence of such kind of embeddings for any nonassociative algebra having a right identity element. On the other hand, one tries to determine all embeddings of a given nonassociative algebra having right identity element.

## **SUR LES ANNEAUX DEDEKIND-FINIS. APPLICATIONS A L'ALGEBRE LINEAIRE**

**Ion D. Ion**

**Meri Constantinescu**

**Résumé.** On démontre que l'anneau  $R[[X]]$  des séries formelles, et en particulier l'anneau des polynômes  $R[X]$ , à coefficients dans un anneau Dedekind-fini  $R$  est un anneau Dedekind-fini. Le même résultat est vrai pour un anneau des matrices triangulaires  $T_n(R)$  à coefficients dans un anneau Dedekind-fini. On obtient aussi une généralisation du théorème fondamental des matrices semblables.

## **LA SOUS-MESURABILITE ET L'INTEGRABILITE DES FONCTIONS PROBABILISTIQUES**

**Octavian Lipovan**

**Sommaire.** En considérant le point de vue statistique proposé par K. Menger [18], dans ce travail on étudie la sous-mesurabilité probabilistique et la  $\Gamma_\mu$ -intégrabilité des applications à valeurs dans l'espace  $\mathcal{R}^+$  de toutes les fonctions de répartitions. Cette démarche est topologique et s'inspire de Bartle [1], ainsi que de Dunford, Schwartz [2] et J.C. Masse [17].

## **BLOCKS OF NORMAL SUBGROUPS AND MORITA EQUIVALENCES**

**Andrei Marcus**

### **1. Introduction**

Let  $N$  be a normal subgroup of a finite group  $H$ ,  $G = H/N$ , and let  $R = \mathcal{O}^\alpha H$  be a twisted group algebra, where  $\mathcal{O}$  is a complete discrete valuation ring with algebraically closed residue field  $k = \mathcal{O}/J(\mathcal{O})$  of characteristic  $p > 0$ , quotient field  $\mathcal{K}$  of characteristic 0 which is also a splitting field for all the  $\mathcal{K}$ -algebras considered below, and  $\alpha \in Z^2(H, \mathcal{O}^*)$  is a cocycle.

E. Dade stated in [3] the so called Inductive Conjecture, which involves Clifford extensions associated to blocks of  $\mathcal{O}^\alpha N$  and to simple  $\mathcal{K}^\alpha N$ -modules.

In Section 2 we show that these Clifford extensions are invariant under Morita equivalences induced by graded bimodules, bringing in this way a structural point of view to Dade's combinatorial conjectures. While this will be done in the more general case of symmetric crossed products, in Section 3 we investigate the local structure of graded Morita equivalences between indecomposable graded subalgebras of  $\mathcal{O}H$ . We use Linckelmann's approach to generalize the results of [7, Section 3], obtained in the case of principal blocks.

## ON A NON-STANDARD ALGEBRAIC HYPERSTRUCTURE AND ITS APPLICATION TO THE COHERENT PROBABILITY ASSESSMENTS

**Antonio Maturo**

**Abstract.** In this paper a particular non-standard algebraic hyperstructure is considered. We prove that, if we use such hyperstructure, the problems on the coherent assessments of probability and their solutions can be put in a form very useful and simply. Moreover, in this new algebraic contest, we can introduce new algorithms of control of coherence.

## ABOUT THE SIMPLIFIABLE CYCLIC SEMIHYPERGROUPS

**Violeta Leoreanu**

**Abstract.** Some results (necessary, sufficient conditions) about simplifiable cyclic semihypergroups and about simplifiable semihypergroups are presented in this paper. Finally, it is given a new proof for the characterization of semihypergroups for which the relation  $\beta$  is transitive.

## CR-STRUCTURES ON HYPERSURFACES OF THE TANGENT BUNDLE

**Marian-Ioan Munteanu**

### 1. Introduction

Let  $(M, g)$  be a Riemannian manifold and  $(TM, \bar{g})$  its tangent bundle endowed with the Sasaki metric  $\bar{g}$ . Using the Levi Civita connection of  $g$  it can be defined on  $TM$  the usual almost complex structure  $J$ ; then  $(TM, J, \bar{g})$  is almost Kaehlerian. The restriction of  $J$  to a real hypersurface of  $TM$  induces an almost  $CR$ -structure (generally, the complex involutivity condition from the definition of  $CR$ -structure is not fulfilled).

There are known some results concerning to the possibility to have a  $CR$ -structure on the tangent sphere bundle ([1], [5], [6], [9]).

In this paper there are found the conditions under which we have induced  $CR$ -structures on the hyperplane tangent bundle and the translated unit tangent bundle thought of as hypersurfaces of  $TM$ .

## BÄCKLUND TRANSFORMATIONS OF CONSTANT TORSION CURVES IN 3-DIMENSIONAL CONSTANT CURVATURE SPACES

S.Z. Németh

**Abstract.** K. Tenenblat extended the Bäcklund transformation of two surfaces in  $\mathbf{R}^3$  (see [3] or [7]) to space forms (see [6]). This transformation can be restricted to corresponding asymptotical lines (see [8]). Surfaces in 3-dimensional constant curvature spaces, in pseudospherical correspondence, have constant Gaussian curvature, and consequently by Enneper their asymptotical lines have constant torsion. This leads to the idea of defining the Bäcklund transformation for such curves. The euclidean case was considered by A. Calini and T. Ivey in [2]. We shall generalize their theorem and the converse of it, proved in my previous paper [5], for constant torsion curves in 3-dimensional constant curvature spaces.

## ABOUT A CATEGORY OF CANONICAL HYPERGROUPS

Cosmin Pelea

**Abstract.** Considering the class of the quasicanonical hypergroups and the class of the very good homomorphisms among quasicanonical hypergroups together with the product of the mappings, knowing that the product of two very good homomorphisms is a very good homomorphism and for any quasicanonical hypergroup  $H$ ,  $1_H : H \rightarrow H$ ,  $1_H(h) = h$  is a very good homomorphism among quasicanonical hypergroups, we can construct a category, denoted by QCH. Similarly, we can construct a category of canonical hypergroups (in fact a full subcategory of QCH), denoted by CH. We will see about CH that it is an exact category. Moreover, in the final part of the paper we will prove that the cartesian product of a family of quasicanonical hypergroups (defined as in [1]) is the product of this family in QCH. Two of the most used results of the paper are those which state that the monomorphisms of CH are the 1 – 1 very good homomorphisms and the epimorphisms of CH are the onto very good homomorphisms; if the result which talks about monomorphisms can be easily extended to the quasicanonical hypergroups the extension of the result which refers to epimorphisms for quasicanonical hypergroups is still a problem to be studied.

## COMBINATORICS IN ALGEBRA AND GEOMETRY

Dorin Popescu

**A little Geometry.** In the fourth century AD, Pappus from Alexandria wrote in a book the following well known theorem

**Theorem 1.** (Pappus) *Let  $L, M$  be two lines in the plane,  $p_1, p_2, p_3$  distinct points of  $L$  and  $q_1, q_2, q_3$  distinct points of  $M$  all distinct from the point  $L \cap M$ . Then the intersection points  $r_{ij}$  ( $i \neq j$ ) of the lines  $p_i q_j$  resp.  $p_j q_i$  are colinear.*

## POINT TRANSITIVE AFFINE PLANES

Sandro Rajola

Maria Scafati Tallini

**Abstract.** In this paper we study the affine planes  $\alpha_q$ ,  $q$  even and the projective planes  $\pi_q$  obtained adding to  $\alpha_q$  the line at infinity. We prove that if  $q$  is even,  $q \neq 8m$ ,  $m \in \mathbb{Z}$ ,  $m \geq 2$  and the group  $\mathcal{G}$  of the affinities of  $\alpha_q$  is transitive on the points, then  $\pi_q$  contains some Fano plane, although  $q$  is not a power of 2 (save the case  $q = 8$ ). In 1955 H. Neumann [2] conjectured that every finite projective plane  $\pi_q$  of order  $q$  contains some Fano plane, save for the desarguesian planes of order  $q$  odd. Here we prove H. Neumann conjecture when  $q$  is even,  $q \neq 8m$ ,  $m \in \mathbb{Z}$ ,  $m \geq 2$  and the group  $\mathcal{G}$  is transitive on the points. Further, we consider affine planes  $\alpha_q$  such that  $\pi_q$  is without Fano planes and define in  $\alpha_q$  the parallelograms with centre at a point  $x$ . We prove that if  $\pi_q$  is without Fano planes and the following holds:  $S'$  is the complement in  $\alpha_q$  of the set  $S$  of the centres of the parallelograms and  $S'$  is not empty,  $X$  is the set of the pairs  $\{x, x'\}$ ,  $x \in S$ ,  $x' \in S'$ , and  $\mathcal{G}$  is transitive on such pairs, then  $S'$  is a set of type  $(0, n)$  either in  $\alpha_q$  or in  $\pi_q$ .

## EXTENSIONS OF GROUPS WITH NON-ABELIAN KERNELS

M. Ștefănescu

V.M. Gontineac

**Abstract.** The subject of this paper is the study of group-extensions in connection with the cohomological constructions that we have done in some previous papers

(see [2], [3], [4]).

## ON SOME CONNECTIONS BETWEEN HYPERSTRUCTURES AND FUZZY SETS

**I. Tofan**

**A.C. Volf**

**Abstract.** This paper studies the hyperoperation associated to an  $L$ -fuzzy set, where  $L$  is a lattice possessing 1. If this lattice is distributive, one obtains in this way a commutative hypergroup. Conversely, if  $(H, *)$  is a hyperstructure possessing certain properties, then it exists a lattice  $L$  and a structure of  $L$ -fuzzy set on  $H$  such that  $*$  is the associated hyperoperation.

## **SUBMODULES AND QUOTIENT MODULES OF THE MODULES WITH THE DIRECT SUMMAND INTERSECTION PROPERTY**

**Dumitru Vălcan**

**Abstract.** It is known that if an  $R$ -module  $A$  has the direct summand intersection property (for short D.S.I.P.), then any direct summand  $B$ , of  $A$ , and  $A/B$  have the same property. In this work we will study the necessary and/or the sufficient conditions for which some submodules of  $R$ -module  $A$  (with D.S.I.P.), which are not necessary direct summands and the corresponding quotient modules, to have D.S.I.P. Thus, being given an  $R$ -module  $A$ , with D.S.I.P. the following submodules of  $A$  are investigated here:  $mA$ ,  $A[m]$ ,  $m^{-1}A$  (in this case,  $A$  is a submodule of an  $R$ -module  $G$ , and  $m$  is any element of ring  $R$ -associative, commutative, with unity),  $F(A)$  – the Frattini submodule and  $B_A$  – the  $p$ -basic submodule, for any  $p$ -prime element of  $R$ , as well as quotient  $R$ -modules corresponding to. All through this paper we will mark with  $R$  an associative, commutative ring with unity, the considered modules will be over  $R$  and we will mark with  $P$  the set of all unassociated prime elements from  $R$ . Other conditions about the ring  $R$  or  $R$ -modules  $A$  will be imposed whenever the case.

## **TWO PROPERTIES OF BLOCH FUNCTIONS**

**Žarko Pavićević**

**Abstract.** In terms of strong analytic normality, for a family of holomorphic functions in the unit disc, we give a characteristic property of a Bloch function and a sufficient condition for a Bloch function to be bounded on boundary angles.